USPAS 2019: Knoxville

Nonlinear Lattice Characterization: Dynamic Aperture, Momentum Aperture, Lifetime, Frequency Map Analysis, Resonance Driving Terms

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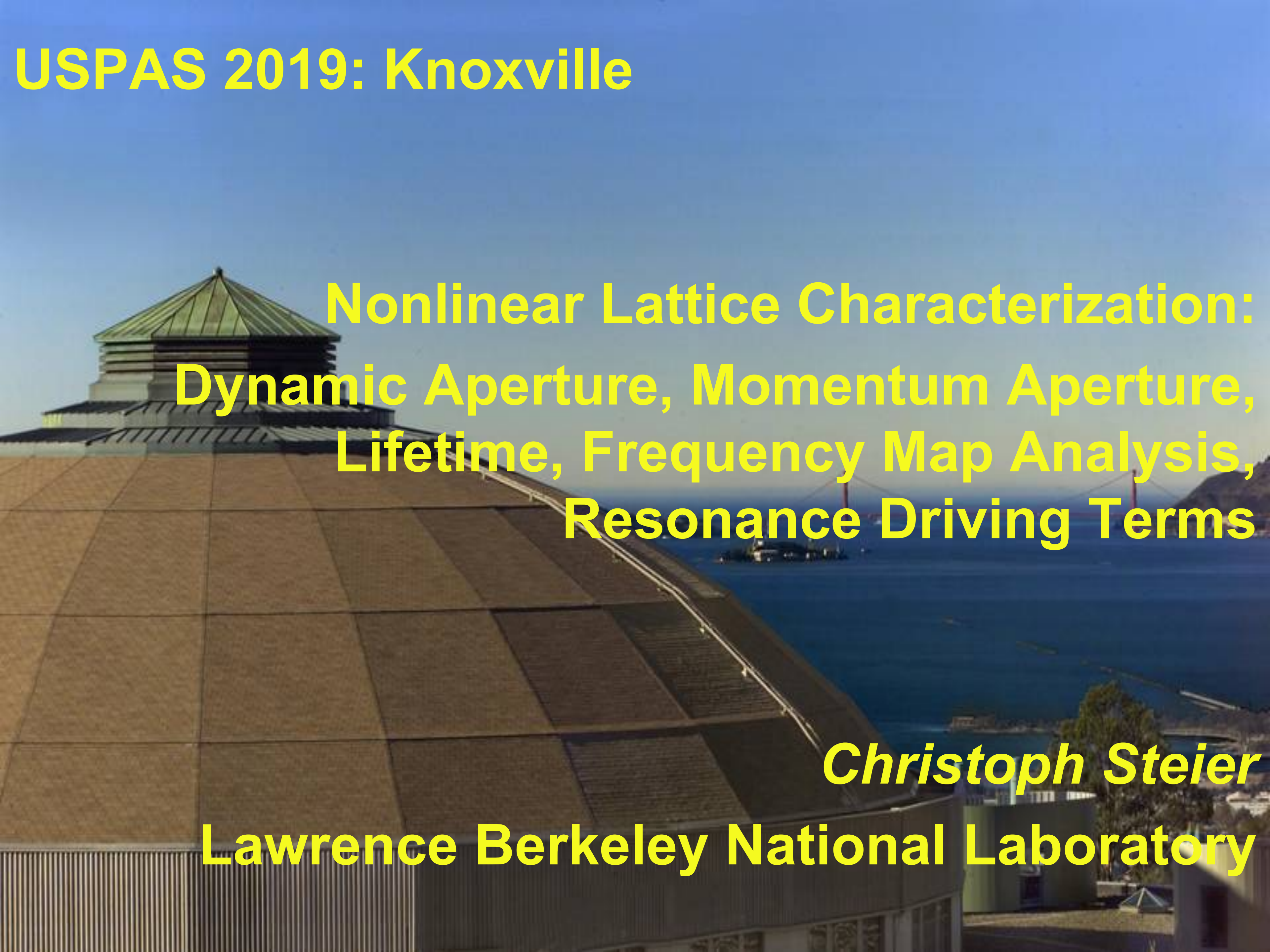


Outline

(Transverse) single particle dynamics often determines injection efficiency, lifetime, …

– Motivation

– Nonlinear Dynamics – Dynamic Aperture

- Tunescans, Frequency Maps

– Lifetime limiting processes

- Momentum aperture: Touschek Lifetime

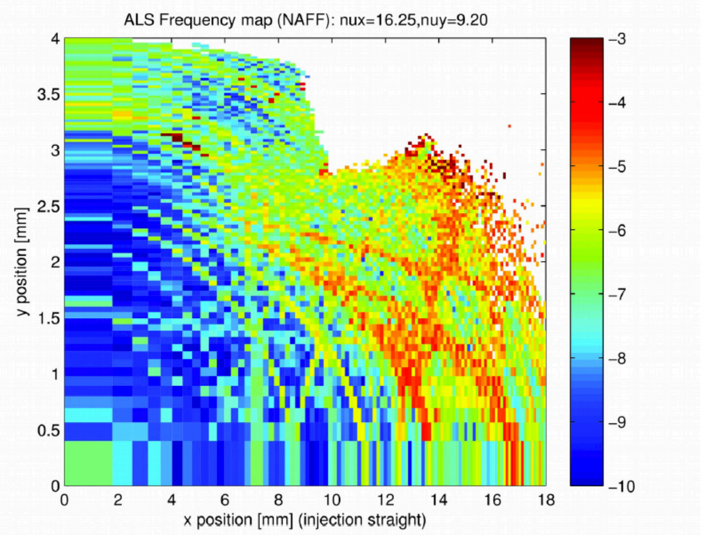
– Nonlinear Lattice Correction

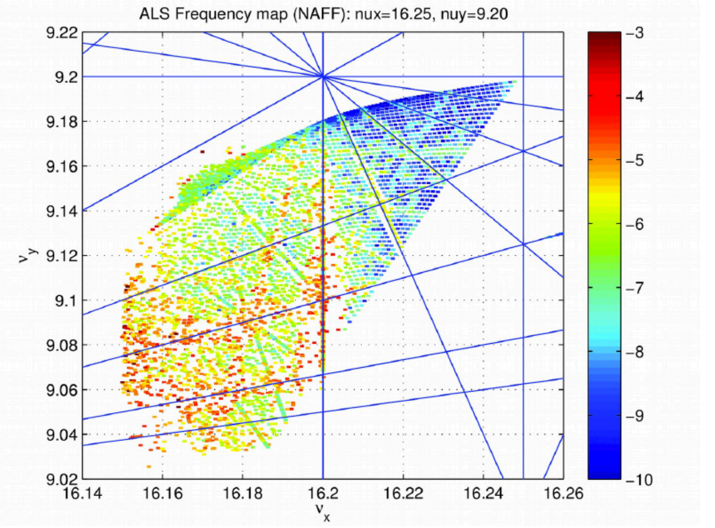
- Resonance Driving Terms

– Summary

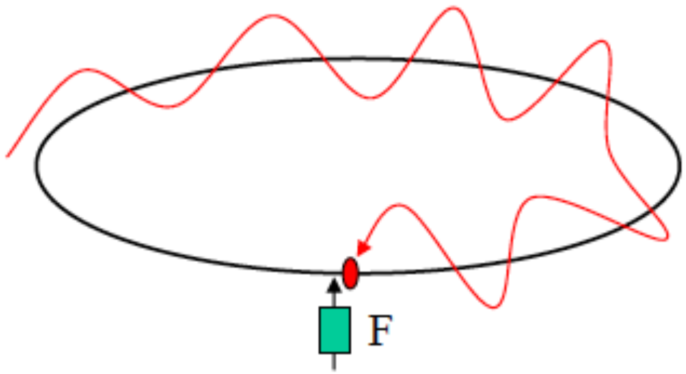
Motivation

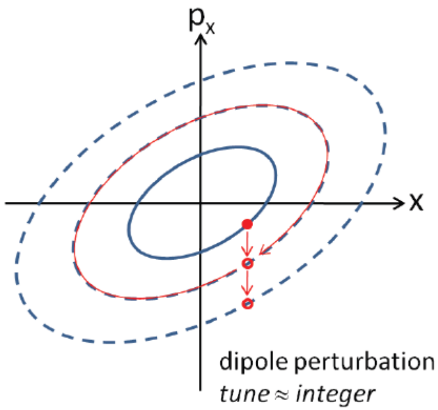
- Particles are lost in accelerators because of finite apertures, potentially limiting
  - Injection efficiency, or
  - Beam lifetime
- Limiting apertures can be *physical* or *dynamic*:
  - Vacuum chamber → physical aperture
  - Nonlinear single particle dynamics → dynamic (energy) aperture
- Loss process typically involves two steps:
  - Scattering process (or injection) launching particles to large amplitudes outside core of beam
  - Resonant or diffusive processes (nonlinear dynamics) leading to growth of oscillation amplitudes

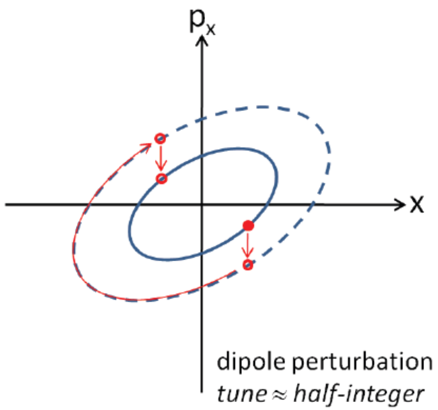




Betatron Resonances





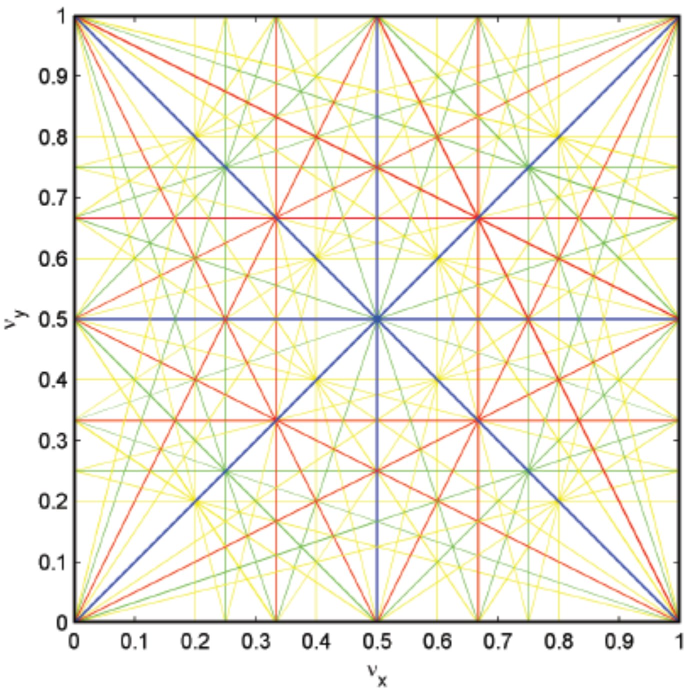


- Resonances can occur when the tunes satisfy:

$$m\nu_x + n\nu_y = q$$

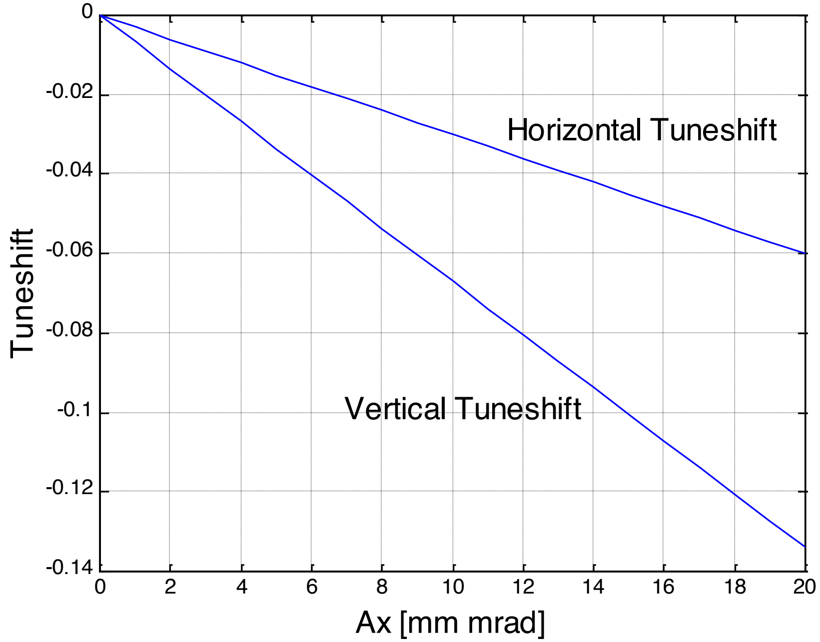
where m, n and q are integers

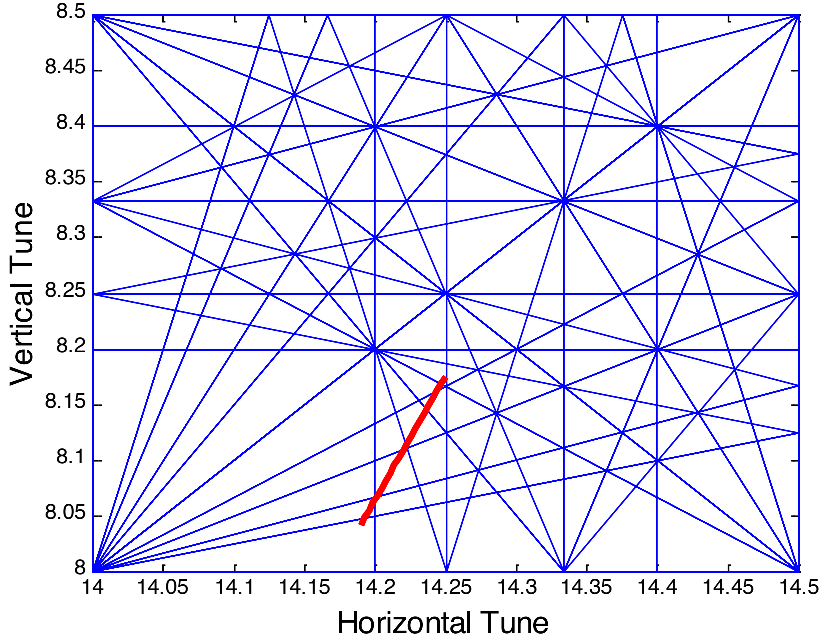
- Generally resonances are weaker the higher their order
- Integer resonances driven by dipole errors, half-integer by quadrupole errors, third-integer by sextupoles, …



Tune shift with amplitude

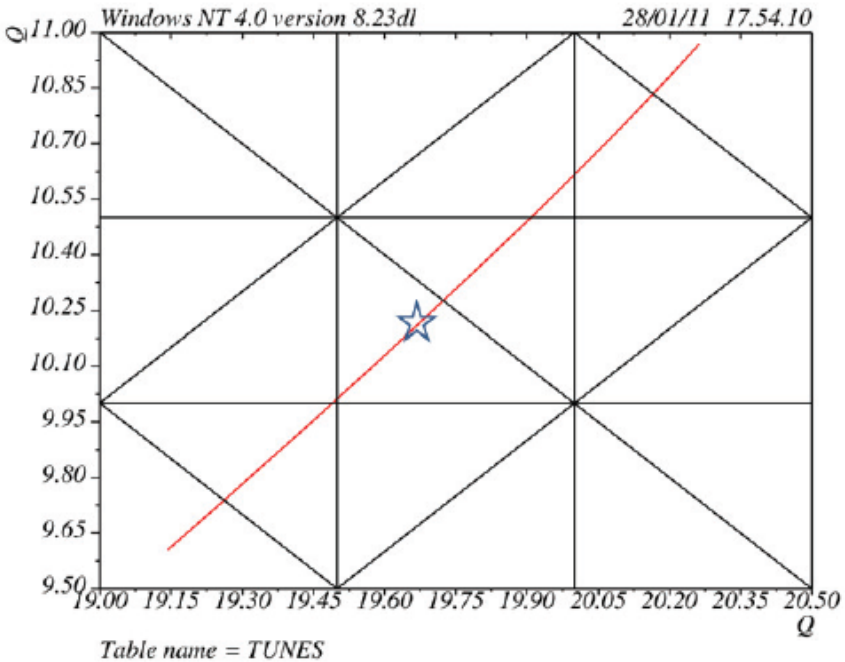
- Particle tune get shifted with amplitude
- After injection, or after a scattering event (gas scattering or Touschek scattering) particles are at large amplitudes



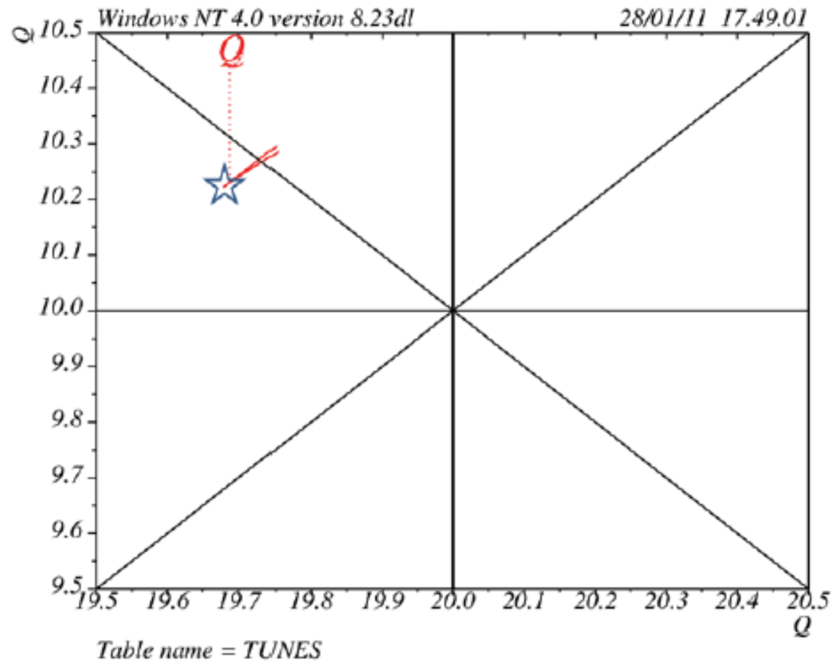


Tune shift with energy

- Particle tune get shifted with particle energy/momentum – Chromaticity
- Sextupoles are used to correct Chromaticity (linear or low order)
  - However, higher order terms often remain (even when using many sextupole families)
- Example below: Double Bend Achromat, 2 sextupole families, +/-2% energy range



