USPAS 2019: Knoxville

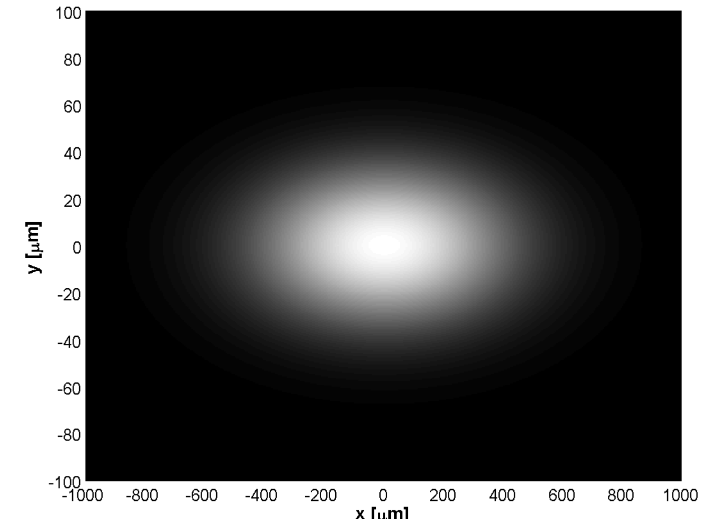
Coupling Measurement and Correction

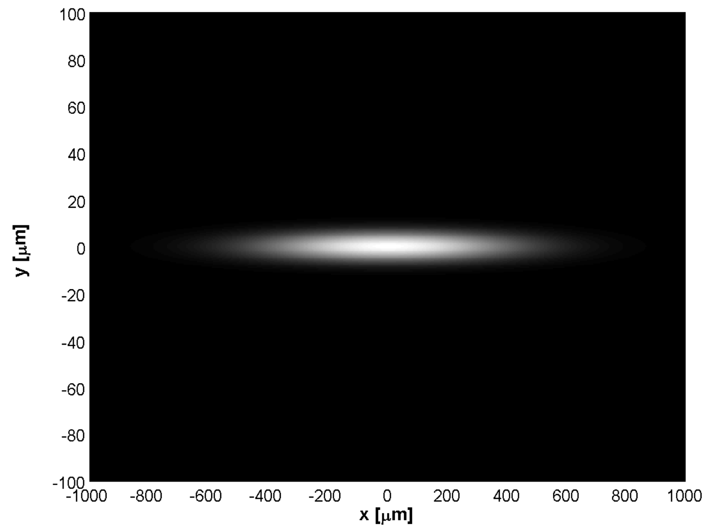
Christoph Steier

Lawrence Berkeley National Laboratory

Outline

- Introduction/Motivation
- Coupling Description
- Correction Methods
- Measurement / Coupled Model Calibration
  - Optimizing the correction algorithm
- Experimental Results
- Summary





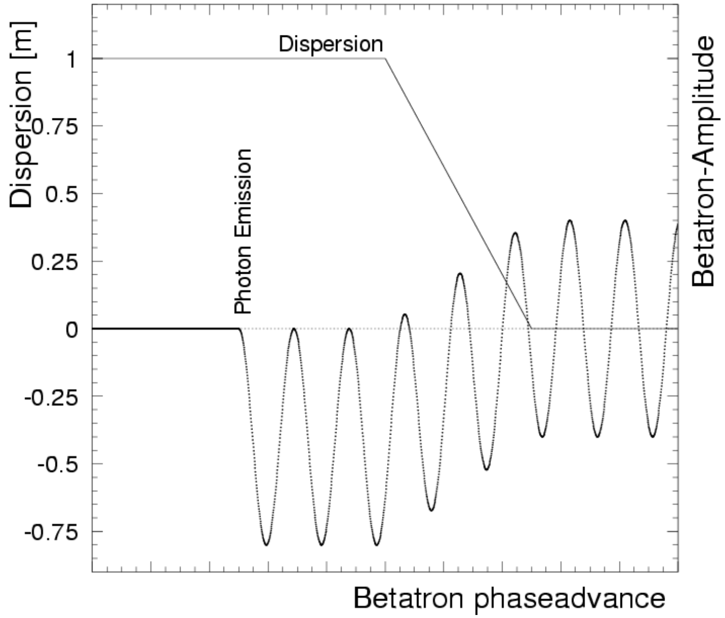
Motivation: Reducing Vertical Emittance

Vertical emittance of ideal, flat accelerator is very small (for ALS of order of 0.5 pm) – correcting coupling errors can help to significantly increase brightness, luminosity, dynamic and momentum aperture, etc.

- Simplest coupling errors are tilts of quadrupoles and offsets in sextupoles
  - These errors cause:
    1. Global coupling
    2. Local coupling
    3. Vertical dispersion
- To optimize performance, all three effects have to be corrected
  - Methods include orbit manipulation, skew quadrupoles, moving of sextupoles, …
- Most successful strategy at light sources: Do not target the three quantities individually, instead use combined approach

Reminder: Quantum excitation

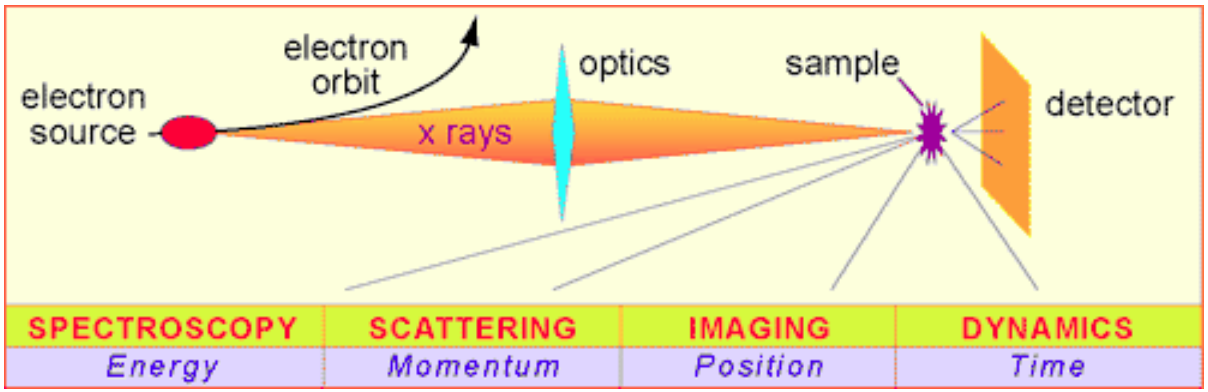
Particles, which change their energy in a region of dispersion due to the emission of a photon, will see an increase of their transverse oscillation amplitude. The balance of quantum excitation and radiation damping gives the equilibrium emittances.



For an ideal, flat accelerator the vertical dispersion is zero, i.e. there is an extremely small vertical emittance. In an accelerator with errors, coupling and spurious vertical dispersion increase the vertical emittance.