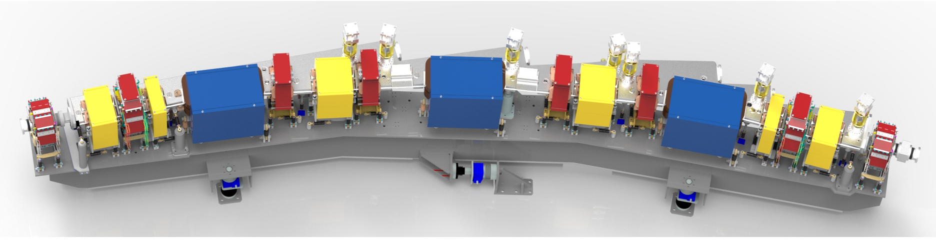
Without sextupoles



With sextupoles

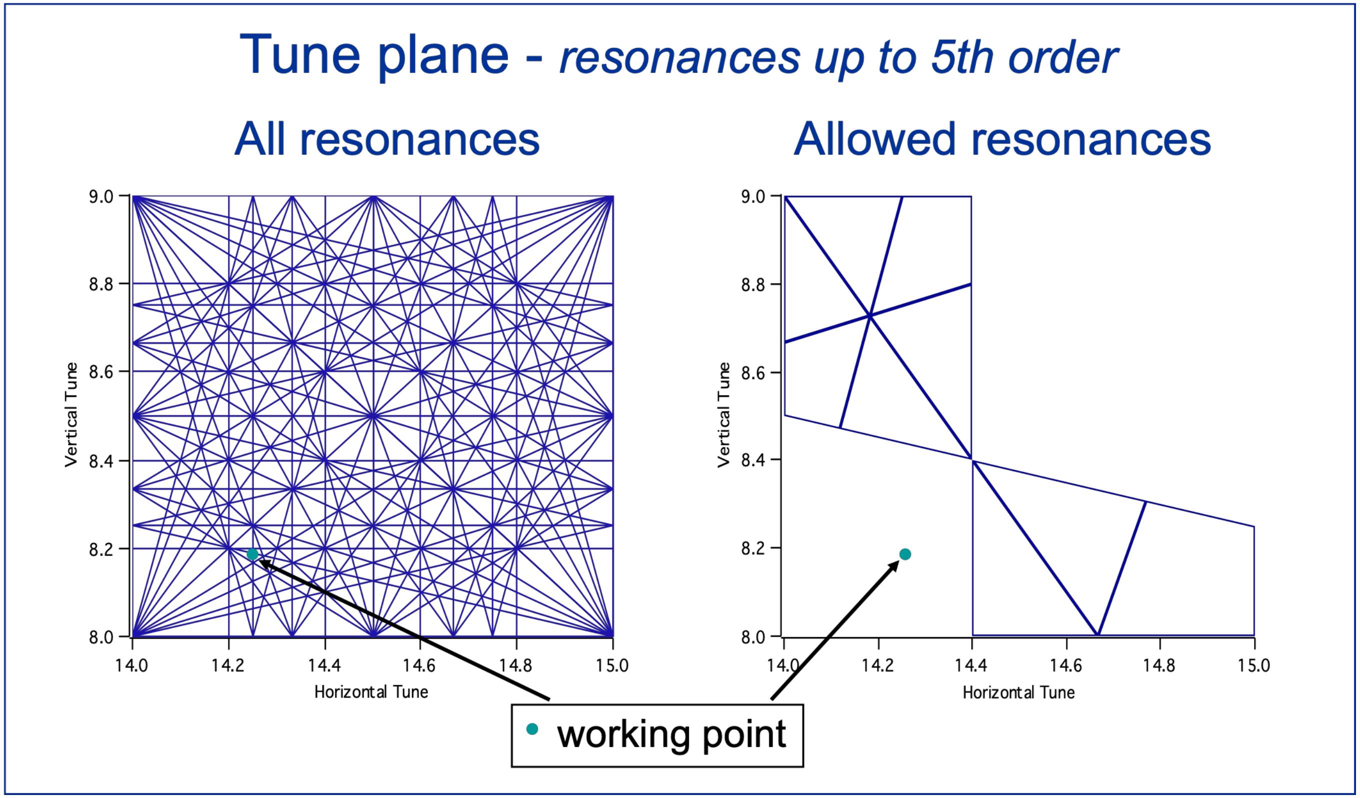
Benefits of Periodicity

- ALS consists of 12 sectors
  - 12-fold periodicity ⟹ *Suppression of resonances*



$$m\nu_x + n\nu_y = \mathbf{12} \times q$$

where m, n and q are integers



Resonance Excitation

Resonances can lead to irregular and chaotic behavior for the orbits of particles which eventually will get lost by diffusion in the outer parts of the beam.

Rule of thumb => Avoid low order resonances **(<~ 12th for protons and <~ 4th for electrons)**

One can study the strength of resonances by using a tracking code or through measurements

=> Tune scans
=> Frequency Map Analysis

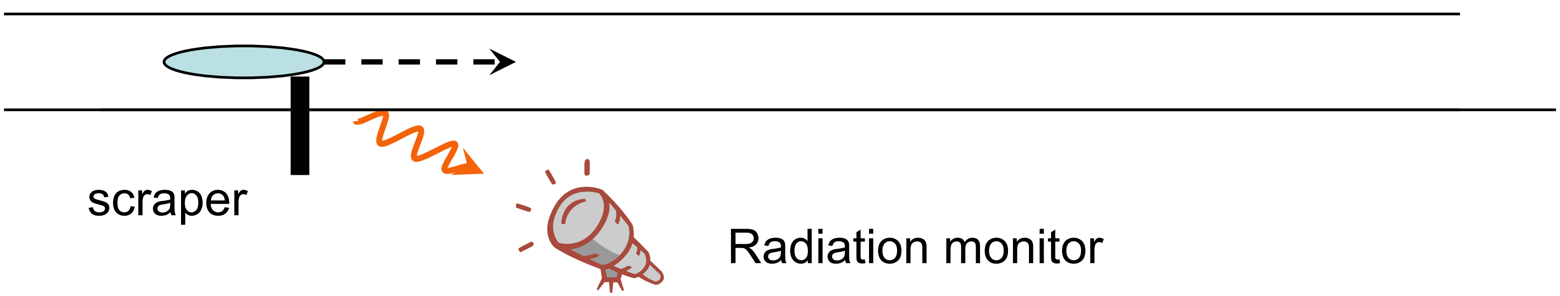
Tune scan

When resonances are present they may change the distribution of the beam at large amplitudes.

- In the case of a resonance island →particles may get trapped at large amplitudes

Technique:

- By Introducing a scraper and a loss monitor

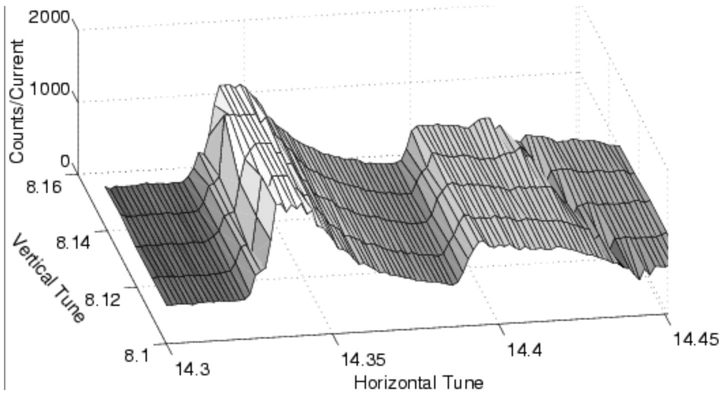


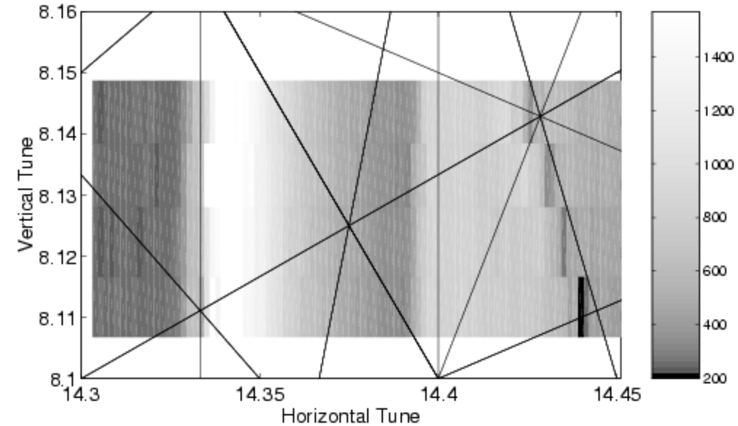
- Scan the tunes and measure the change in the count rate

Developed by A. Temnykh (Proc. Of the IXth ALL-Union Meeting on Accelerators of Chaged Particles, Dubna, 1984, INP Peport No. INP 84-131

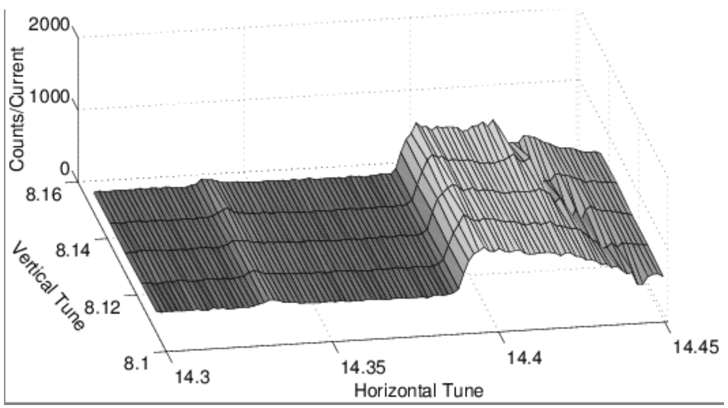
Tune scans (with and without large beta beating)

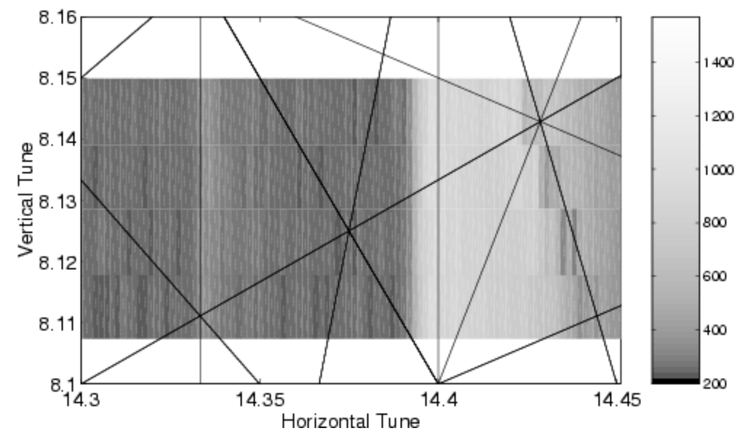
Uncorrected lattice





Corrected lattice





Three resonances are present:

$5\nu_x = 72$ **(allowed)**

$3\nu_x = 43$ **(unallowed)**

$2\nu_x - \nu_y = 37$ **(unallowed)**

KAM Theorem / Frequency Maps

- According to the KAM (Kolmogorov-Arnold-Moser) theorem, in a phase space that is sufficiently close to an integrable conservative system, many invariant tori will persist. Trajectories starting on one of these tori remain on it thereafter, executing ***quasi-periodic motion with a with a fixed frequency vector*** depending only on the torus.
  - $\Rightarrow$ Measuring how quickly frequencies of particle motion change allows quantitative analysis of how irregular a trajectory is
- Frequency Map Analysis – developed by Jacques Laskar – uses a frequency analysis algorithm (NAFF) as a postprocessor for tracking data that computes frequencies for any initial condition.

- Frequency Map: Initial condition $\longrightarrow$ Frequency vector

Regular orbits $\longrightarrow$ Frequency vector remains fixed in time

Nonregular orbits $\longrightarrow$ Frequency vector changes in time

Tunes and Diffusion Rates

TRACKING CODE + FREQUENCY ANALYSIS POSTPROCESSOR

Track particle for **N** turns

↓

Compute horizontal and vertical tunes ν_{x1} and ν_{y1}

↓

Track particle for another **N** turns

↓

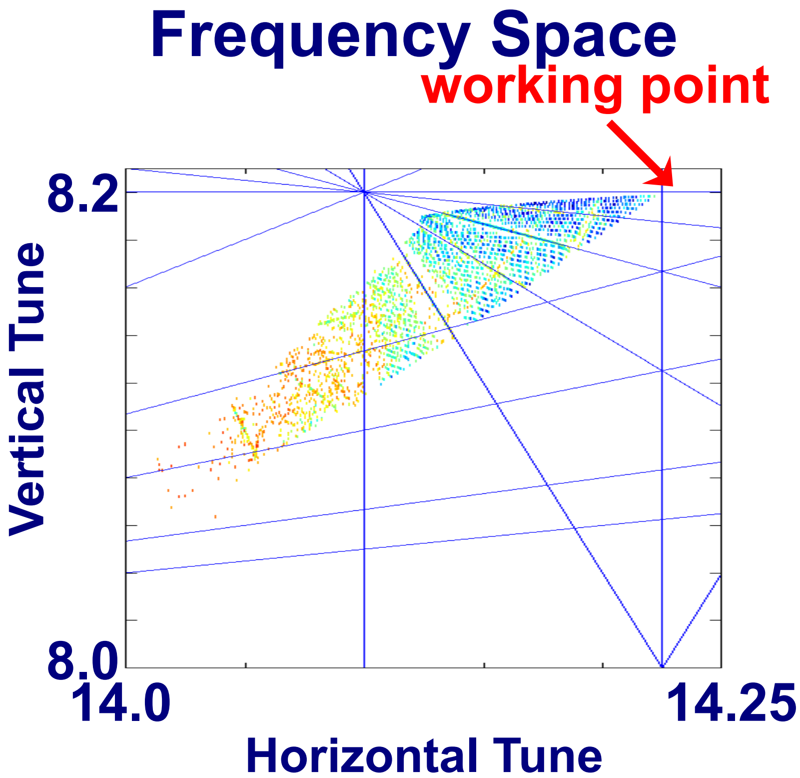
Compute horizontal and vertical tunes ν_{x2} and ν_{y2}

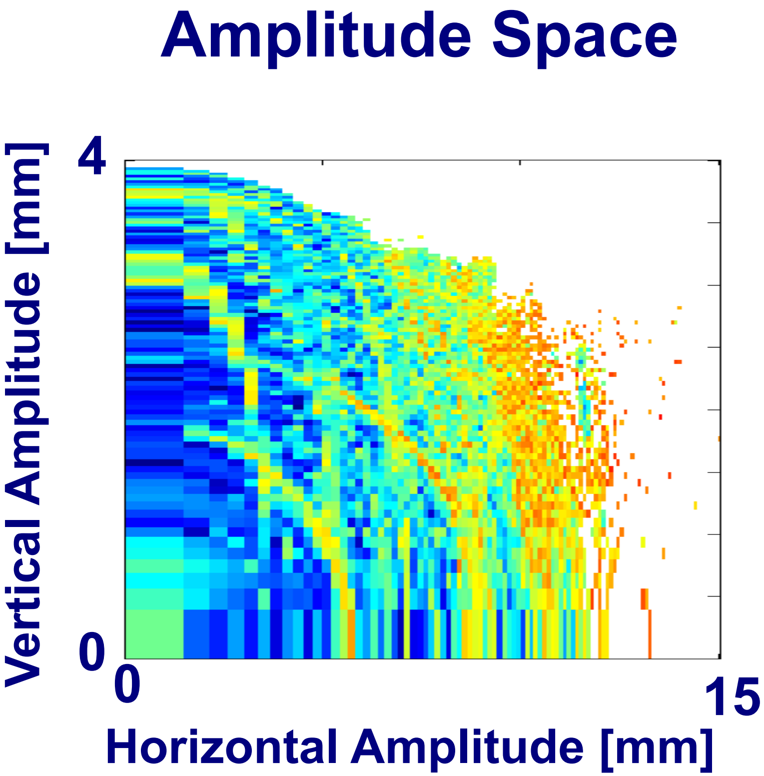
Compute diffusion rates

$$\frac{\partial \nu_x}{\partial \tau} \approx \frac{\nu_{x2} - \nu_{x1}}{N}$$

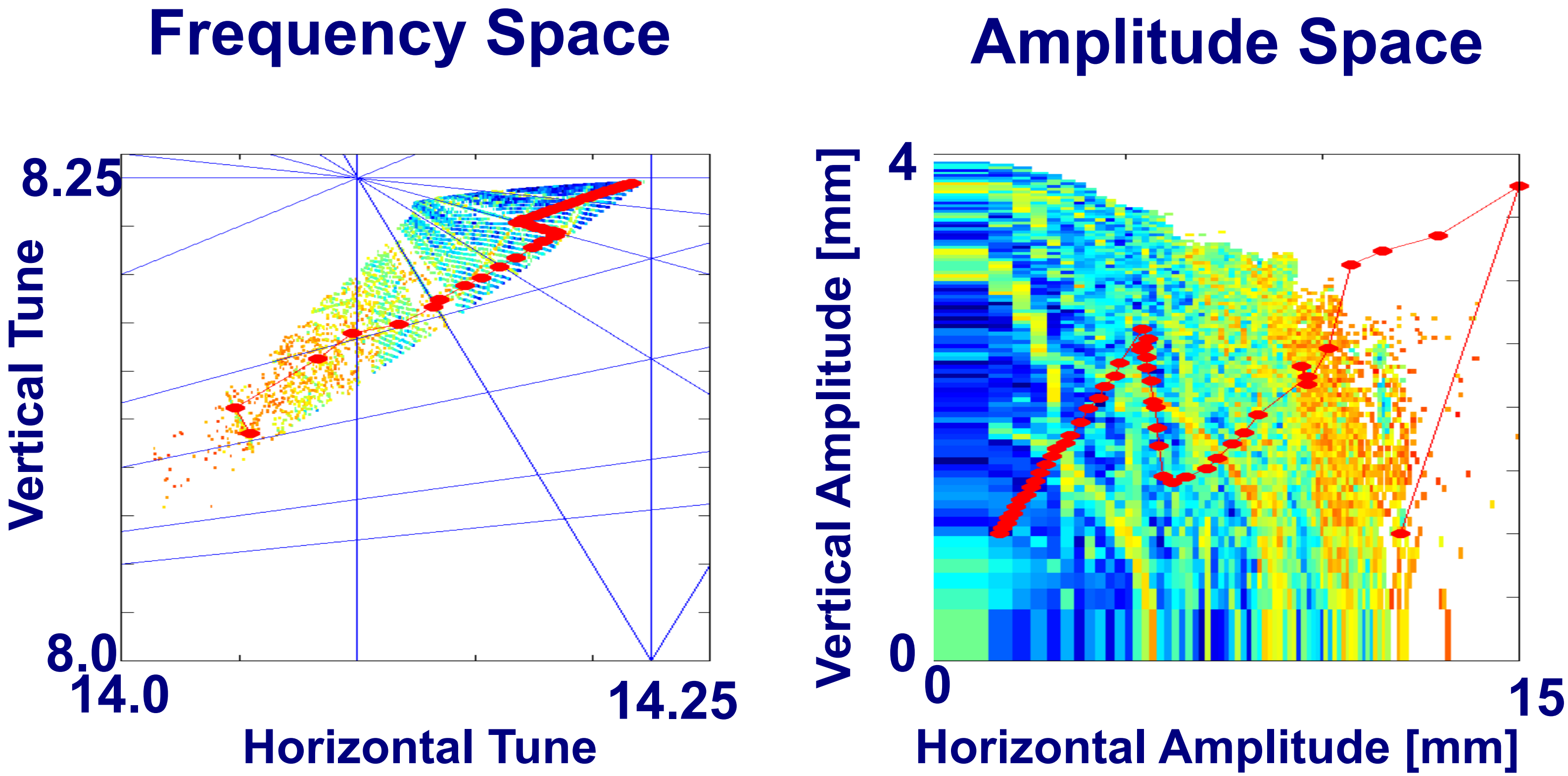
$$\frac{\partial \nu_y}{\partial \tau} \approx \frac{\nu_{y2} - \nu_{y1}}{N}$$

Frequency Map Analysis



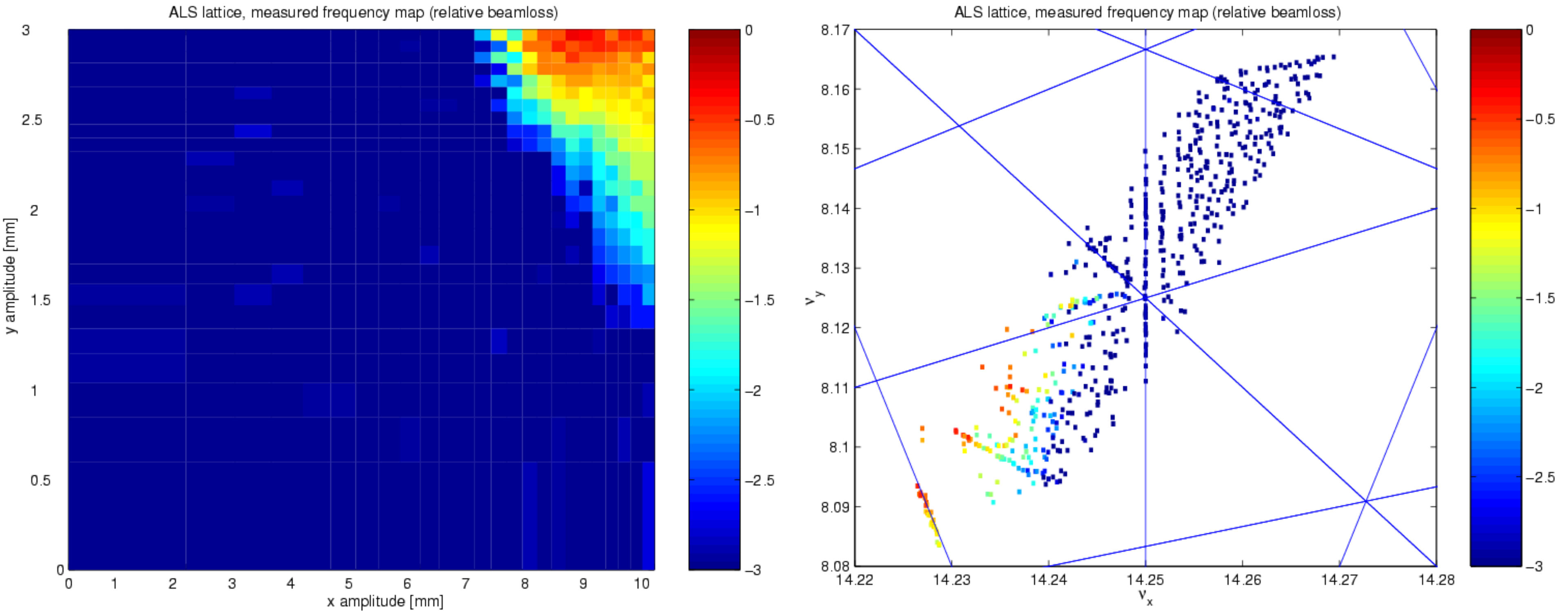


Frequency Map Analysis



- In this earlier lattice of the ALS, the limited dynamic aperture strongly influenced the injection efficiency
- Example trajectory is simulated for 10000 turns with synchrotron radiation, i.e. including damping

Measured Frequency Map/Beam Loss

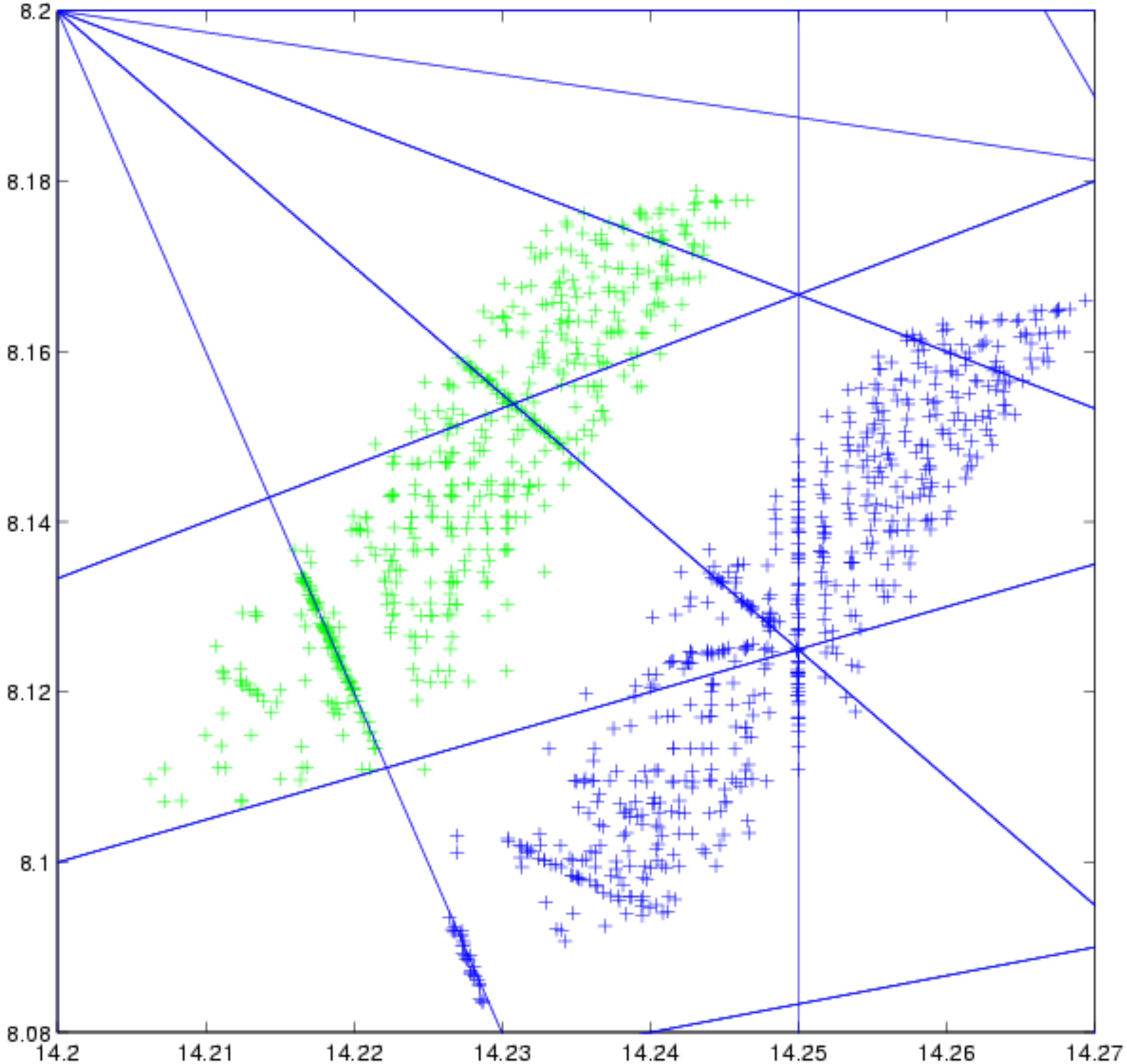


- Frequency maps can be measured and compared to simulated ones
  - Decoherence is challenge for measurement
- Partial beam loss often when particles pass through resonance intersection
  - Isolated resonances not dangerous.

Side remark: Spectra contain more information than just fundamental frequencies – other resonance lines – resonance strength versus amplitude

Phys. Rev. Lett 85, 3 (2000) 558

Model independent evaluation of dynamics



- Frequency map analysis can provide model independent evaluation of how regular beam motion is
- Based on this example, the tune of ALS was moved away from the original design point

Definition of Lifetime

- In a loss process, the number of particles lost at the time *t* is proportional to the number of particles present in the beam at the time *t*:

$$dN = -\alpha N(t)dt \qquad with \quad \alpha \equiv constant$$

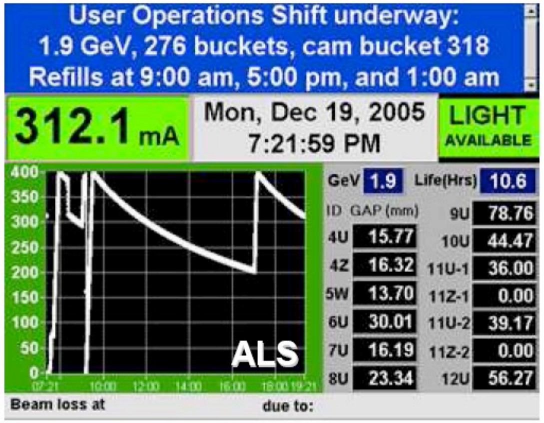
- By defining the lifetime τ as: $\tau = \frac{1}{\alpha}$ ⇒ $N = N_0 e^{-t/\tau}$

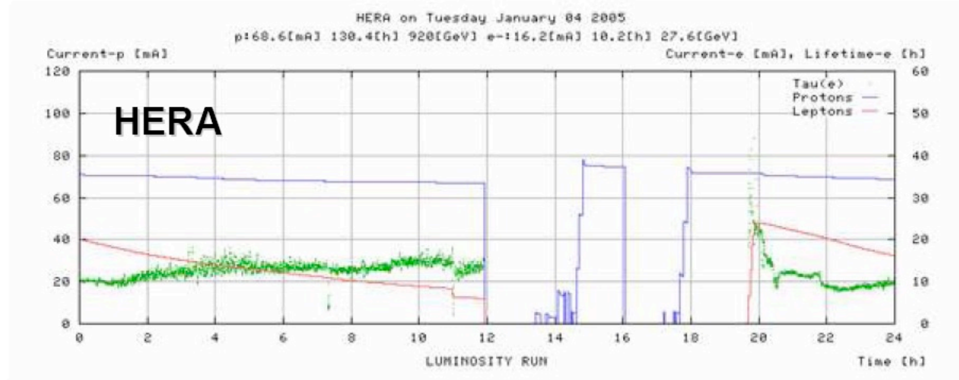
- From the last equation, one can see that the lifetime is defined as the time required for the beam to reduce its number of particles to 1/*e* of the initial value.
- Lifetime due to the individual effects (gas, Touschek, …) can be similarly defined. The total lifetime will be then obtained by summing the individual contributions:

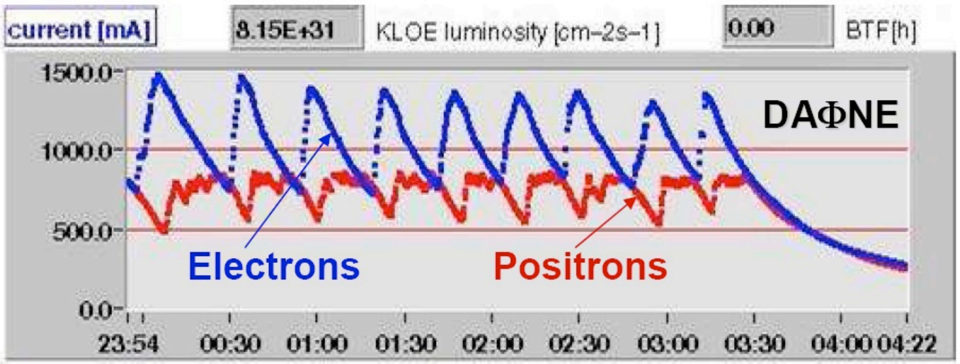
$$\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} + \ldots.$$

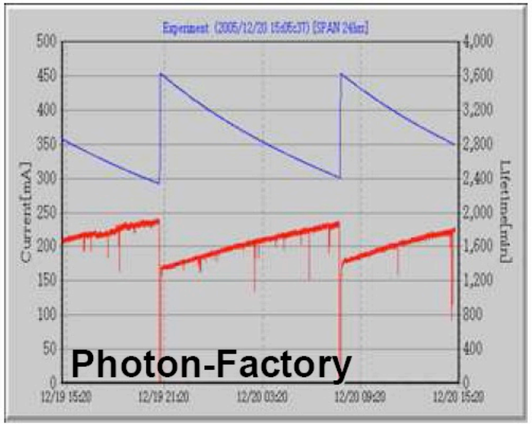
- With this definition, the problem of calculating the lifetime is reduced to the evaluation of the single lifetime components.

Example Lifetimes in different Accelerators





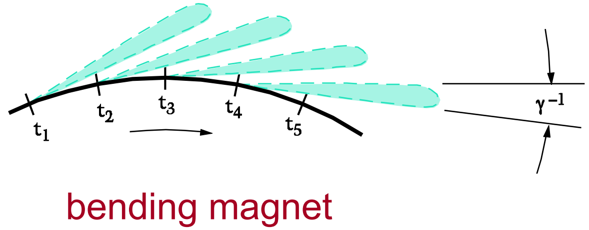


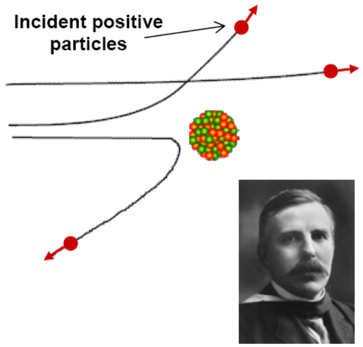


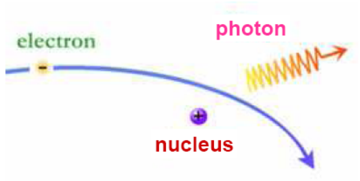
- In proton and heavy ions storage rings no damping is present: Any perturbation can build up and can eventually lead to particle loss.
- In synchrotron light sources Touschek scattering usually dominates.
- In colliders, the interaction between the colliding beams, the so-called *beam-beam effect*, often becomes the main mechanism of losses.

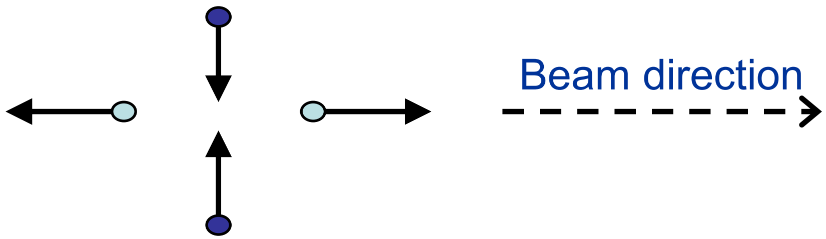
Some types of scattering events influencing beam lifetime

- **Electron-Photon Scattering**
  - **Quantum Lifetime**
- **Electron-Gas Scattering**
  - **Gas Lifetime**
- **Electron-Electron Scattering**
  - **Touschek Lifetime**









Quantum Lifetime

- Emission of synchrotron radiation is quantized
  - Transverse distribution of radiation is approximately Gaussian
  - A Gaussian distribution of particles is produced
- Tails of distribution are lost
  - Redistribution on time scale of damping time

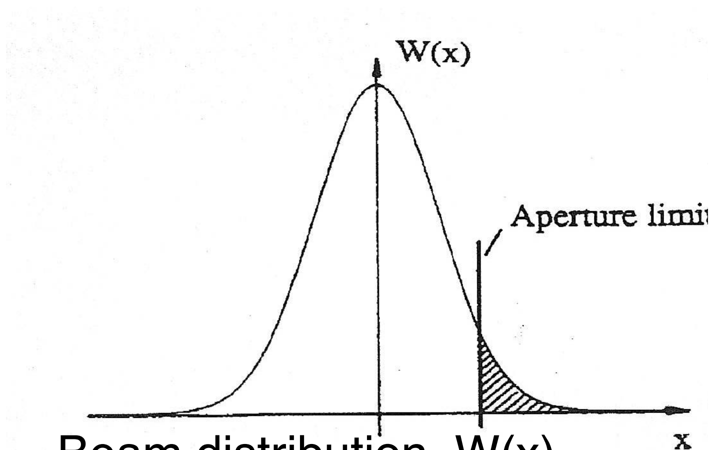
$$\tau_{Q_T} \cong \tau_{D_T} \frac{\sigma_T^2}{A_T^2} \exp\left(A_T^2 / 2\sigma_T^2\right) \quad T = x, y$$

Transverse quantum lifetime

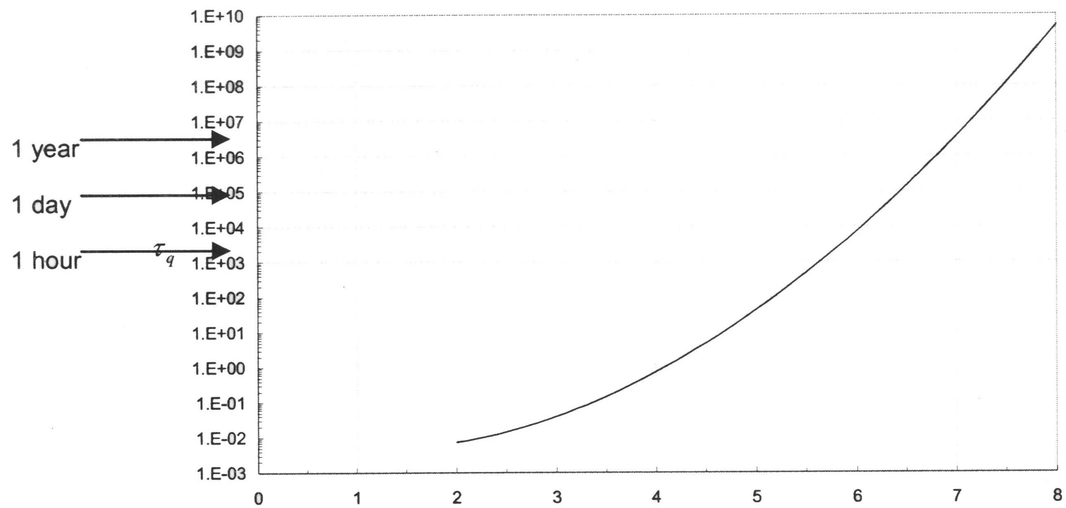
$$\text{where} \quad \sigma_T^2 = \beta_T \varepsilon_T + \left(\eta_T \frac{\sigma_E}{E_0}\right)^2 \quad T = x, y$$

$$\tau_{Q_L} \cong \tau_{D_L} \exp\left(\Delta E_A^2 / 2\sigma_E^2\right)$$

Longitudinal quantum lifetime



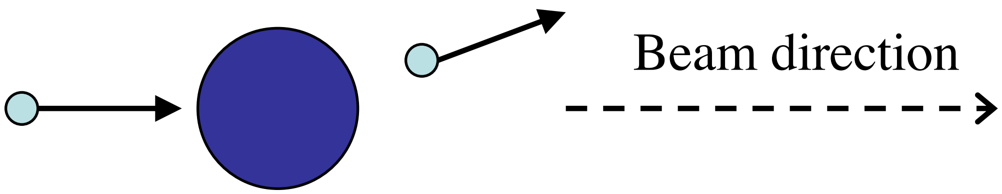
Beam distribution, W(x)

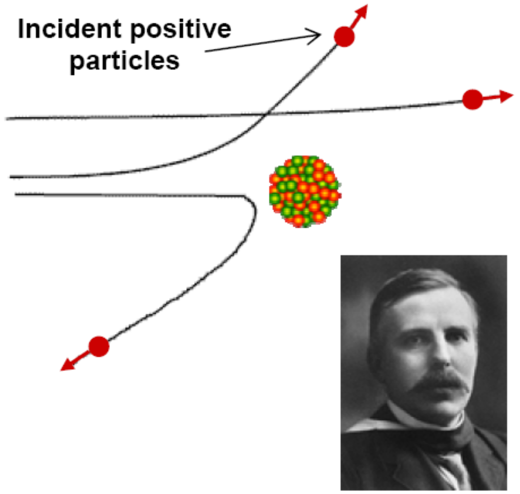


Quantum Lifetime versus aperture-to-beamsize ratio a/σ_a

Gas-scattering lifetime

Particles scatter elastically or inelastic with residual gas atoms. This introduces betatron or synchrotron oscillations.