Experiments requiring small vertical emittance

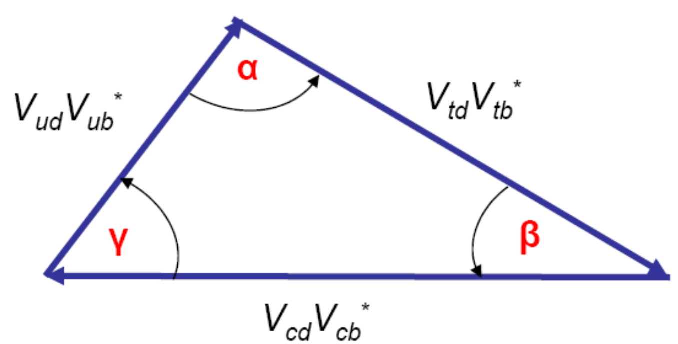
- Synchrotron light sources: High brightness (photon flux/sizes/divergences) enables high resolution experiments and provides partial transverse coherence



- Colliders: Particle physics experiments require high statistics – high luminosity – small vertical beamsize at IP

$$L = \frac{f N_1 N_2}{4\pi\sigma_x\sigma_y}$$

$$R = L\sigma_{total}$$



Coupled Transfer Matrix

Skew quadrupole field errors generate betatron coupling between horizontal and vertical equations of motion.

4x4 transfer matrix for a quadrupole rotated by a small angle φ

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{\text{final}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -k & 1 & -2k\varphi & 0 \\ 0 & 0 & 1 & 0 \\ -2k\varphi & 0 & k & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{\text{initial}}$$

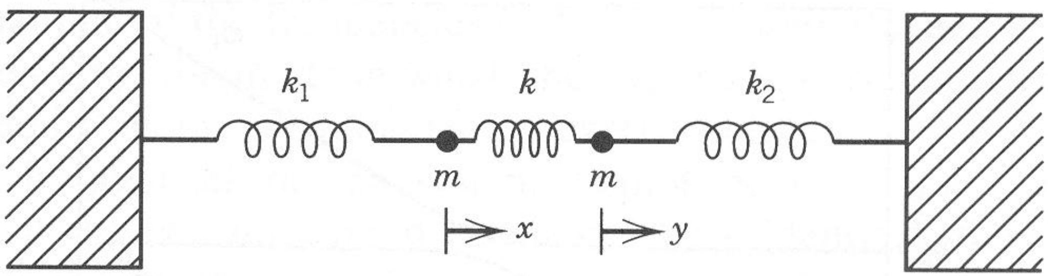
Local/Global Coupling, Vertical Dispersion

- Coupled (Hills) equations of motion :

$$x'' - Kx = -K_s y \qquad y'' + Ky = -K_s x$$

- With $K = \frac{1}{B\rho}\frac{\partial B_y}{\partial x} \qquad K_s = \frac{1}{B\rho}\frac{\partial B_x}{\partial x}$

- Analogy with mechanical coupled harmonic oscillators (with springs)



$$m\ddot{x} + (k_1 + k)x - ky = 0,$$

$$m\ddot{y} + (k_2 + k)y - kx = 0,$$

Resonance Description of Global Coupling

- Global coupling is typically described using a resonance theory
- Difference coupling resonance

$$\kappa = \frac{1}{4\pi}\int ds\, K_s \sqrt{\beta_x \beta_y}\, e^{i\phi_D}$$

$$\frac{\phi_D}{2\pi} = \mu_x(s) - \mu_y(s) - \frac{s}{C}\Delta_r \qquad \Delta_r = (\nu_x - \nu_y - N)$$

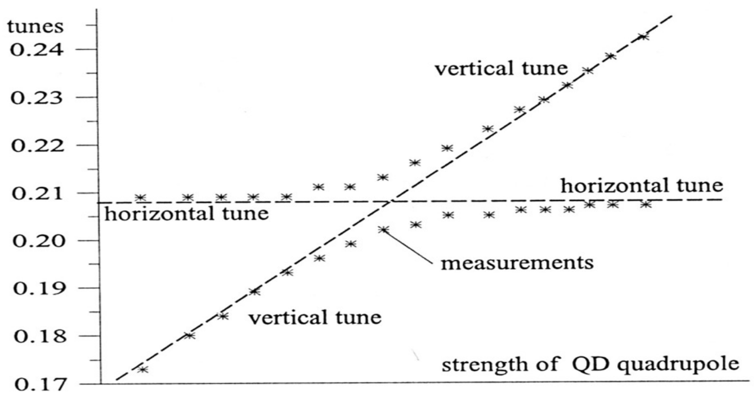
  - Vertical emittance near difference resonance:

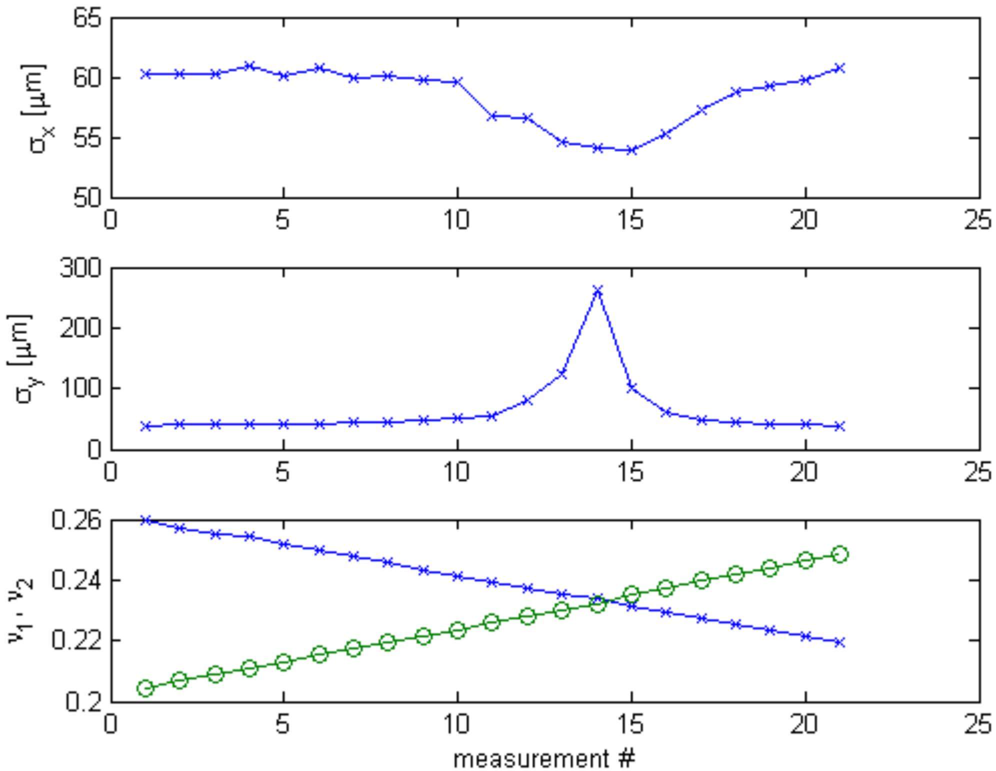
$$\frac{\varepsilon_y}{\varepsilon_x} = \frac{|\kappa|^2}{|\kappa|^2 + \Delta_r^2/2}$$

κ is resonance strength, Δ_r is distance from resonance.

Scan of Difference Resonance

- There are sum resonances as well (phase advance proportional to sum of horizontal and vertical phase advance) and of course higher order resonances.
- One can create orthogonal knobs of skew quadrupoles directly acting on one of those coupling





- Minimum tune split (on resonance):

$$(\nu_x - \nu_y)_{\min} = 2|\kappa|$$

Normal mode Analysis: C matrix

- On Monday, we only discussed the uncoupled case (and mostly looked at 2x2 matrices). If there are coupling errors, one can do a so-called normal mode analysis (diagonalizing matrix)
- Start with 4x4, one-turn matrix $R_{\text{one-turn}}$, which maps the 4 transverse coordinates **x**=(x,x',y,y'). Normal mode form:

$$\mathbf{R}_{\text{one-turn}} = \mathbf{V}\mathbf{U}\mathbf{V}^{-1}, \text{normal mode matrix } \mathbf{U} = \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{pmatrix},$$

$$\text{with } \mathbf{A} = \begin{pmatrix} \cos\phi_a + \alpha_a \sin\phi_a & \beta_a \sin\phi_a \\ -\gamma_a \sin\phi_a & \cos\phi_a - \alpha_a \sin\phi_a \end{pmatrix},$$

$$\mathbf{V} \text{ is of the form (Edwards + Teng)}$$

$$\mathbf{V} = \begin{pmatrix} \gamma\mathbf{I} & \mathbf{C} \\ -\mathbf{C}^{+} & \gamma\mathbf{I} \end{pmatrix},$$

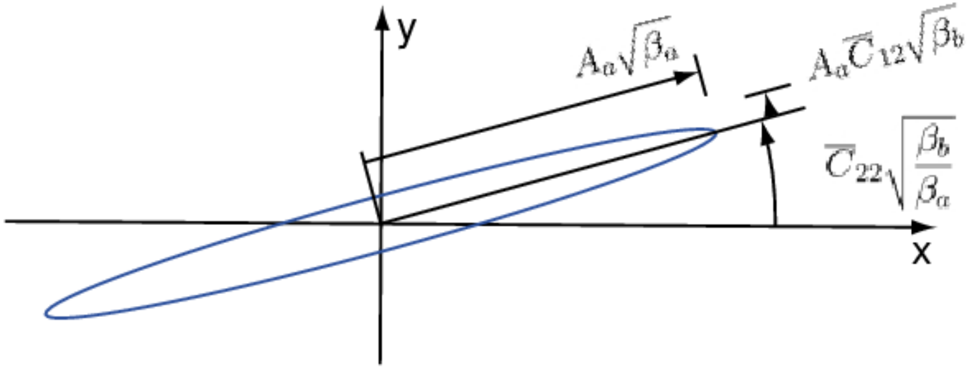
$$\text{with } \gamma^2 + \left\|\overline{\mathbf{C}}\right\| = 1. \text{The magnitude of C is a measure of local coupling.}$$

Local Coupling

Often the normalized matrix $\overline{\mathbf{C}}$ is used :

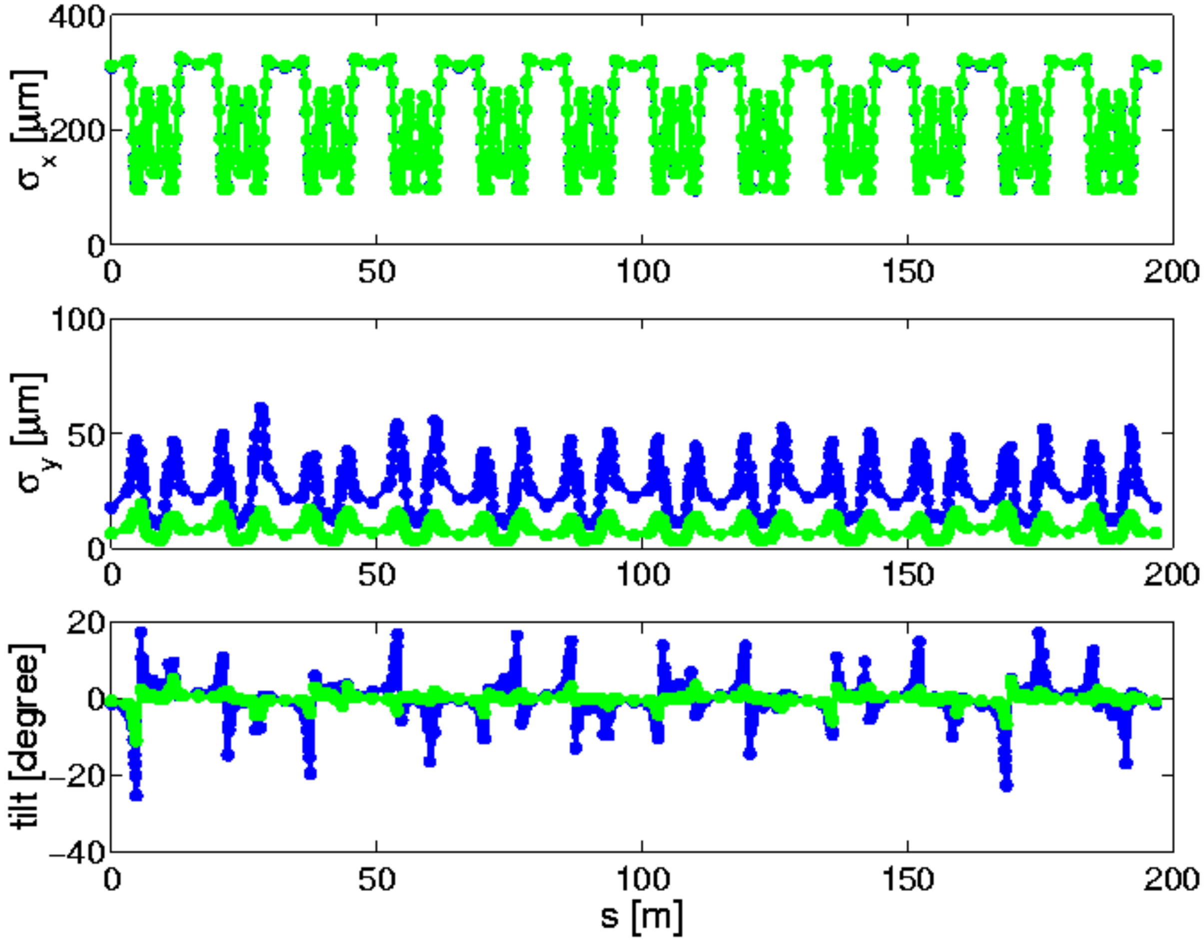
$$\overline{\mathbf{C}} \equiv \mathbf{G}_a \mathbf{C} \mathbf{G}_b^{-1}, \text{where } \mathbf{G}_a = \begin{pmatrix} \frac{1}{\sqrt{\beta_a}} & 0 \\ \frac{\alpha_a}{\sqrt{\beta_a}} & \sqrt{\beta_a} \end{pmatrix}.$$

- Locally there is torsion in addition to the global invariant vertical emittance, resulting in a larger projected emittance:



- Again driving terms scale like the sqrt of the product of the beta functions at the location of the skew errors.

ALS Example: Local Coupling



- Even for very well corrected global coupling, local coupling can still be significant (as shown here by local tilt angles).
- Projected emittance can change significantly around ring.