The scattering process can be described by the classical Rutherford scattering with differential cross section per atom in cgs units

$$\frac{d\sigma}{d\Omega} = \left(\frac{zZe^2}{2\beta cp}\right)\frac{1}{\sin^4\frac{\theta}{2}}$$

Gas-scattering lifetime

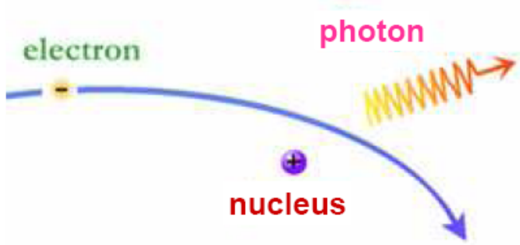
If the new amplitudes are outside the aperture the particles are lost.

•The elastic scattering lifetime is proportional to the square of the transverse aperture *A*:

$$\frac{1}{\tau_{el}} \propto \frac{1}{E^2} \times \left(\frac{\beta_x}{A_x^2} \langle P\beta_x \rangle + \frac{\beta_y}{A_y^2} \langle P\beta_y \rangle \right)$$

• The inelastic scattering lifetime is proportional to the logarithm of the energy/momentum aperture ε:

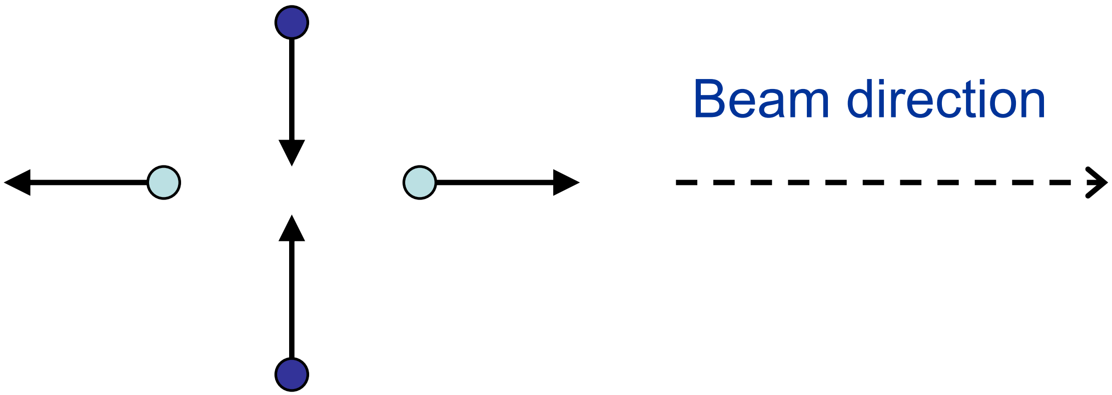
$$\frac{1}{\tau_{inel}} \propto \langle P \rangle \times \ln(\varepsilon)$$



- For typical electron ring parameters, one finds that the requirement on vacuum is for dynamic pressures of the order of a few nTorr to achieve many hours of vacuum lifetime.

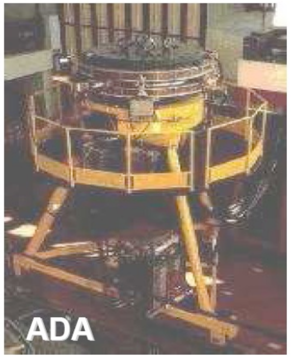
Touschek Lifetime

- Particles perform betatron oscillations. If two collide within a bunch they can transform transverse momenta into longitudinal momenta.



- If they fall outside the momentum aperture, ε, the particles are lost. The lifetime is proportional to the square of ε

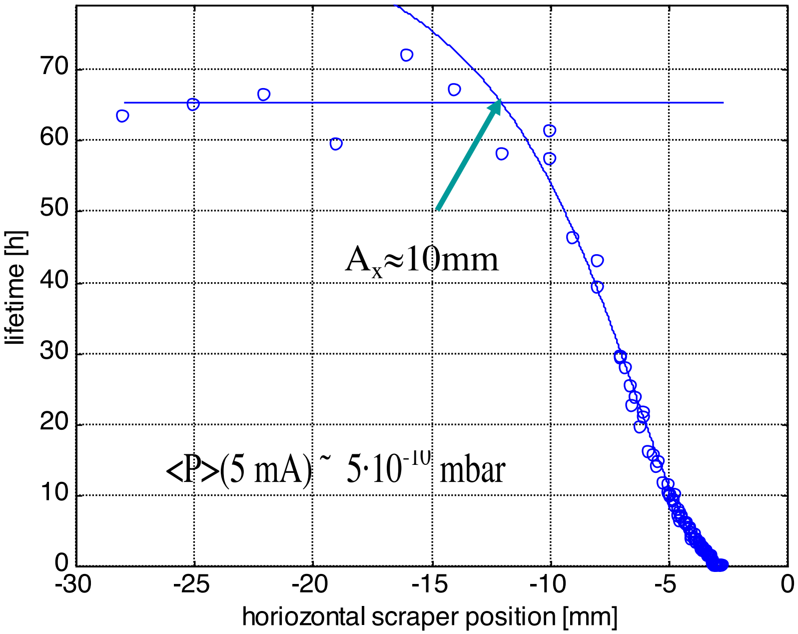
$$\frac{1}{\tau_{tou}} \propto \frac{1}{E^3} \frac{I_{bunch}}{V_{bunch}\sigma_x^{'}} \frac{1}{\varepsilon^2} f\left(\varepsilon, \sigma_x^{'}, E\right)$$

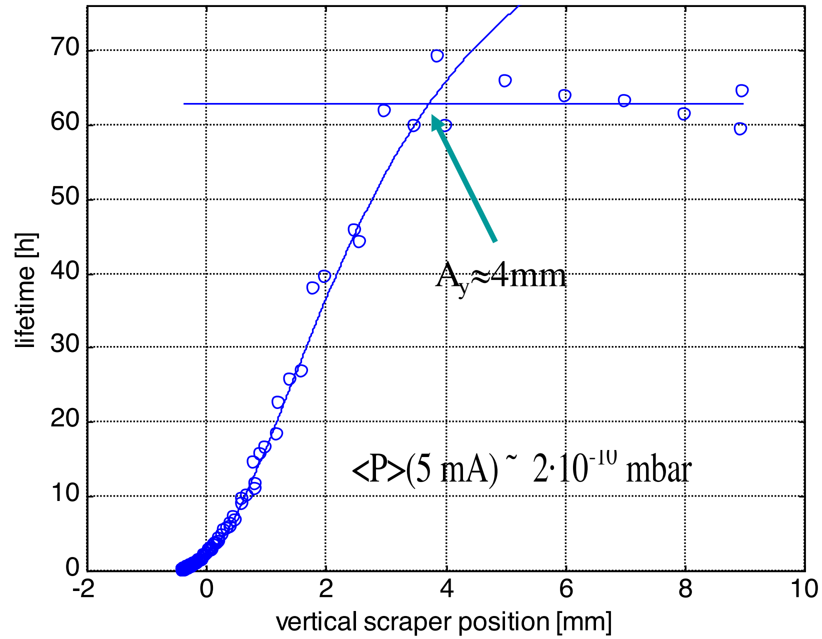


- First observation in the early 60s in Frascati at ADA.
- Touschek effect is the dominant lifetime contribution in many modern electrons storage rings.

Acceptance Scans and Gas Lifetime

- move scraper into beam and record lifetime -> acceptance, gas pressure

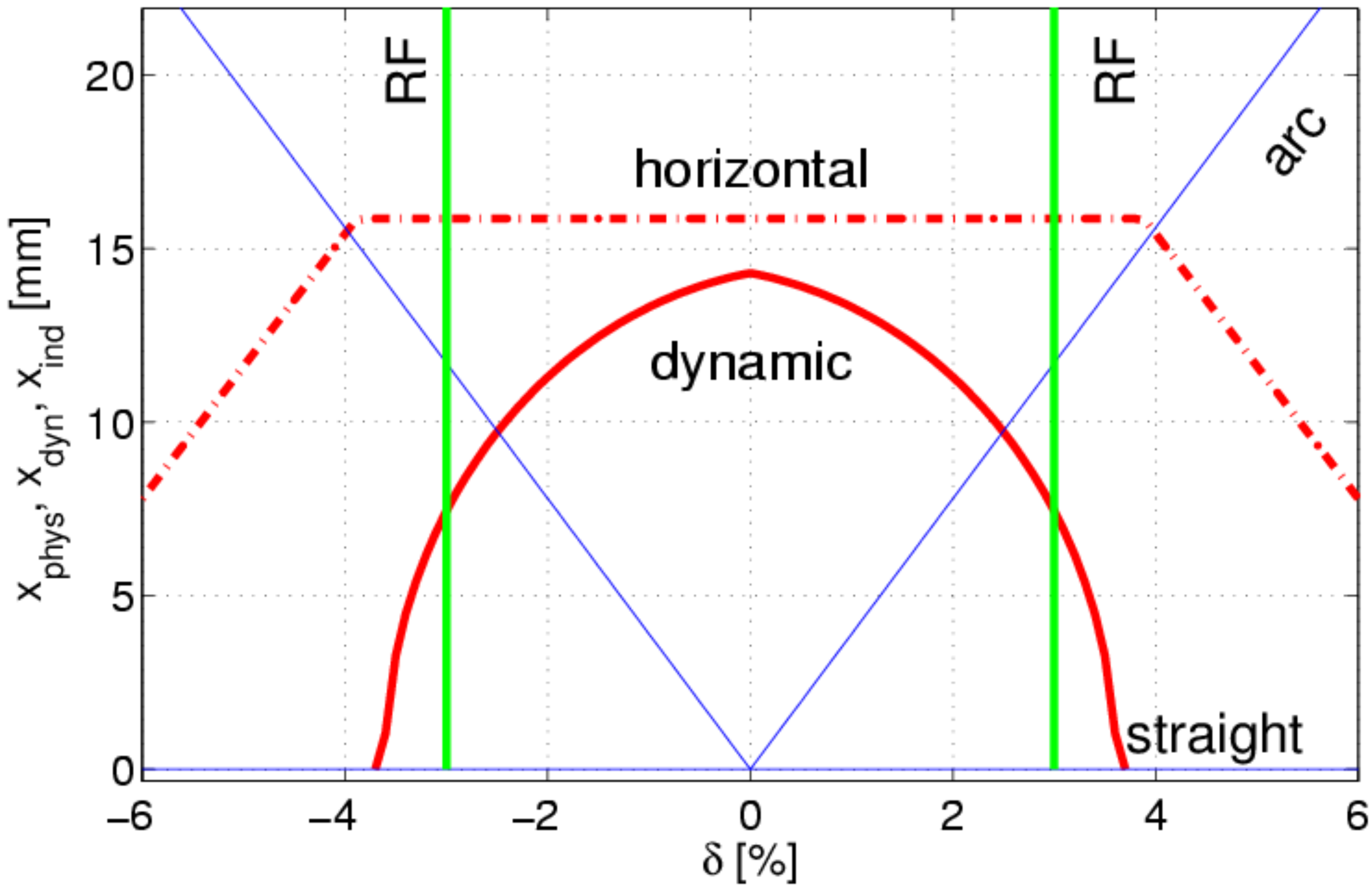




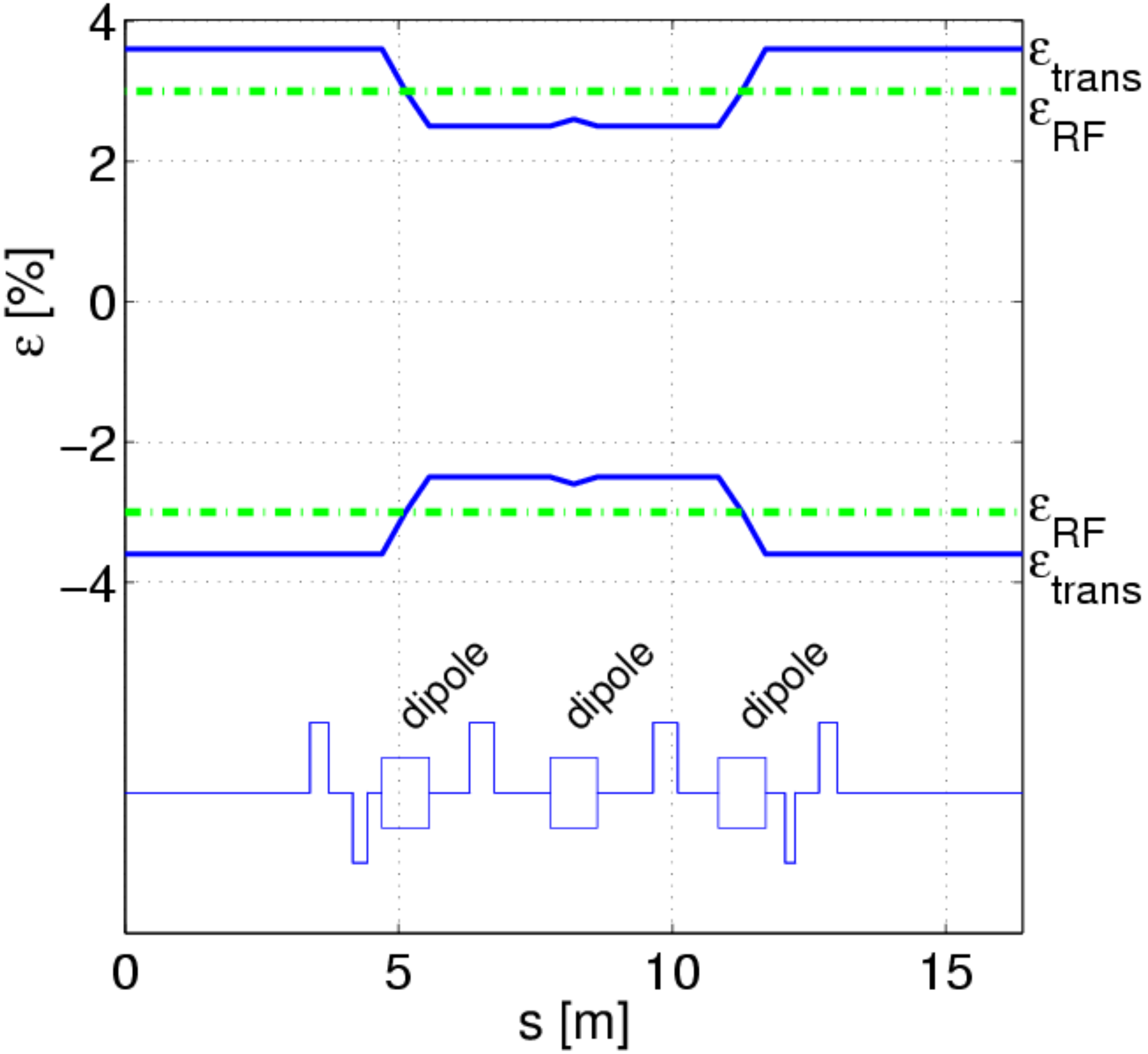
$$\frac{1}{\tau}(\Delta_x) = \begin{cases} const. & \text{if } \Delta_x > A_x \\ \dfrac{1}{\tau_{tou+inel}} + C_{el}\dfrac{1}{E^2}\langle P\rangle\left(\langle\beta_x\rangle\dfrac{\beta_x}{\Delta_x^2} + \langle\beta_y\rangle\dfrac{\beta_y}{A_y^2}\right) & \text{if } \Delta_x < A_x \end{cases}$$

Assuming different distribution of the gas, i.e. higher pressure in the straight sections: **$3*10^{-10}$ mbar**