Vertical Dispersion

- There are two main terms that can create vertical dispersion:

$$\eta_y'' + K\eta_y = \frac{1}{\rho_y} - K_s\eta_x$$

  - Dipole errors (steering magnets, misalignments, …) or intentional vertical bending magnets
  - Skew quadrupole fields at the location of horizontal dispersion (due to quadrupole tilts, or vertical offsets in sextupoles)

$$\kappa_{\eta_y} = \int ds\, K_s \eta_x \sqrt{\beta_y}\, e^{i\phi_{\eta_y}}$$

$$\frac{\phi_{\eta_y}}{2\pi} = \mu_y(s) - \frac{s}{C}(\nu_y - 5)$$

- Vertical dispersion directly causes increase of the vertical emittance by quantum excitation

Correction Techniques

- One can correct the three coupling effects using skew quadrupoles, vertical offsets (movers or orbit bumps) in sextupoles, steering magnets, …
- The corrections can either target global quantities, local quantities at individual points of the ring, or local quantities everywhere.
  - Distribute in difference coupling resonance phase

$$\kappa = \frac{1}{4}\pi\int ds K_s \sqrt{\beta_x \beta_y} e^{i\Phi_D} \qquad \frac{\Phi_D(s)}{2\pi} = (\mu_x(s) - \mu_y(s)) - \frac{s}{C}(\nu_x - \nu_y - N)$$

  - In sum coupling resonance phase

$$\frac{1}{4}\pi\int ds K_s \sqrt{\beta_x \beta_y} e^{i\Phi_S} \qquad \frac{\Phi_S(s)}{2\pi} = (\mu_x(s) + \mu_y(s)) - \frac{s}{C}(\nu_x + \nu_y - M)$$

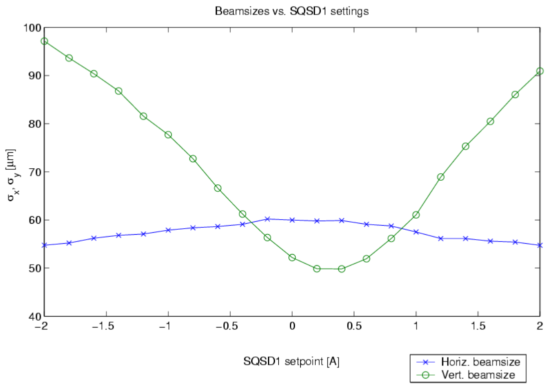
  - And in η_y phase

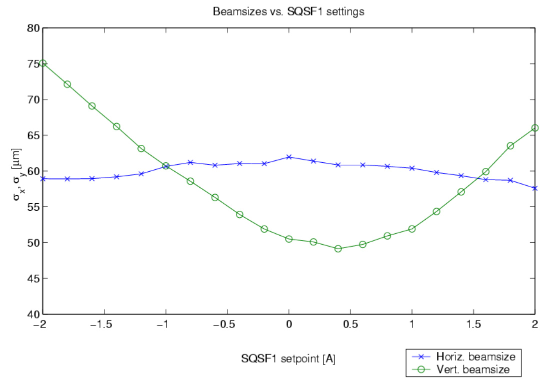
$$\eta_x \sqrt{\beta_y} e^{i\Phi_{\eta_y}} \qquad \frac{\Phi_{\eta_y}(s)}{2\pi} = \mu_y(s) - \frac{s}{C}(\nu_y - N)$$

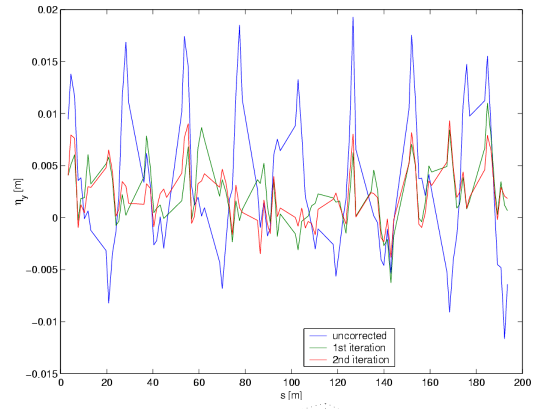
Need some skew quadrupoles at non-zero η_x

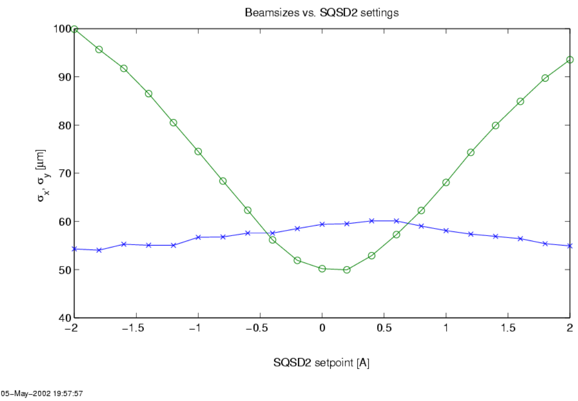
Unsuccessful Correction Attempts … for ALS

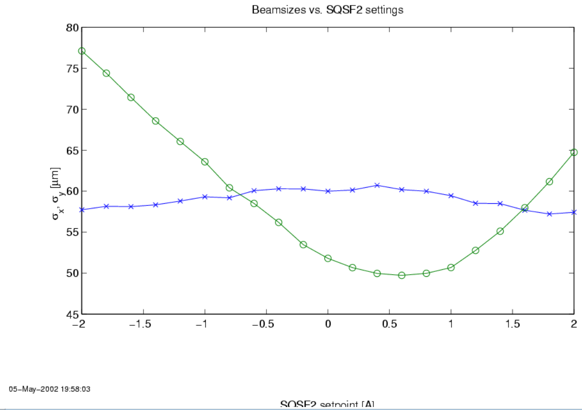
- Originally, we tried three different approaches at ALS:
  1. Coupling correction with skew quad chains (single resonance)
  2. Dispersion correction using orbit correctors (TBA, chromaticity)
  3. Dispersion correction using skew quadrupoles without minimizing the coupling simultaneously

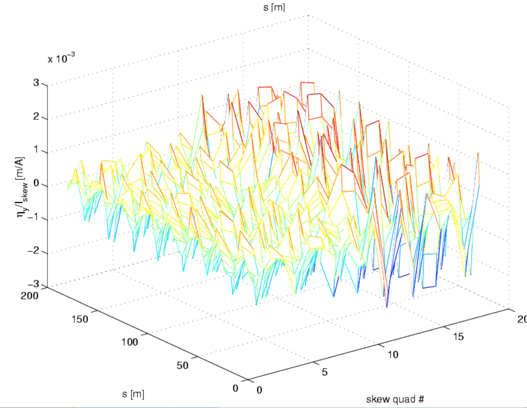












Separated approaches can work …

- In case of FODO lattices, or if there are no user requirements to keep orbit fixed, the separated approach of coupling correction can work well (i.e. in colliders):
  - FODO lattice is simple and allows dispersion correction via orbit correction/bumps.
  - In addition one can often minimize the global coupling with only four orthogonal skew (families) mostly independently from dispersion.
  - The local coupling in most colliders is only relevant at the interaction point and can be compensated there with a few local skew quadrupoles.
- However, usually the integrated approach is better and next part of lecture will focus on it:
  - Targeting local and global coupling, as well as vertical dispersion simultaneously
  - Method has proven extremely powerful at light sources.

Integrated coupling correction

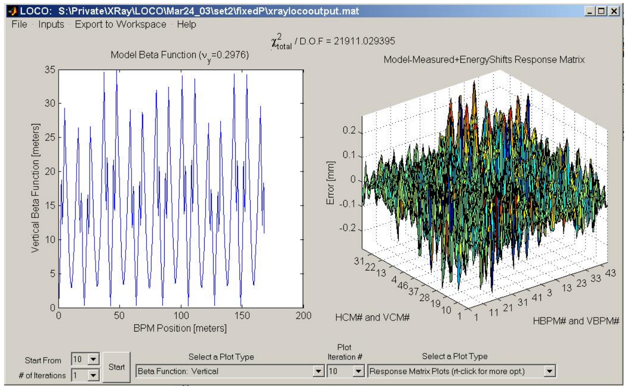
Use accelerator toolbox, Matlab and LOCO for simulations (see lecture by James on LOCO)

- Simulate many random skew error seeds
- Try to find effective skew corrector distributions and to optimize correction technique in simulation, using two correction approaches:
  1. Response Matrix fitting – 'deterministic', small number of iterations
  2. Direct minimization (nelder-simplex, …) – easy to do on the model, would be difficult on real machine

Surprisingly both approaches gave about the same performance in the model calculations

- For response matrix analysis you have to optimize several parameters of the code as well (weight of dispersion, number of SVs, use of effective model/full model …)

Recap: LOCO Method



The orbit response matrix is defined as

$$\begin{bmatrix} \vec{x} \\ \vec{y} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \vec{\theta}_x \\ \vec{\theta}_y \end{bmatrix}$$

The parameters in a computer model of a storage ring are varied to minimize the χ^2 deviation between the model and measured orbit response matrices (M_{mod} and M_{meas}).

$$\chi^2 = \sum_{i,j} \frac{(M_{ij}^{\text{meas}} - M_{ij}^{\text{model}})^2}{\sigma_i^2} \equiv \sum_{k=i,j} E_k^2$$

The σ_i are the measured noise levels for the BPMs; E is the error vector.

The χ^2 minimization is achieved by iteratively solving the linear equation

$$E_k^{new} = E_k + \frac{\partial E_k}{\partial K_l} \Delta K_l = 0$$

Adjust fit parameters (like gradients, …) until calculated response matrix matches measured one

$$-E_k = \frac{\partial E_k}{\partial K_l} \Delta K_l$$

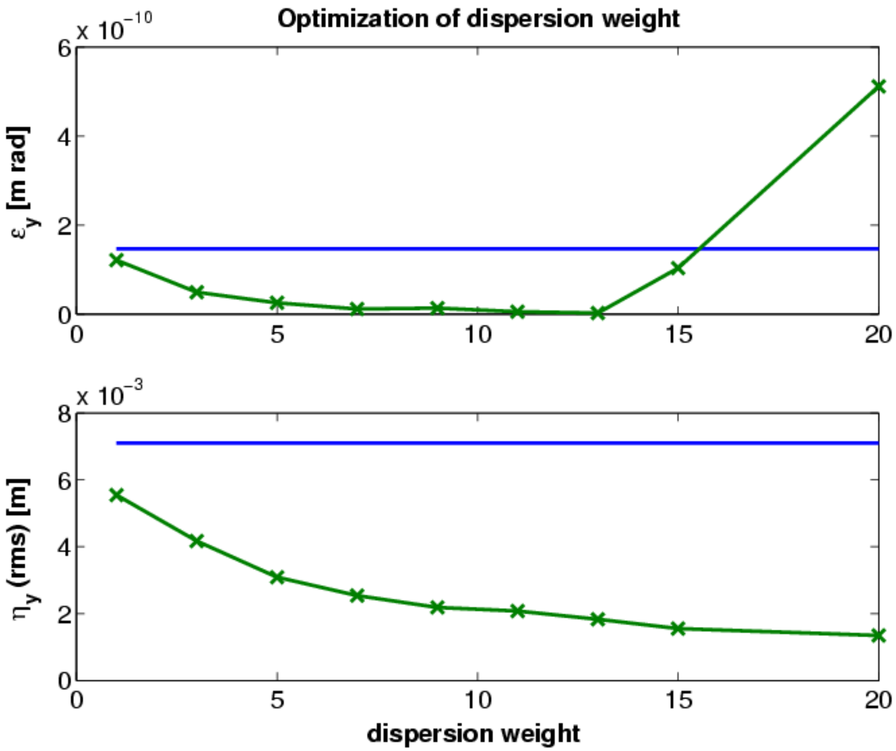
$$\frac{\partial E_k}{\partial K_l} = J_{kl} = Jacobian$$

For the changes in the model parameters, K_l, that minimize $||E||^2 = \chi^2$.

Gauss-Newton minimization.

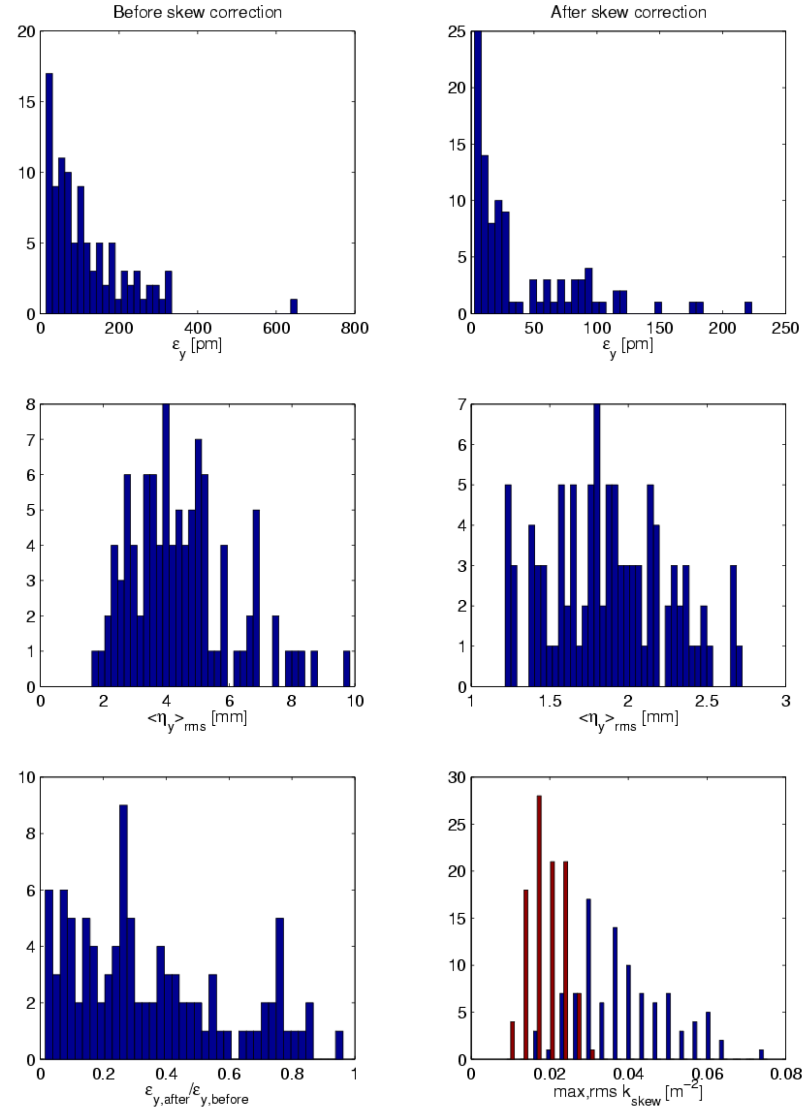
Weight of dispersion in LOCO fit

- The relative contribution of vertical dispersion and coupling to the vertical emittance depends on the particular lattice (and the particular error distribution).
- Therefore the optimum weight for the dispersion in the LOCO fit has to be determined (experimentally or in simulations).
- The larger the weight factor, the better the vertical dispersion gets corrected, but eventually the coupling ‘explodes’.
- Set weight to optimum somewhat below that point.