What determines the momentum aperture

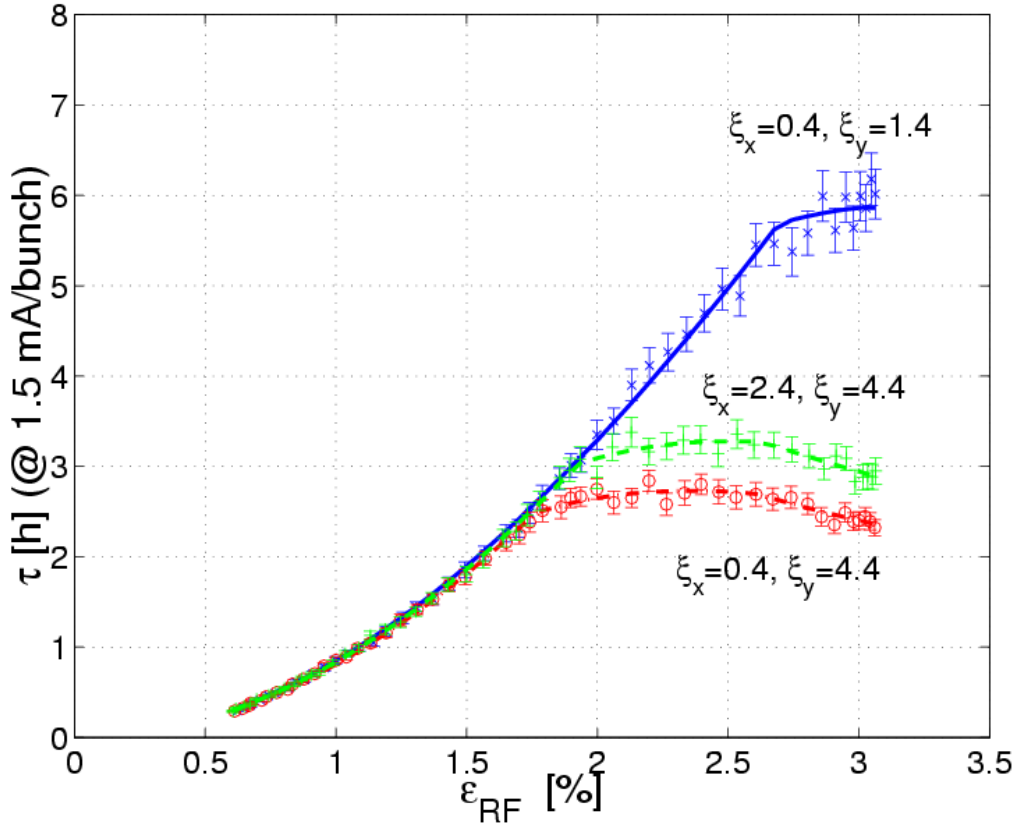


Longitudinal variation of momentum aperture



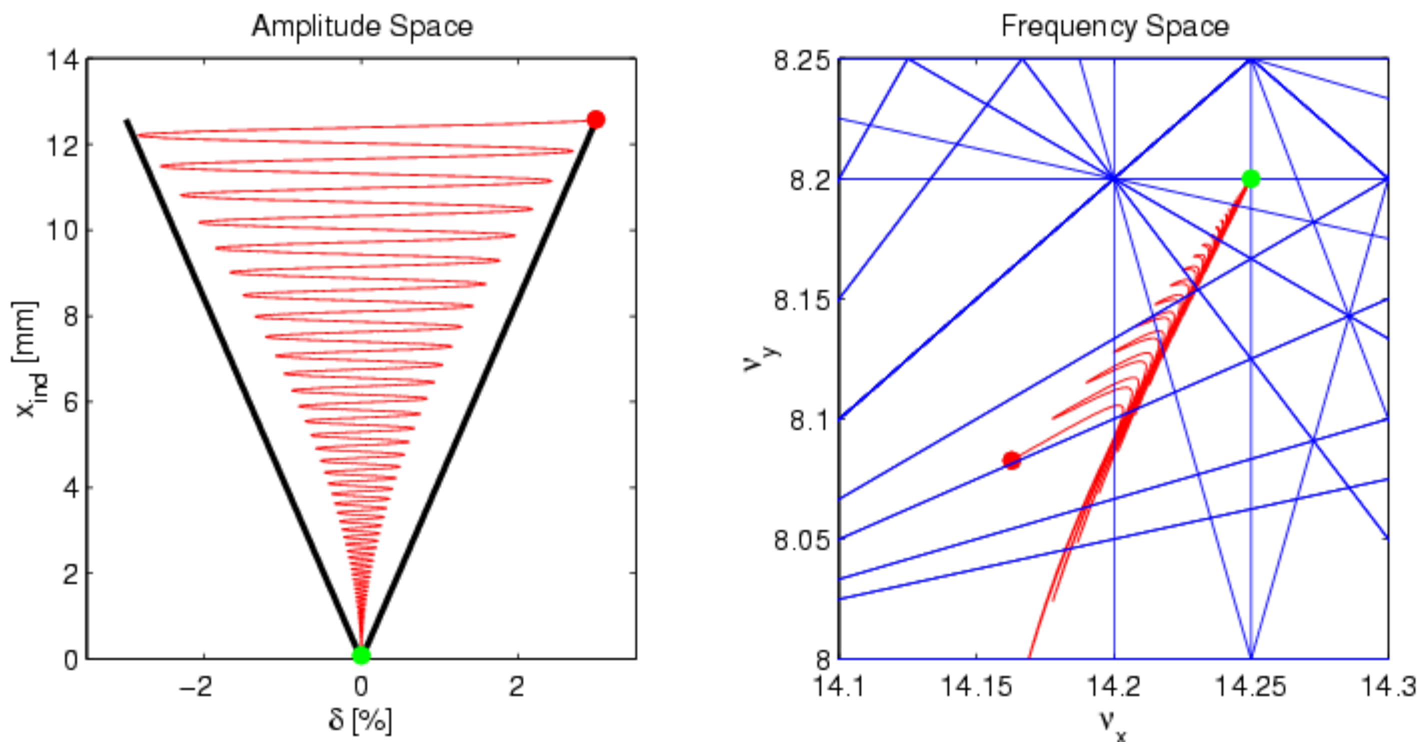
- Because of variation in H-function, momentum aperture will vary around the ring (depending on scattering location)
- Not necessarily symmetric for positive and negative momentum deviation (asymmetric bucket)

ALS example: 3 Chromaticities



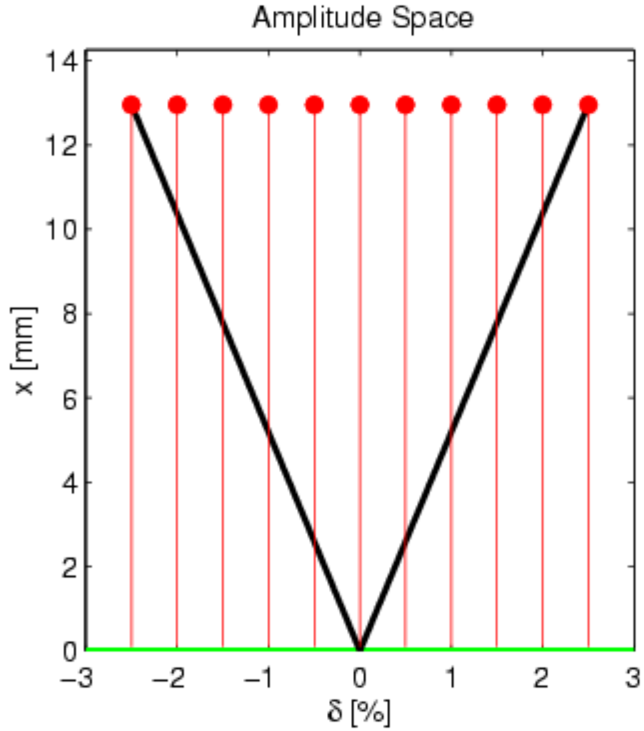
- Momentum aperture in ALS is impacted by nonlinear dynamics
- Sensitivity to chromaticity is at first surprisingly large (sextupole strength only different by a few percent).

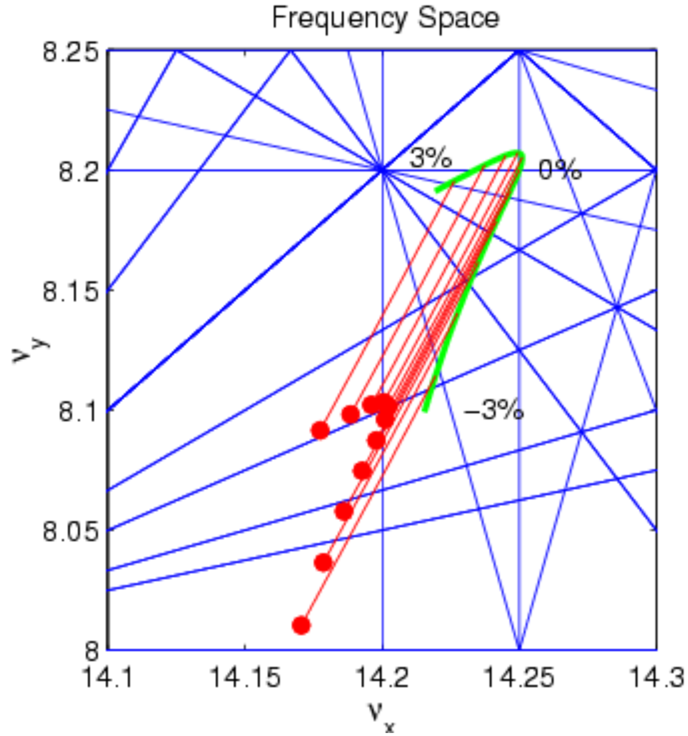
Touschek Scattering – Tune Shift – Particle Loss



- Particle losing/gaining energy – horiz. oscillation (dispersion/H-function) + long. Oscillation
- Particle changes tune
  - Synchrotron oscillations (chromaticity)
  - Radiation damping (detuning with amplitude and chromaticity)
- During damping process particle can encounter region in tune space where motion gets resonantly excited.

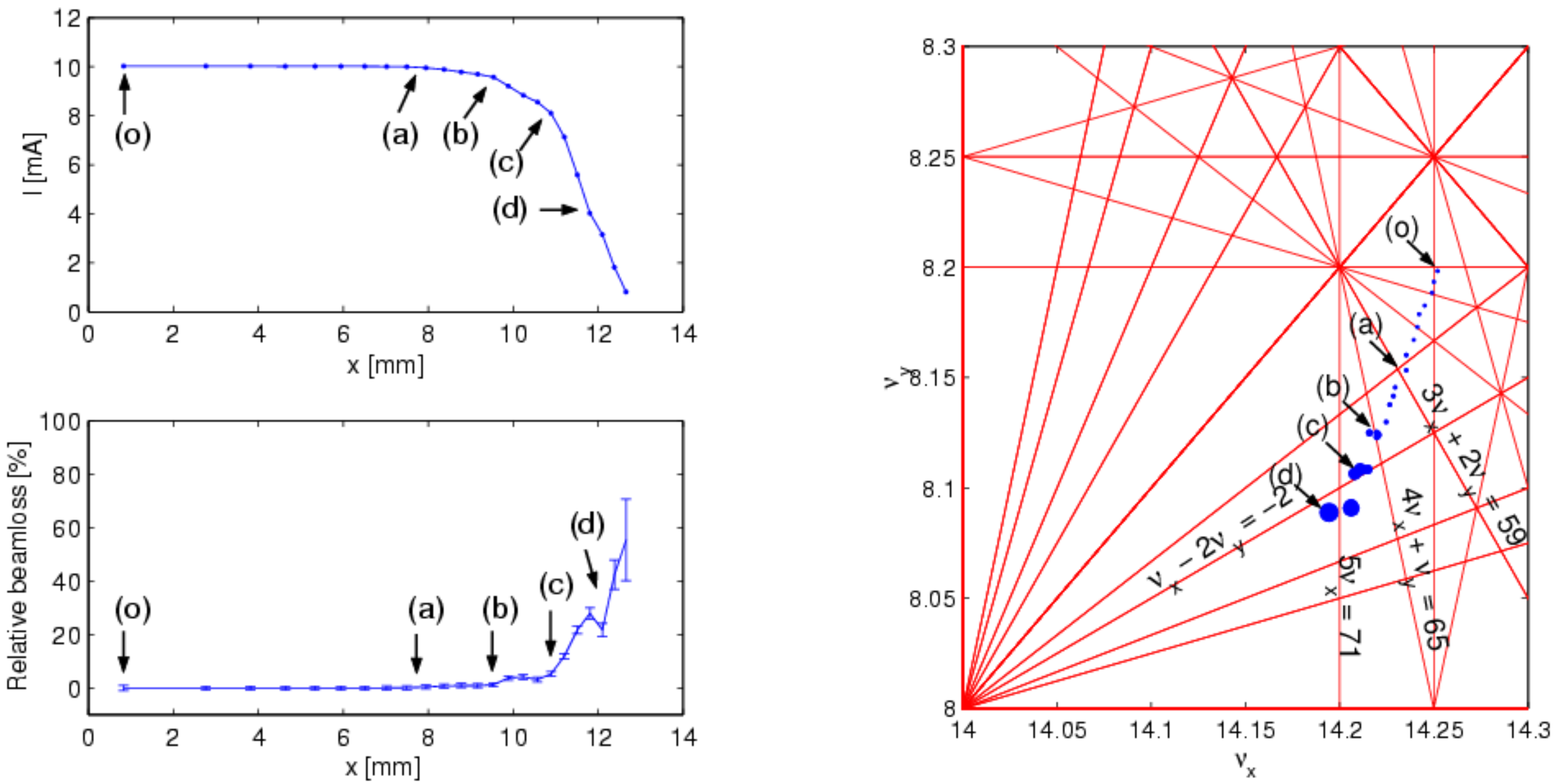
Measurement principle





- Experimentally very difficult to exactly simulate Touschek scattering (simultanous kicks) – also difficult to measure tunes during synchrotron oscillations
  - Some positive results (Y. Papaphilippou et al.)
- Still possible to locate loss regions when scanning only transverse amplitude while keeping energy offset fixed

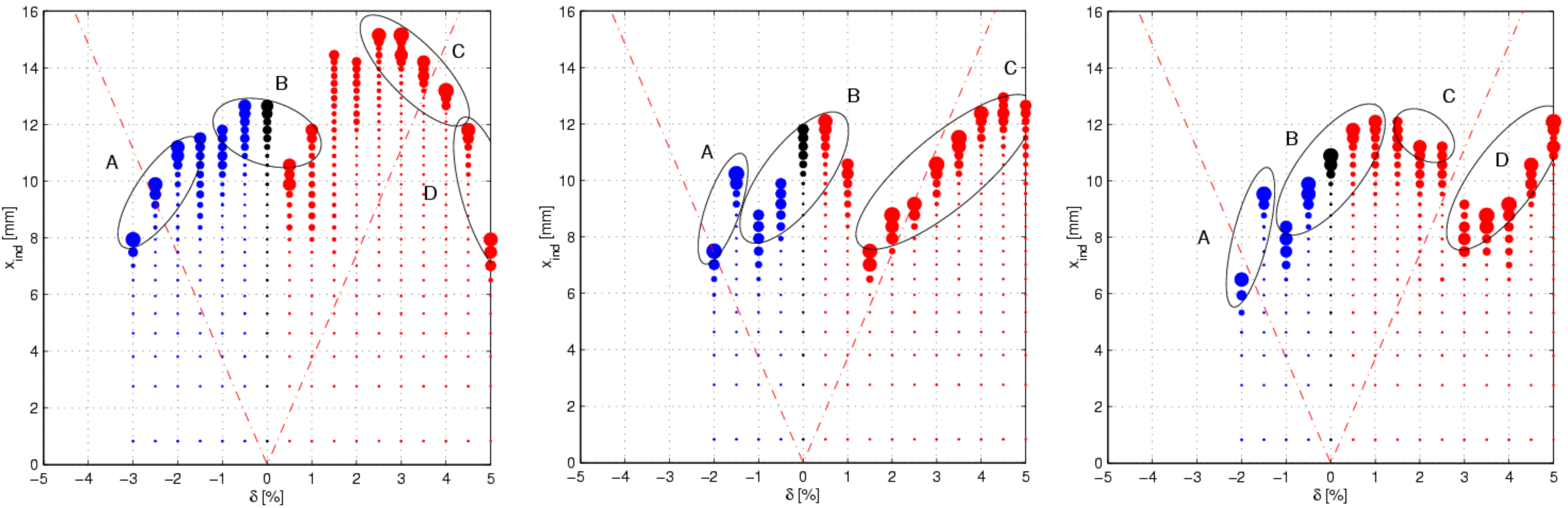
Measurement Detail



- Use single turn kicker to excite beam with increasing amplitude
- Use current monitor to record relative beam loss after kick
- Use turn-by-turn BPMs to record oscillation frequencies

Aperture Scan for 3 Different Chromaticities

Small horiz. Chromaticity Small vert. | **Small horiz. Large vert.** | **Large horiz. Large vert.**



ε > 3 % straight
2.65 % arcs

ε = 2.6 % straight
1.75 % arcs

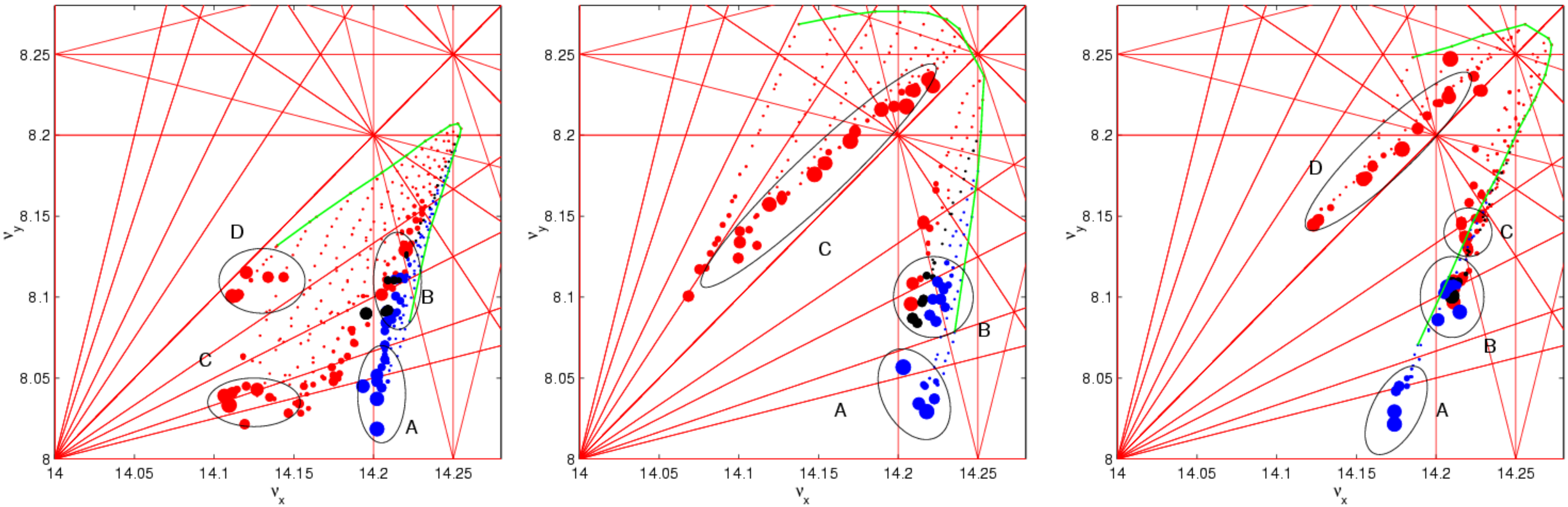
ε = 2.6 % straight
1.9 % arcs

Aperture Scan for 3 Different Chromaticities

Small horiz. Chromaticity
Small vert.

Small horiz.
Large vert.

Large horiz.
Large vert.