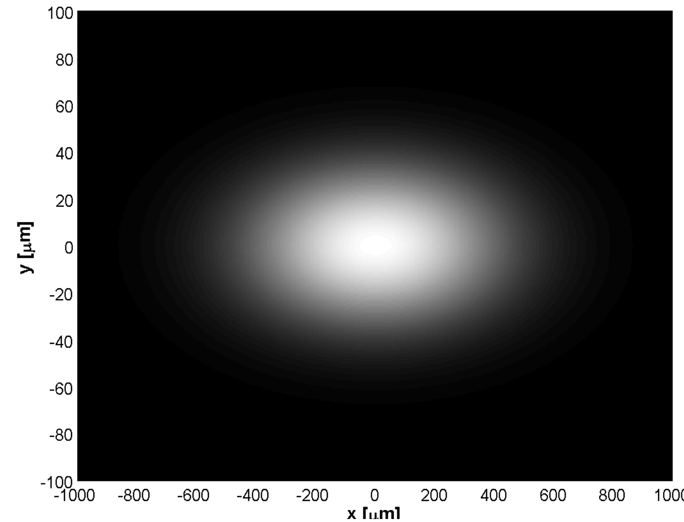
Finding an Effective Skew Quadrupole Set

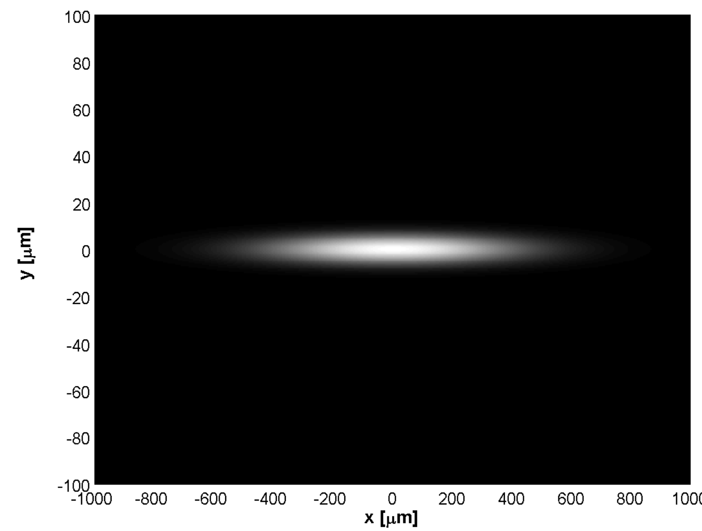
- To find an effective skew quadrupole distribution, we used several correction methods, first in simulations – best method was orbit response matrix fitting (using LOCO)
- Predictive method, can be easily used on real machine
- Issues are:
  - Cover set of phases relative to dominant coupling resonance(s)
  - Magnets should be distributed around the ring in order to avoid excessive local coupling/vertical dispersion
  - Need different values of dispersion/beta function to be effective both for coupling and vertical dispersion correction
- Set of 12 skew quadrupoles was reasonably efficient



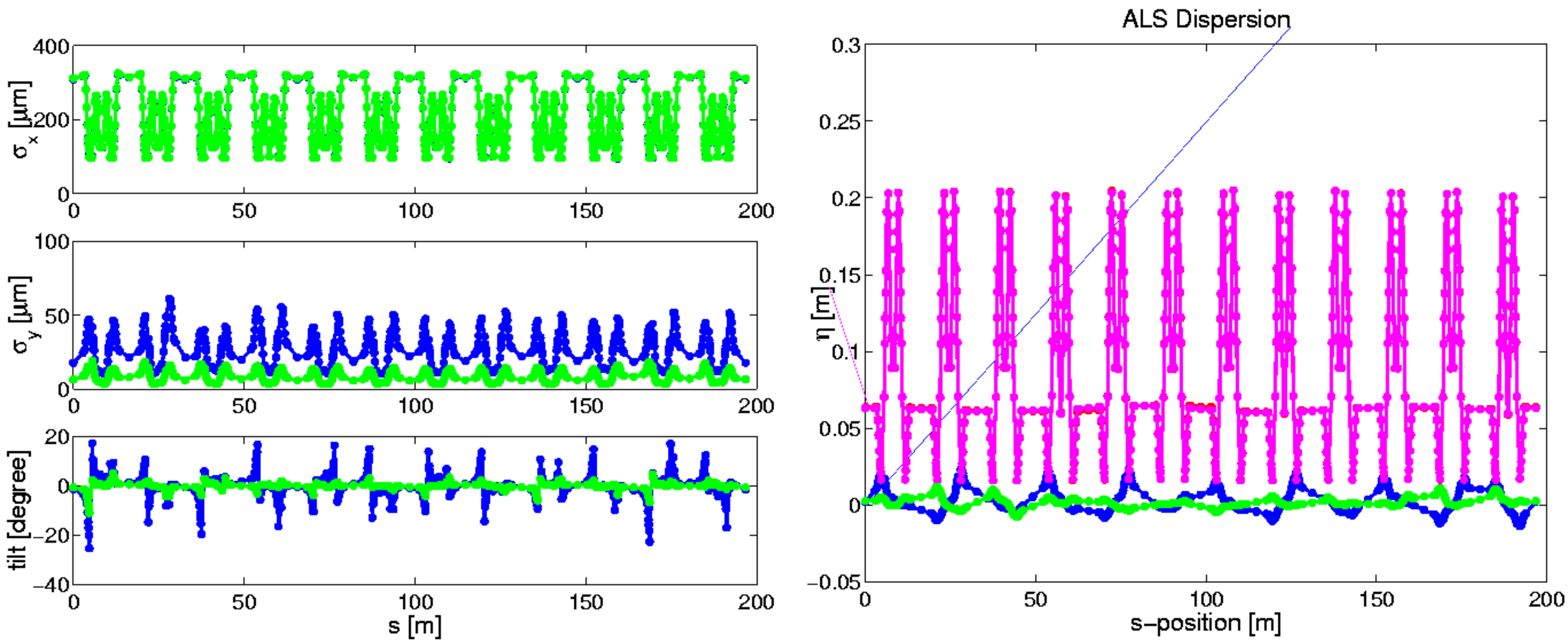
ALS example: Emittance Reduction

- Achieved an emittance reduction from 150 pm (routine ALS operation) to about 4 pm (pictures on the right illustrate size reduction for insertion device straights).
- Touschek lifetime requires to not make full use of the possibility: Nowadays in top-off we operate at 20-40 pm.
- 4 pm was a world record in 2003 and about the NLC damping ring design value
- Correspondingly the brightness would increase by factor 30 (for hard x-rays – because of diffraction limit less for soft x-rays)
- Recently repeated with more skew quadrupoles, now reach values below 1 pm, close to the ‘quantum limit’.





ALS correction example: Sizes, tilt, dispersion

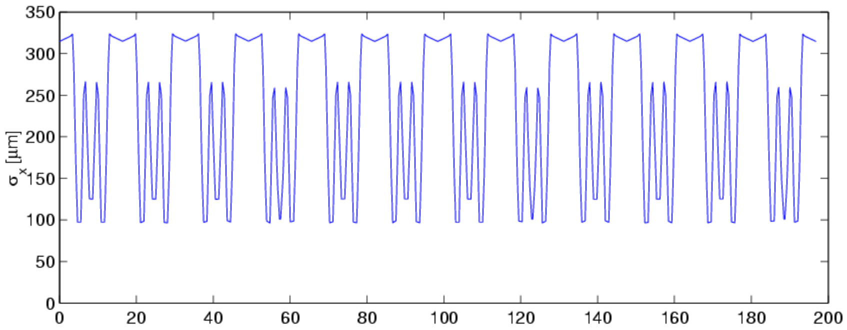


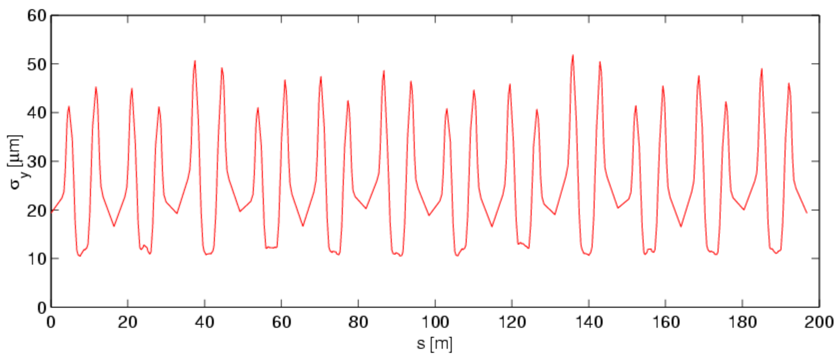
- In this example vertical beamsize was reduced by factor of more than 4 (emittance by factor 20)
- Spurious vertical dispersion reduced from 7 mm rms to below 3 mm rms
- Tilt of phase space reduced significantly everywhere

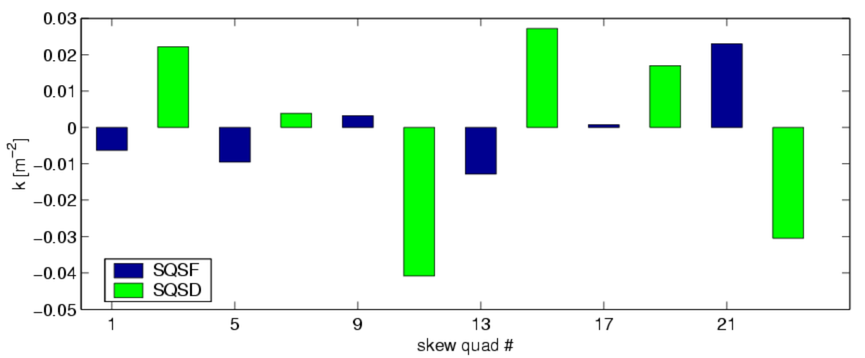
Ways to Increase the Vertical Emittance …

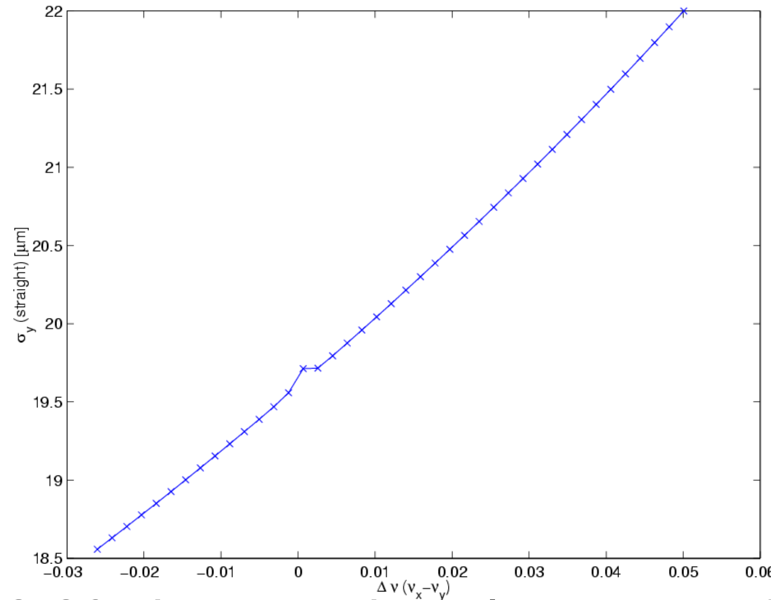
- Low energy third generation light sources usually increase the vertical emittance intentionally to achieve acceptable lifetime.
- Historically at the ALS we used a family of skew quadrupoles to excite linear coupling resonance.
- In 2003 we switched to a mode where we correct the coupling and dispersion as well as possible and then blow up the vertical emittance using a global vertical dispersion wave.
- Method has many advantages (beamsize stability, dynamic momentum aperture, …)