ε > 3 % straight
2.65 % arcs

ε = 2.6 % straight
1.75 % arcs

ε = 2.6 % straight
1.9 % arcs

Summary: Lifetime Limiting Processes

- Elastic Scattering

$$\frac{1}{\tau_{el}} \propto \frac{1}{E^2} \times \left(\frac{\beta_x}{A_x^2} \langle P\beta_x \rangle + \frac{\beta_y}{A_y^2} \langle P\beta_y \rangle \right) \quad (1)$$

- Touschek Effect

$$\frac{1}{\tau_{tou}} \propto \frac{1}{E^3} \frac{I_{bunch}}{V_{bunch}\sigma'_x} \frac{1}{\varepsilon} f\left(\varepsilon, \sigma'_x, E\right) \quad (2)$$

- Quantum Lifetime

$$\frac{1}{\tau_q} \propto \frac{A^2}{\sigma^2} \times \exp\left(-\frac{A^2}{2\sigma^2} \right) \quad (3)$$

- Inelastic Scattering

$$\frac{1}{\tau_{inel}} \propto \langle P \rangle \times \ln(\varepsilon) \quad (4)$$

$$\frac{1}{\tau} = \frac{1}{\tau_{el}} + \frac{1}{\tau_{tou}} + \frac{1}{\tau_{ql}} + \frac{1}{\tau_{inell}}$$

Measured vs. model lattice - linear and nonlinear

Amplitudes and phases of the spectral line of the betatron motion can be used to compare and correct the real accelerator with the model

LOCO

Closed Orbit Response Matrix from model Closed Orbit Response Matrix measured	fitting quadrupoles, etc	Linear lattice correction/calibration

R. Bartolini and F. Schmidt in PAC05

Spectral lines + FMA from model Spectral Lines + FMA measured	fitting sextupoles and higher order multipoles	Nonlinear lattice correction/calibration

Combining the complementary information from FM and spectral lines can allow the calibration of the nonlinear model and correction of nonlinear resonances

Frequency Analysis of Betatron Motion and Lattice Model Reconstruction (1)

Accelerator Model

- tracking data at all BPMs
- spectral lines from model (NAFF)
- build a vector of Fourier coefficients

Accelerator

- beam data at all BPMs
- spectral lines from BPMs signals (NAFF)
- build a vector of Fourier coefficients

$$\overline{A} = \left(a_1^{(1)} \quad \dots \quad a_{NBPM}^{(1)} \quad \varphi_1^{(1)} \quad \dots \quad \varphi_{NBPM}^{(1)} \quad a_1^{(2)} \quad \dots \quad a_{NBPM}^{(2)} \quad \varphi_1^{(2)} \quad \dots \quad \varphi_{NBPM}^{(2)} \quad \dots \right)$$

Define the distance between the two vector of Fourier coefficients

$$\chi^2 = \sum_k \left(A_{Model}(j) - A_{Measured}(j) \right)^2$$

Frequency Analysis of Betatron Motion and Lattice Model Reconstruction (2)

Least Square Fit (SVD) of accelerator parameters θ

to minimize the distance χ^2 of the two Fourier coefficients vectors

- Compute the "Sensitivity Matrix" M $\quad \Delta\overline{A} = M\overline{\theta}$
- Use SVD to invert the matrix M $\quad M = U^T W V$
- Get the fitted parameters $\quad \overline{\theta} = (V^T W^{-1} U)\,\Delta\overline{A}$

MODEL → TRACKING → NAFF →

Define the vector of Fourier Coefficients – Define the parameters to be fitted

SVD → CALIBRATED MODEL

Frequency Analysis of Non Linear Betatron Motion

A.Ando (1984), J. Bengtsson (1988), R.Bartolini-F. Schmidt (1998)

The quasi periodic decomposition of the orbit

$$x(n) - ip_x(n) = \sum_{k=1}^{n} c_k e^{2\pi i \nu_k n} \qquad c_k = a_k e^{i\phi_k}$$

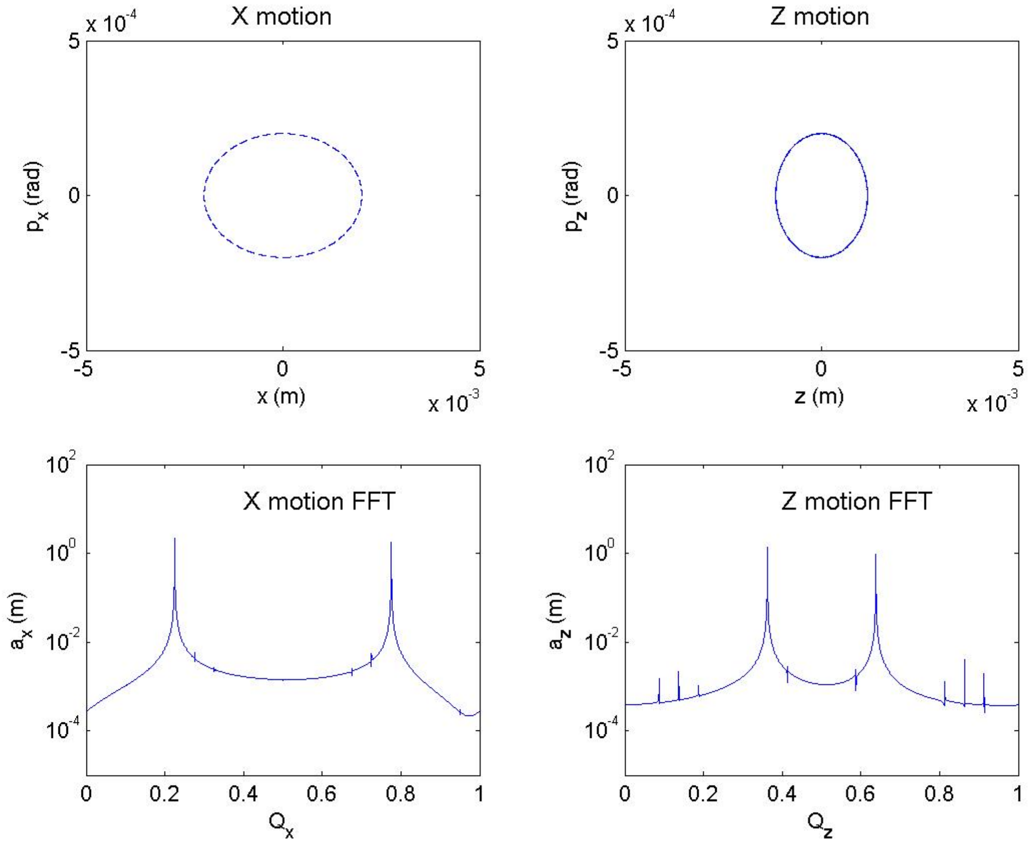
can be compared to the perturbative expansion of the non linear betatron motion

$$x(n) - ip_x(n) = \sqrt{2I_x} e^{i(2\pi Q_x n + \psi_0)} +$$
$$-2i \sum_{jklm} j s_{jklm} (2I_x)^{\frac{j+k-1}{2}} (2I_y)^{\frac{l+m}{2}} e^{i\left[(1-j+k)(2\pi Q_x n + \psi_{x0}) + (m-l)(2\pi Q_y n + \psi_{y0})\right]}$$

Each resonance driving term s_{jklm} contributes to the Fourier coefficient of a well defined spectral line

$$\nu(s_{jklm}) = (1 - j + k)Q_x + (m - l)Q_y$$

Simulated spectra for DIAMOND



Spectral Lines detected with NAFF algorithm

e.g. Horizontal:

• (1, 0)	$1.10\ 10^{-3}$	horizontal tune
• (0, 2)	$1.04\ 10^{-6}$	$Q_x - 2\ Q_z$
• (–3, 0)	$2.21\ 10^{-7}$	$4\ Q_x$
• (–1, 2)	$1.31\ 10^{-7}$	$2\ Q_x + 2\ Q_z$
• (–2, 0)	$9.90\ 10^{-8}$	$3\ Q_x$
• (–1, 4)	$2.08\ 10^{-8}$	$2\ Q_x + 4\ Q_z$

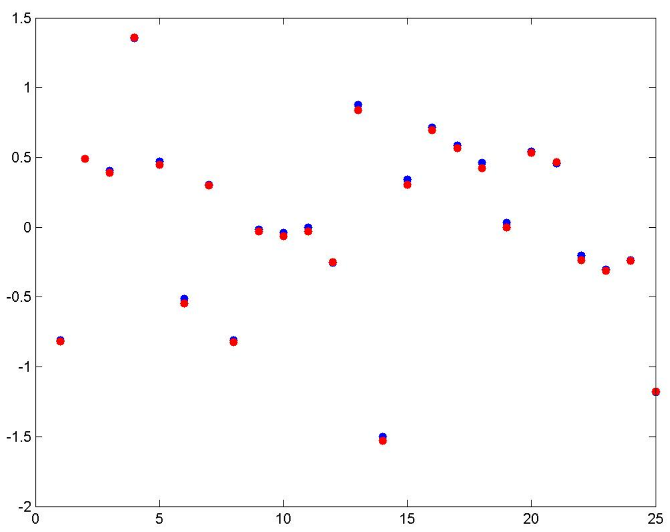
low emittance lattice
(.2 mrad kick in both planes)

R. Bartolini

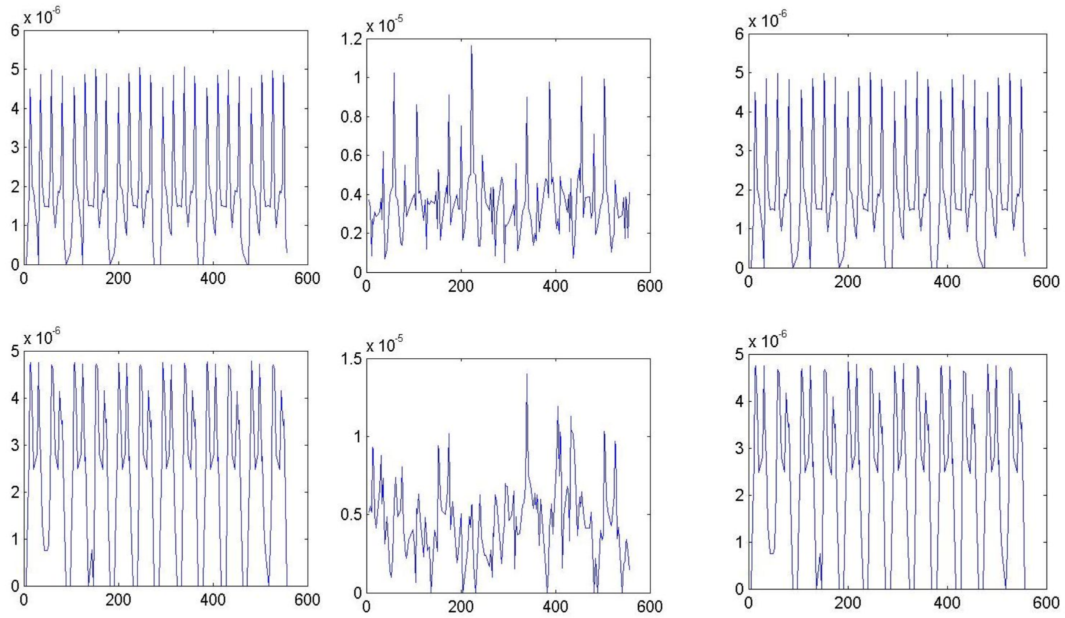
Fitted values for the 24 sextupoles gradients errors obtained from SVD - Simulation

Blu dots = assigned misalignments

Red dots = reconstructed misalignments

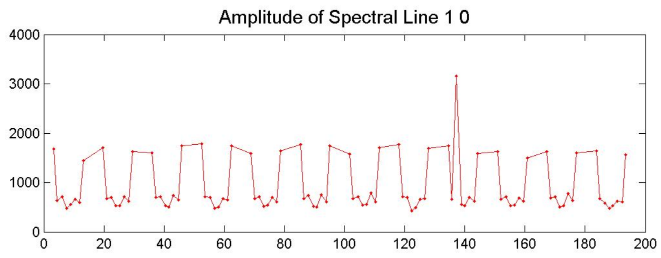


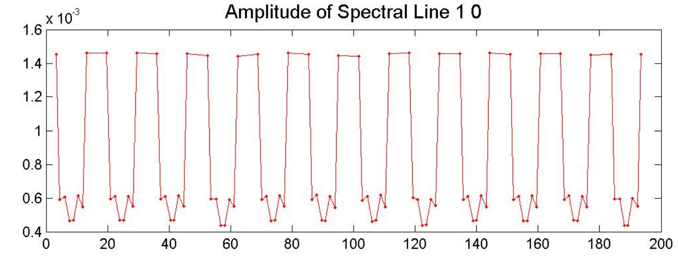
no gradient errors	with gradient errors	with gradient errors and corrections

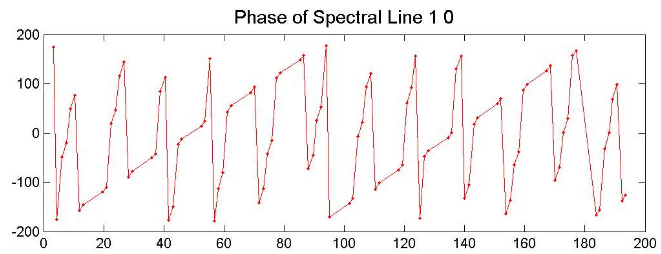


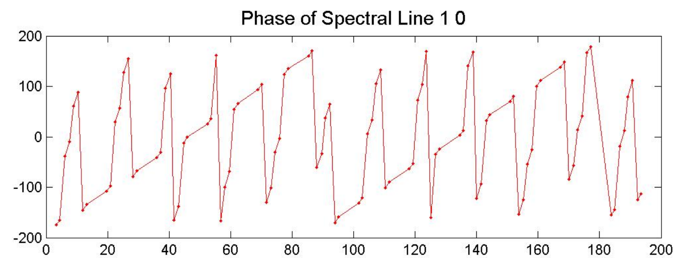
R. Bartolini

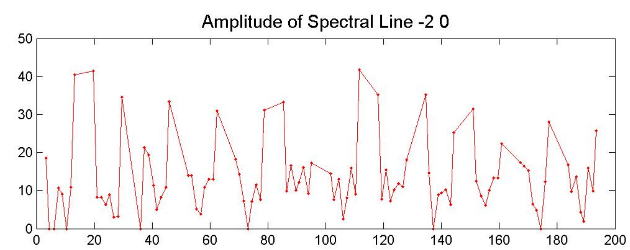
Measurements: ALS example (raw results)

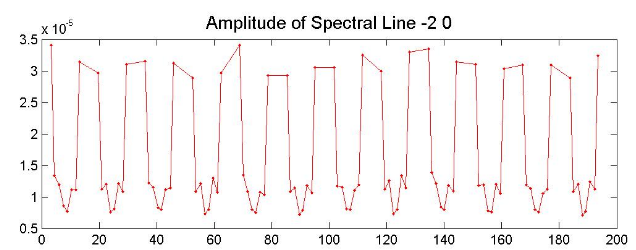


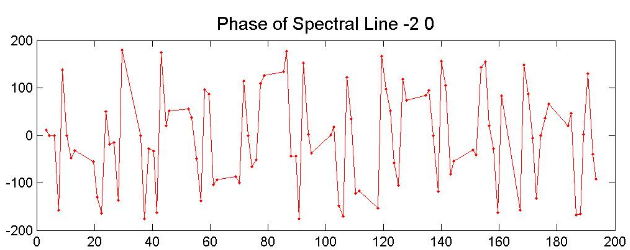


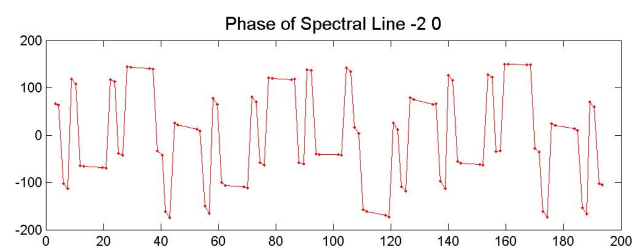












Measurements at Diamond

All BPMs have turn-by-turn capabilities

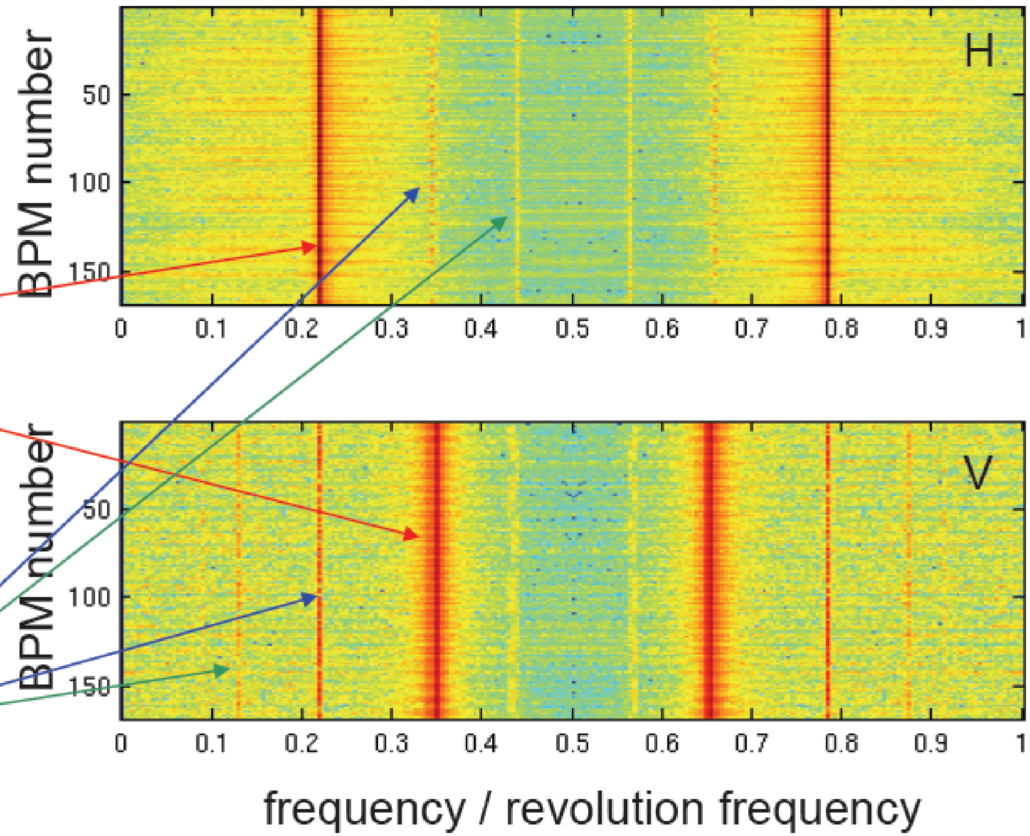
- excite the beam diagonally
- measure tbt data at all BPMs
- colour plots of the FFT

Q_X = 0.22 H tune in H

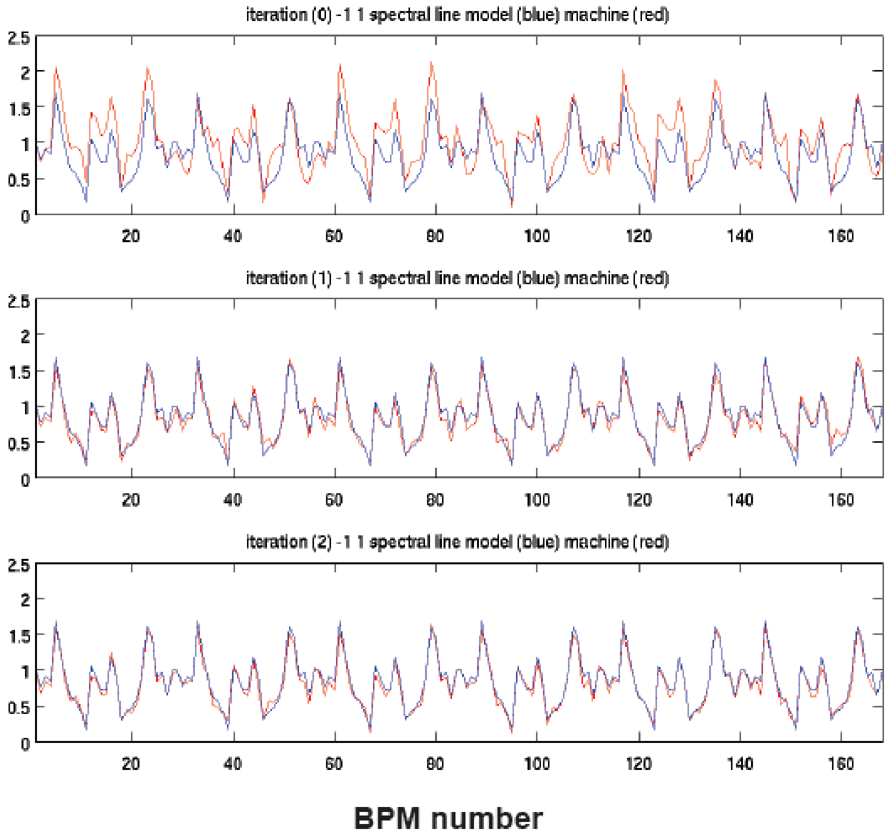
Q_y = 0.36 V tune in V

All the other important lines are linear combination of the tunes Q_x and Q_y

$m\,Q_x + n\,Q_y$



First attempt at correction …



Blue model; red measured

A first attempt to fit the spectral line (-1,1), determined by the resonance (-1,2), improved the agreement of the spectral line with the model

However the lifetime was worse by 15%

The fit produced non realistic large deviation in the sextupoles (>10%);

The other spectral lines were spoiled

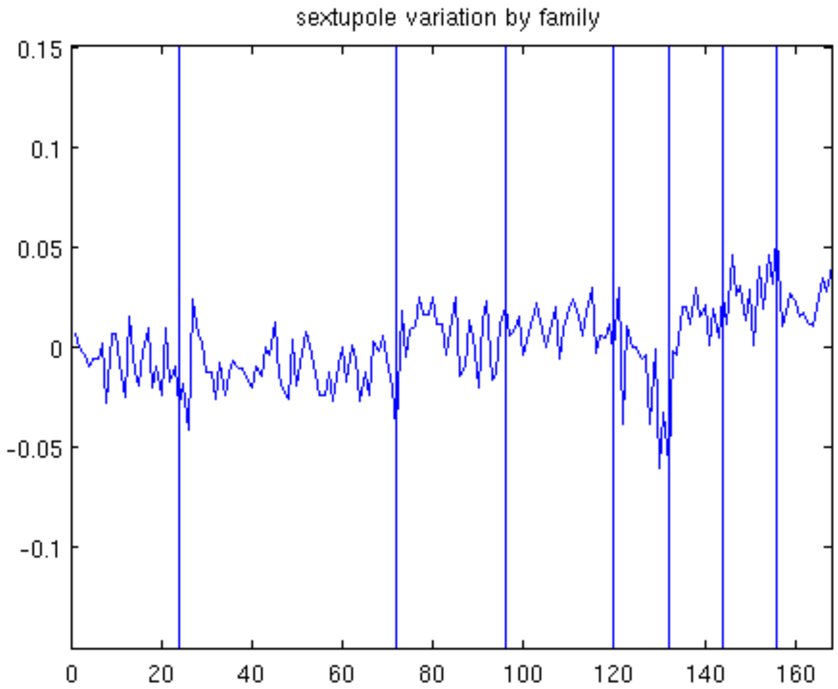
Potential problems could be that:

- problem is underconstrained (too many sextupole knobs, too few BPMs)
- Linear (coupled) lattice errors contributed to variation in driving terms – however correction only involved sextupoles

R. Bartolini

Using multiple resonance lines simultaneously

Simultaneous fit of (-2,0) in H and (1,-1) in V



Now the sextupole variation is limited to < 5%

Both resonances are controlled

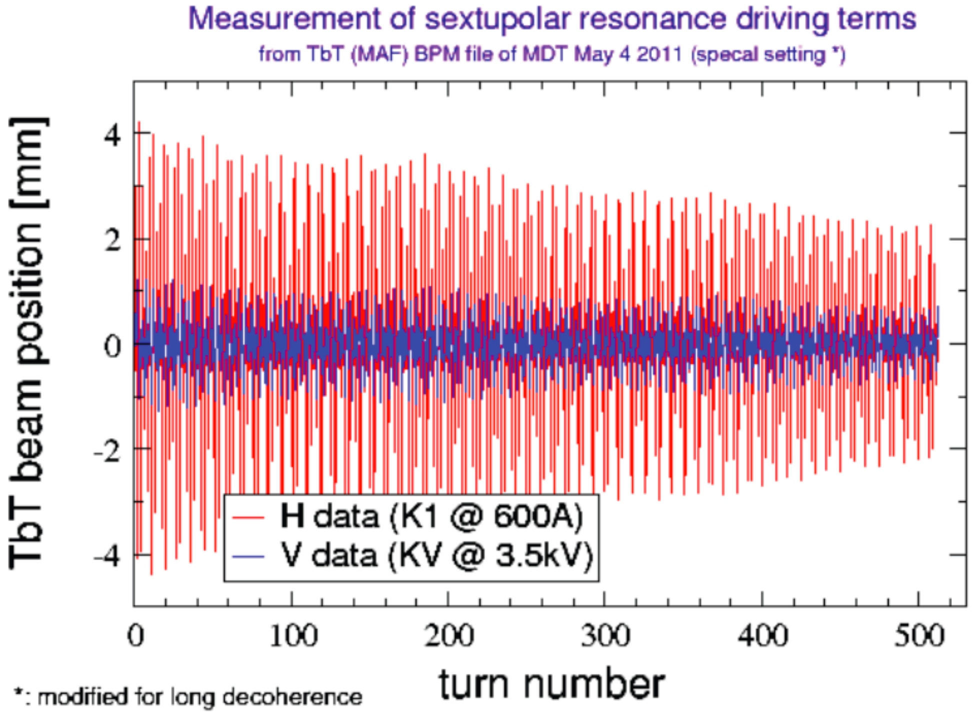
Measured a slight improvement in the lifetime (10%)

Limits: BPM noise, calibration, nonlinearities, frequency dependence + decoherence

R. Bartolini

ESRF example

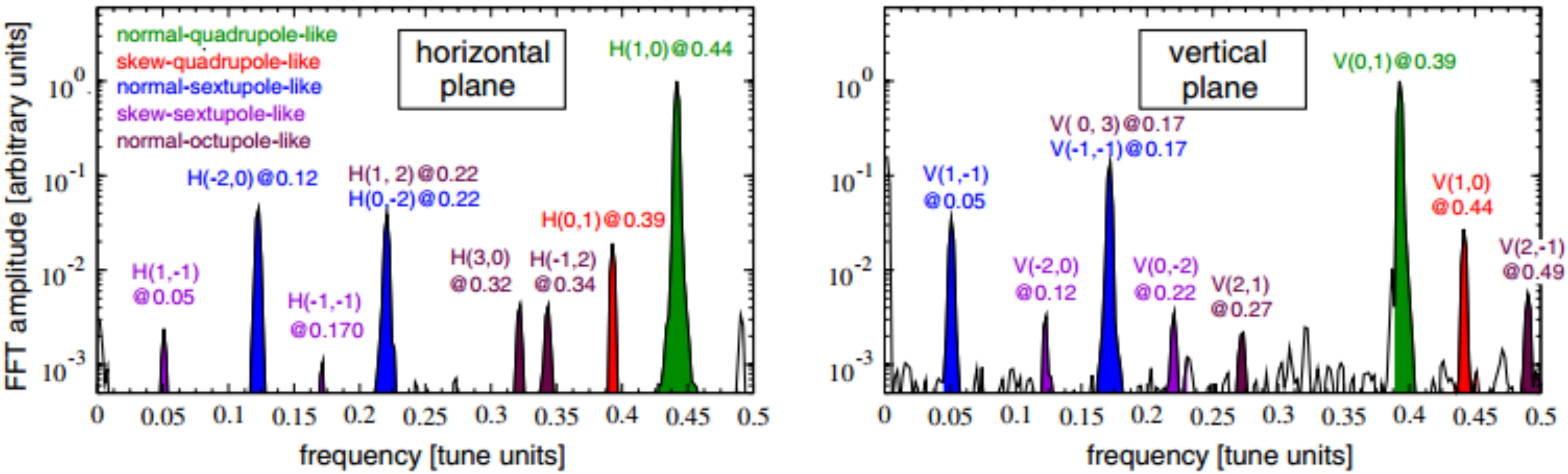
- To overcome BPM resolution and decoherence problem, studying special case: Zero chromaticity, zero detuning with amplitude -> Almost no decoherence -> many turns of BPM data
- Not the case one wants to study, but one can calibrate sextupole strengths here and then apply same correction in nominal lattice



A. Franchi

ESRF example

- Example Spectrum of BPM data is shown below
- Attempt is to reconstruct sextupole strength errors and tilts



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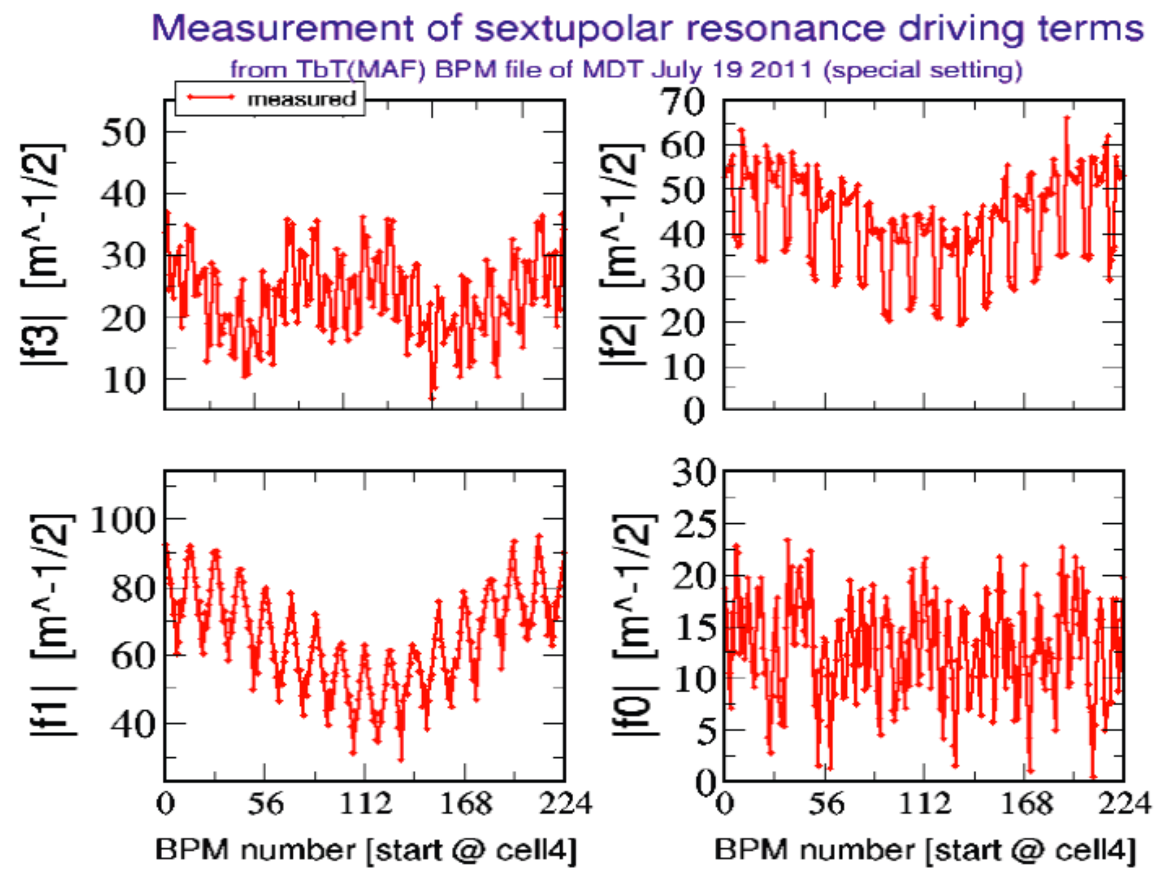
ESRF sextupoles

- Reconstructed sextupole resonance driving terms



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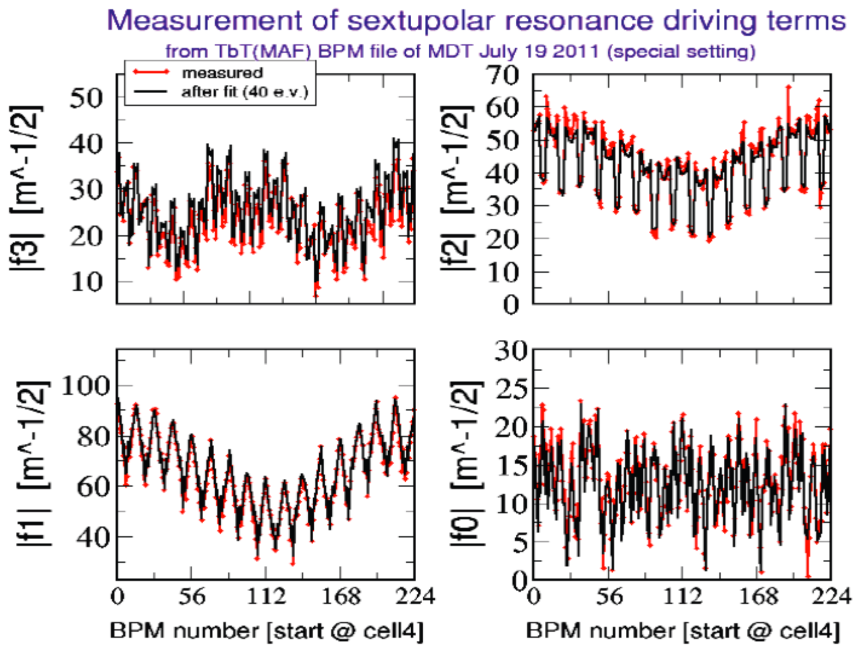
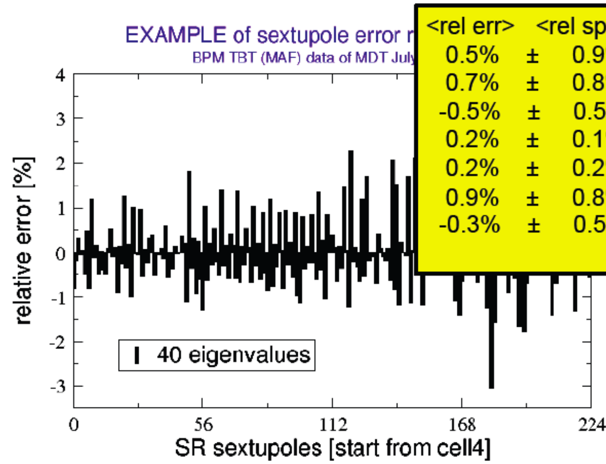
ESRF sextupoles





<rel err>		<rel spread>	sextupole family
0.5%	±	0.9%	S4 (37A)
0.7%	±	0.8%	S6 (-44A)
-0.5%	±	0.5%	S13 (-52A)
0.2%	±	0.1%	S19 (209A)
0.2%	±	0.2%	S20 (-181A)
0.9%	±	0.8%	S22 (-72A)
-0.3%	±	0.5%	S24 (77A)

- After fit of sextupole strength, reached good agreement with model prediction based on calibrated linear lattice (ORM/TBT)
- Sextupole families with bigger error tentatively identified as ones with small error in hysteresis standardization loop

A. Franchi

Summary

- Nonlinear single particle dynamics determine dynamic and momentum aperture
  - Limiting injection efficiency and lifetime
- Nonlinear dynamics can be probed experimentally by tune scans, frequency maps, …
- Dominating lifetime processes differ for different accelerator types
  - Lifetime is in many cases lined to nonlinear dynamics
- Resonance driving term analysis provides quantitative information about nonlinearities in the machine
  - Allows to measure the local distribution of the dominant nonlinearities
  - Theoretically it can provide a method similar to orbit response matrix analysis (or phase advance, …) to correct the nonlinear lattice (sextupoles, …)

Further Reading (Nonlinear Dynamics)

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- W. Decking, and D. Robin, in *Proceedings of the AIP Conference 468*, Arcidosso, Italy, 1998 (Woodbury, New York, 1999), 119–128.
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- J. Safranek, Nucl. Instr & Methods, A388, 27 (1997)
- D. Robin, J. Safranek, and W. Decking, Phys. Rev. ST Accel. Beams 2, 044001 (1999).
- D. Robin, C. Steier, J. Laskar, and L. Nadolski, Phys. Rev. Lett., 85, 3, 558 (2000).
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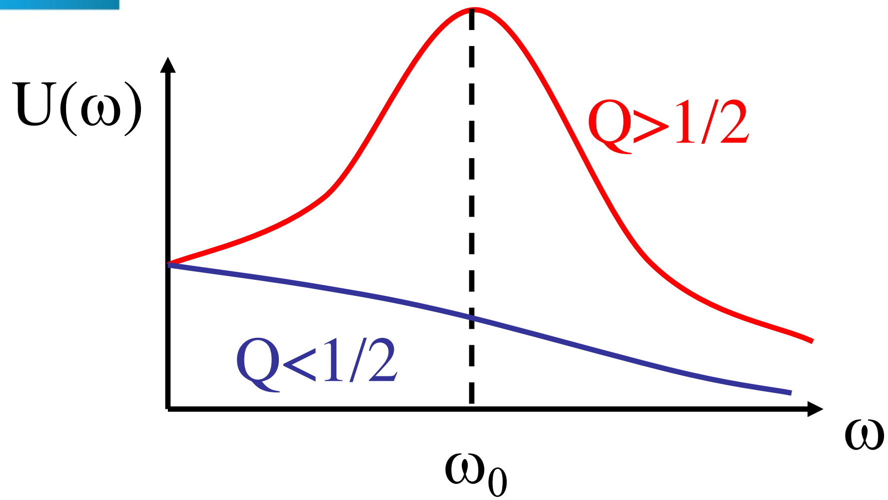
- J. Laskar, in *Proceedings of 3DHAM95 NATO Advanced Study Institutes*, S'Agaro, 1995 (Kluwer Academic Publishers, Dordrecht, The Netherlands, 1999), 134–150.
- C. Steier,D. Robin, J. Laskar, and L. Nadolski, in *Proceedings of the 7th European Particle Accelerator Conference*, Vienna, 2000 (Austrian Academy of Sciences Press, Vienna, 2000), 1077–1079.
- J. Laskar, Physica D, 67, 257-281 (1993)
- C. Steier, et al. Phys. Rev. E 65, 056506 (2002).

Further Reading (Resonance Driving Terms)

- Proceedings of the 2008 workshop on nonlinear dynamics, held at ESRF: http://www.esrf.eu/Accelerators/Conferences/non-linear-beam-dynamics-workshop
- R. Bartolini, et al. 'Measurement of Resonance Driving Terms by Turn-by-Turn Data', Proceedings of PAC 1999, 1557, New York (1999)
- W. Fischer, et al. 'Measurement of Sextupolar Resonance Driving Terms in RHIC', Proceedings of PAC 2003, Portland
- R. Bartolini and F. Schmidt, LHC Project note 132, Part. Accelerators. 59, pp. 93-106, (1998).
- F. Schmidt, R. Tomas, A. Faus–Golfe, CERN-SL-2001-039-AP Geneva, CERN and IPAC 2001.
- M. Hayes, F. Schmidt, R. Tomas, EPAC 2002 and CERNSL-2002-039-AP Geneva, 25 Jun 2002.
- F. Schmidt, CERN SL/94-56 (AP) Update March 2000.
- R. Bartolini and F. Schmidt, CERN SL–Note–98–017 (AP).
- A. Franchi, private communication 2012, to be published
- A. Franchi, et al, PRSTAB 17, 074001 (2014)

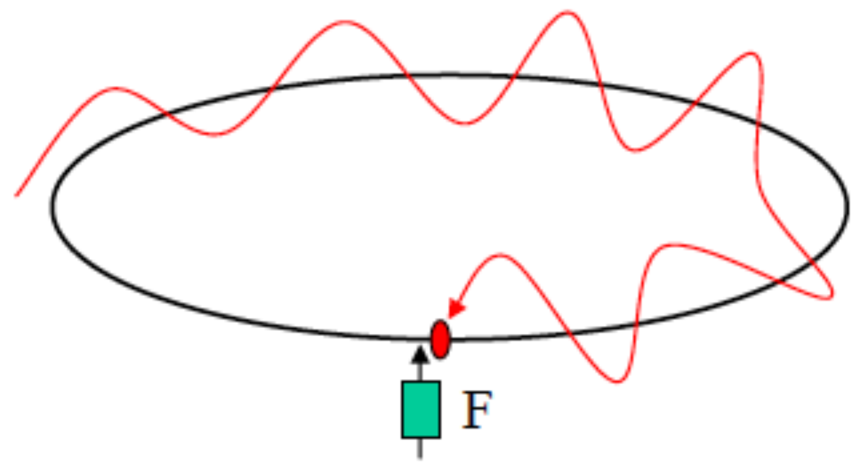
Backup Slides

Resonance effects in Accelerators



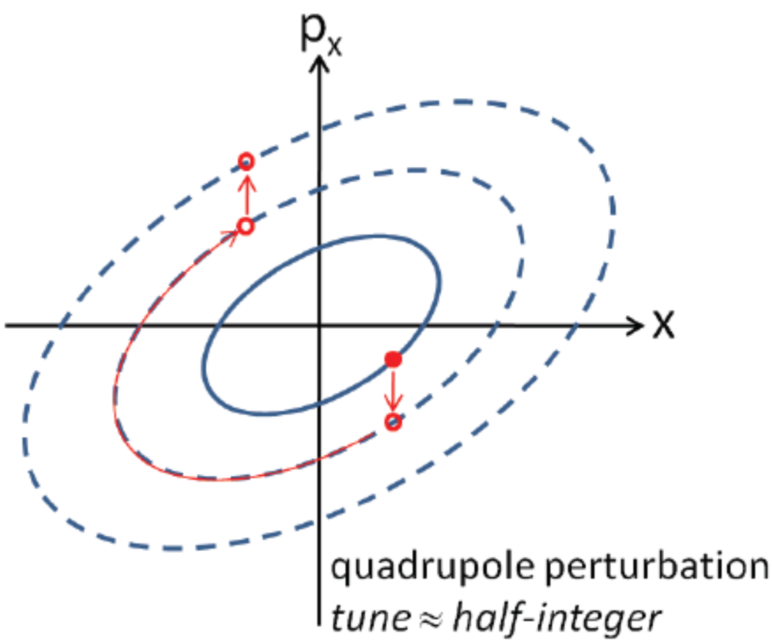
$$U(\omega) = \frac{U(0)}{\sqrt{(1-(\frac{\omega}{\omega_0})^2)^2 + (\frac{\omega}{Q\omega_0})^2}}$$

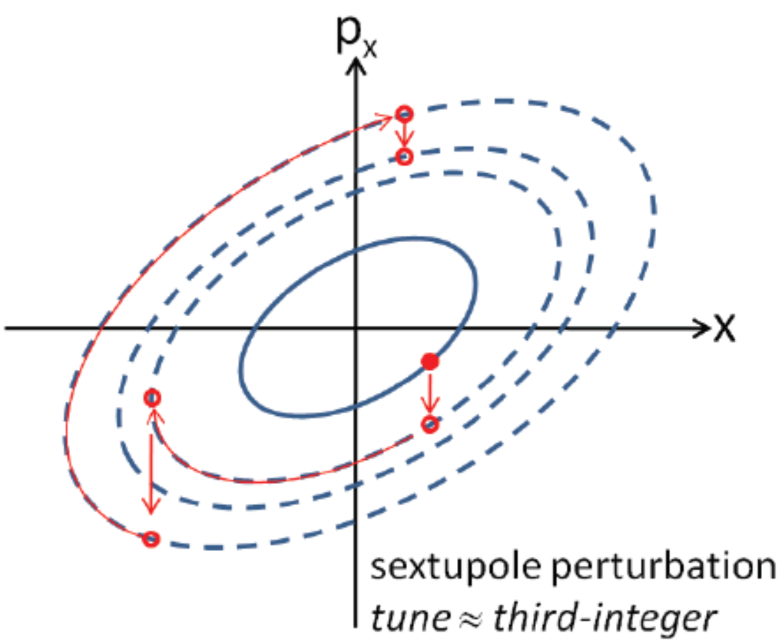
- Driven harmonic oscillator
  - periodic excitations
  - frequency of excitation determined by external source
- Betatron oscillations
  - Excitation due to field error, fixed in space (and usually not time dependent)
  - Excitation frequency is determined by oscillation frequency of beam particles
- Both result in similar driven resonances



Resonances in Phasespace

- A quadupole perturbation (i.e. kick linearly dependent on position) quickly increase betatron amplitude near half-integer resonance, sextupole perturbation (i.e. kick depends quadratically on position) drives third-integer resonance.





NAFF algorithm – J. Laskar (1988)
(Numerical Analysis of Fundamental Frequencies)

Given the quasi–periodic time series of the particle orbit $(x(n); p_x(n))$,

- Find the main lines in the signal spectrum

  $\Rightarrow \nu_1$ frequency, a_1 amplitude, ϕ_1 phase;

- build the harmonic time series

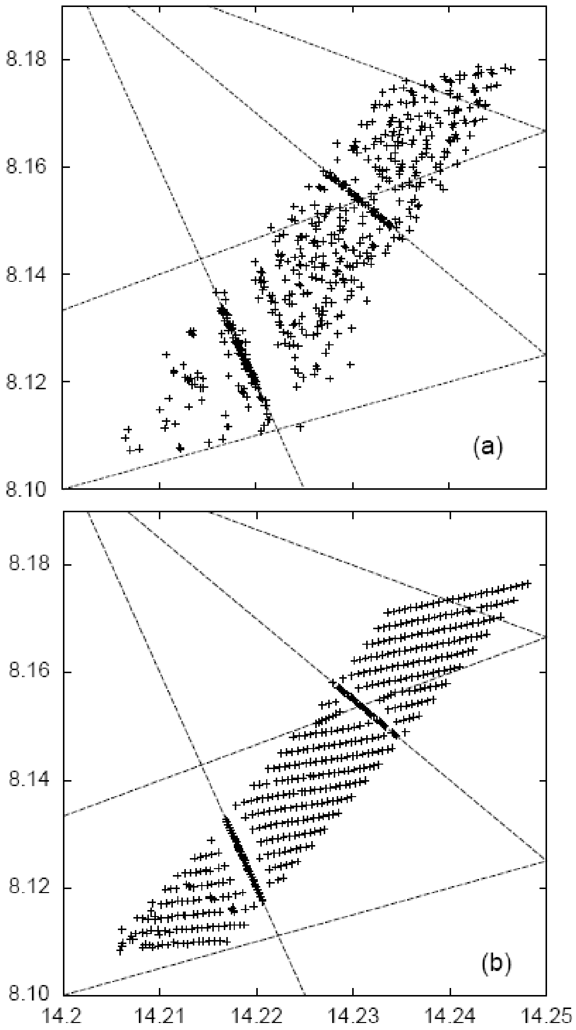
$$z_1(n) = a_1 e^{i\phi_1} e^{2\pi i \nu_1 n}$$

- subtract from the original signal
- analyze again the new signal $z(n) - z_1(n)$ obtained

The decomposition $z(n) = \sum_{k=1}^{n} a_k e^{i\phi_k} e^{2\pi i \nu_k n}$ **allows the**

Measurement of Resonant driving terms of non linear resonances

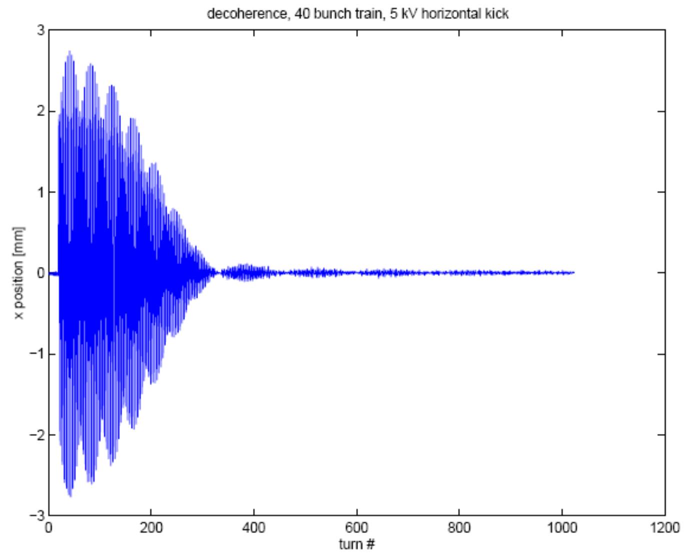
Measured Frequency Map

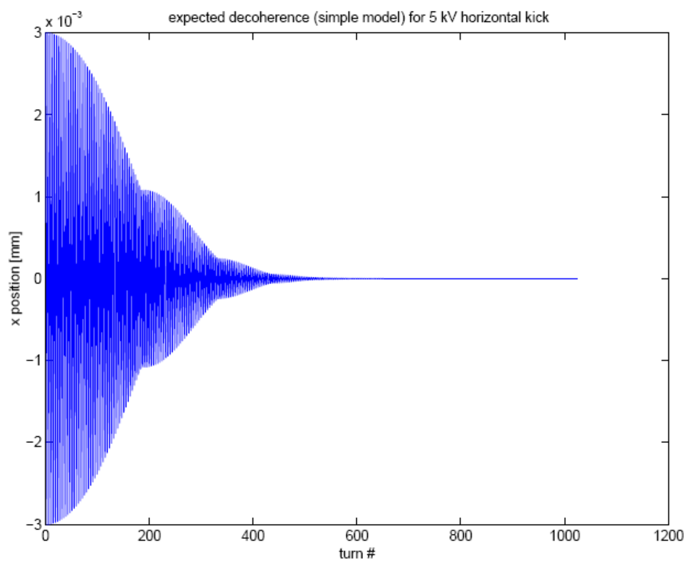


- excellent agreement, using calibrated model (gradient errors), random skew errors, nominal sextupoles

Phys. Rev. Lett. 85, 3, (July 2000), pp.558-561

Fast Decoherence Problem for Experiment





- Detuning with amplitude causes very fast decoherence for larger amplitudes
- Individual particles are still oscillating with same amplitude (radiation damping time >10k turns)
- Makes frequency analysis difficult
  - Small number of turns
  - Signal not quasiperiodic

Gas Lifetime – Vacuum Requirements

- For electrons one can simplify the formulas for gas Bremsstrahlung lifetime (in the approximation of $\langle Z^2 \rangle \sim 50$):

$$\tau_{Brem[hours]} \cong -\frac{153.14}{\ln(\Delta E_A / E_0)} \frac{1}{P_{[nTorr]}}$$

- In the same approximation, the elastic gas scattering lifetime becomes:

$$\tau_{Gas[hours]} \cong 10.25 \frac{E^2_{0[GeV]}}{P_{[nTorr]}} \frac{\varepsilon_{A[\mu m]}}{\langle \beta_T \rangle_{[m]}}$$

- For typical electron ring parameters, one finds that the requirement on vacuum is for dynamic pressures of the order of a few nTorr.

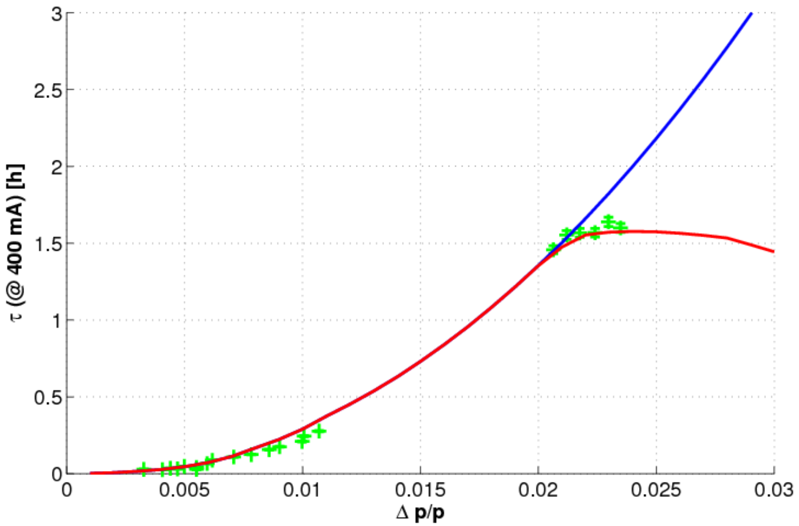
Off-energy dynamics: Touschek Lifetime

- Lifetime is crucial performance parameter for light sources ⇒ for 3rd generation light sources limit is Touschek lifetime ⇒ strong function of momentum aperture ε

$$\frac{1}{\tau_{tou}} \propto \frac{1}{E^3} \frac{I_{bunch}}{V_{bunch}\sigma_x'} \frac{1}{\varepsilon^2} f\left(\varepsilon, \sigma_x', E\right)$$

- Momentum aperture ε is often limited by single particle dynamics
- 3rd generation light sources with their strong focusing to achieve small equilibrium emittances (small dispersion) and very strong sextupoles did originally not achieve their design momentum apertures of about 3%.

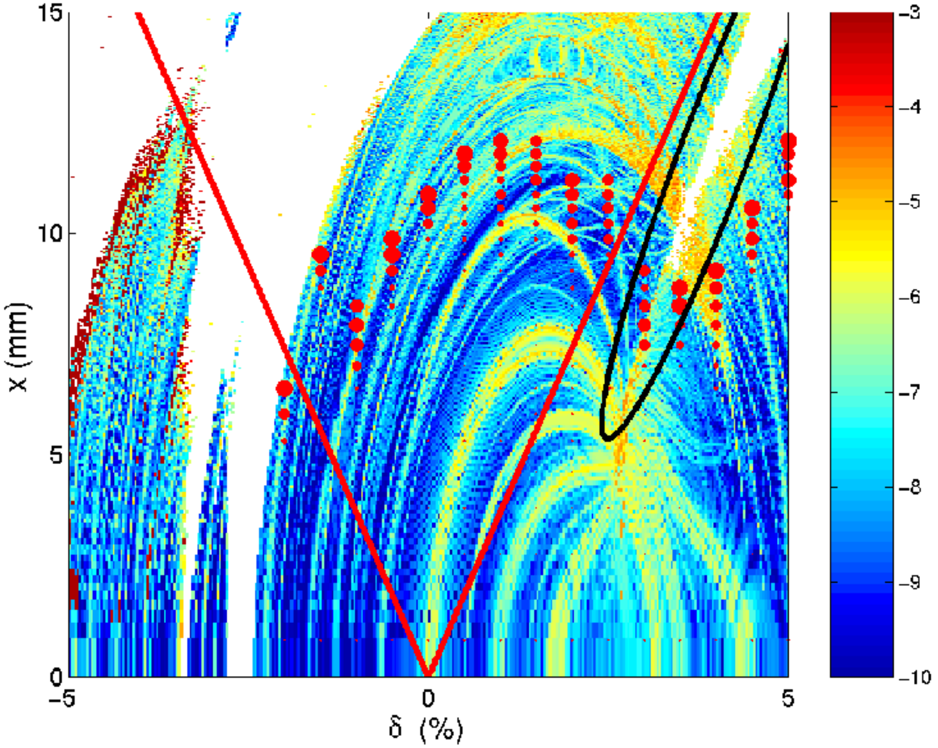
Touschek lifetime scans

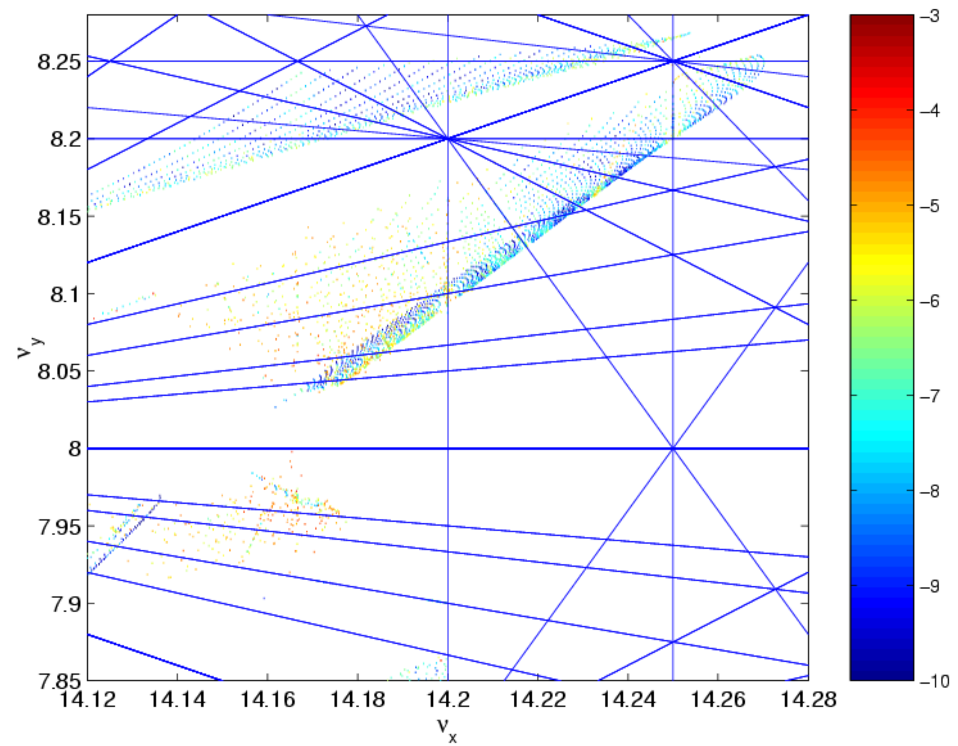


- Calculate RF voltage dependent Touschek lifetime based on calibrated machine model (emittance, beamsized, lattice function, s-position dependent dynamic momentum aperture all calculated from calibrated model)

- Compare measurements (green errorbars) with those calibrated calculation
  - Excellent Agreement

Results agree well with Simulations





- Simulations reproduce shift of beam loss area caused by the coupling resonance to higher momentum deviations

Limits of resonance driving term measurements

BPMs precision in turn by turn mode (+ gain, coupling and non-linearities)

10 μm with ~10 mA

very high precision required on turn-by-turn data (not clear yet is few tens of μm is sufficient); Algorithm for the precise determination of the betatron tune lose effectiveness quickly with noisy data. R. Bartolini et al. Part. Acc. 55, 247, (1995)

BPM gain and coupling can be corrected by LOCO, but nonlinearities remain (especially for diagonal kicks)

Decoherence of excited betatron oscillation reduce the number of turns available
Studies on oscillations of beam distribution shows that lines excited by resonance of order m+1 decohere m times faster than the tune lines. This decoherence factor m has to be applied to the data R. Tomas, PhD Thesis, (2003)