Vertical Dispersion Wave





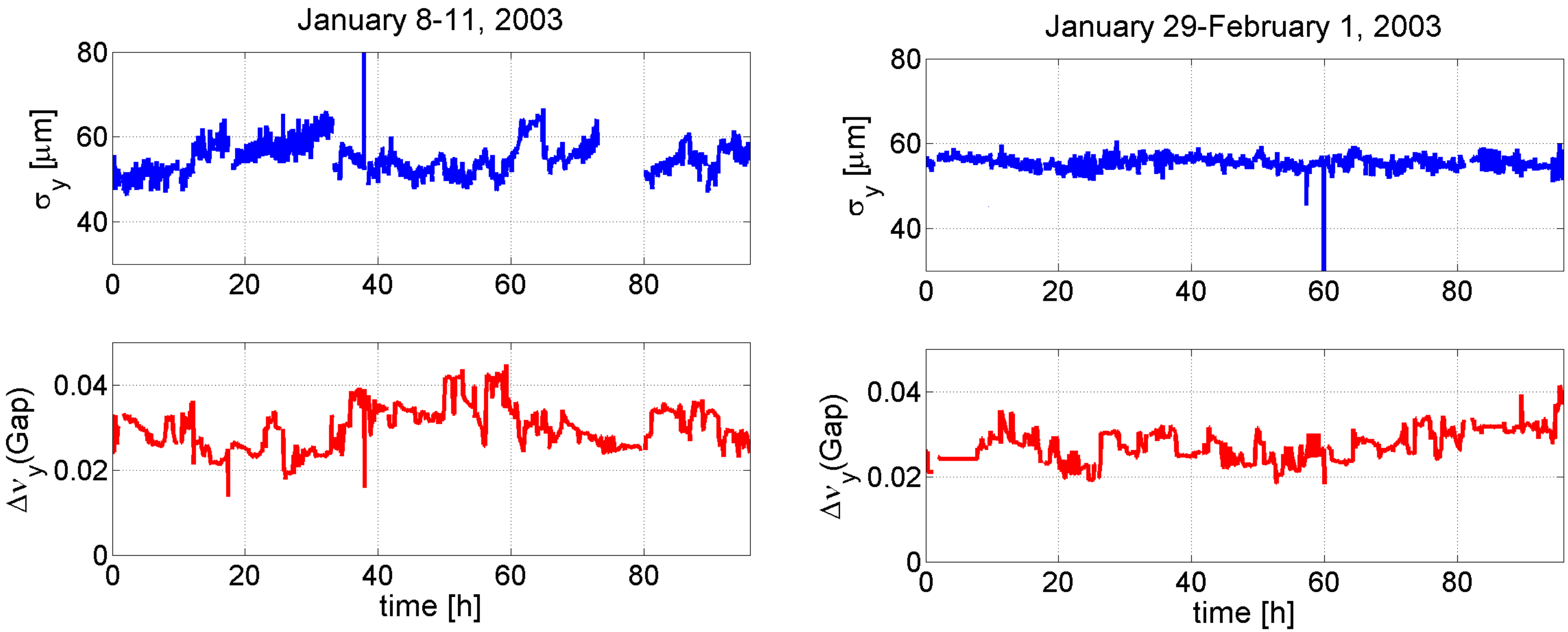




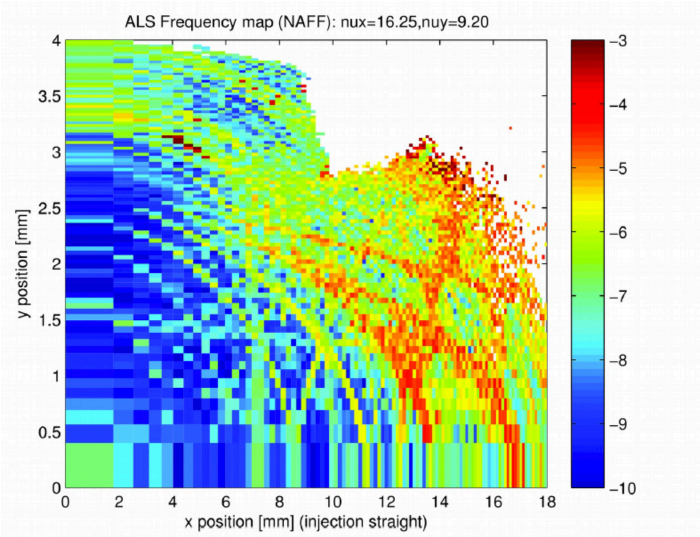
- 12-20 skew quadrupoles are used such, as to generate a global vertical dispersion wave, without exciting nearby coupling resonances
- Vertical emittance is directly generated by quantum excitation
- Local emittance ratio around the ring is fairly constant, local tilt angles are small

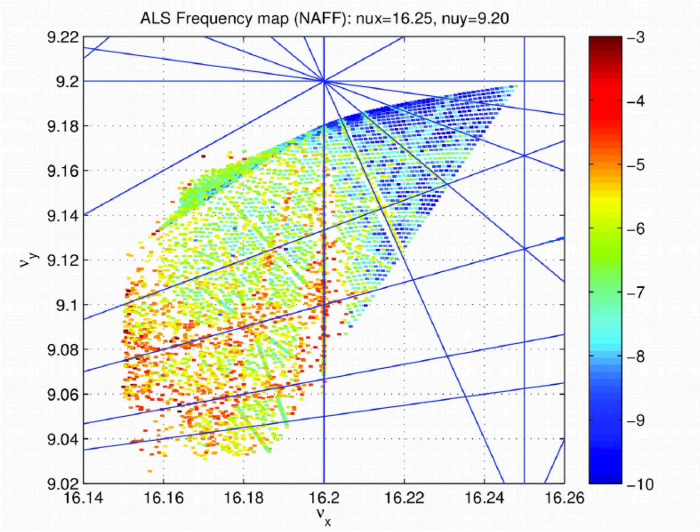
Vertical Beamsize Stability

- The stability of the (vertical) beamsize is important for users (not all effects of varying beamsize can be normalized out)
  - Main issues affecting the beamsize are residual tuneshifts (after feedforward compensation) when scanning undulators or skew errors inside those undulators (especially EPUs)
- Using dispersion wave instead of coupling resonance to increase vertical emittance improves beamsize stability



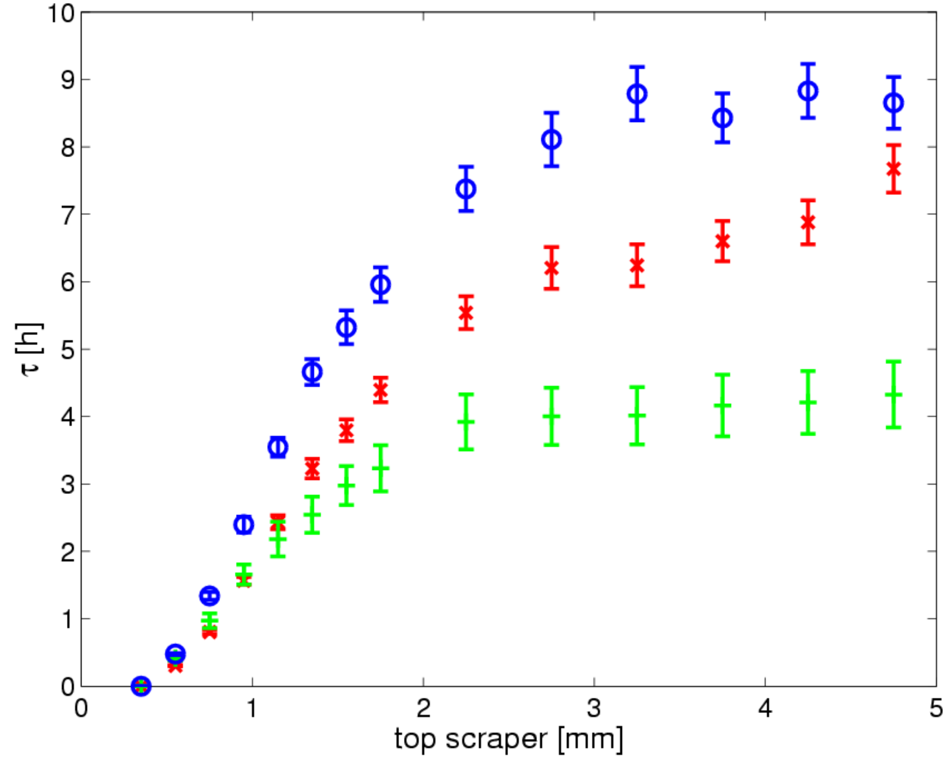
Impact of Coupling Errors on Dynamic Aperture





- Coupling errors (not just linear coupling) have strong impact on amplitude at high amplitude
- Excitation strength of ‘high’ order resonances, that usually dominate dynamic aperture, depends on skew errors
- Example on the left shows dynamic aperture and frequency map of ALS, revealing many high order coupling resonances close to the limit of dynamic aperture
- Correction of skew errors followed by controlled setting of emittance improves dynamic and momentum aperture

Lifetime vs. Vertical Physical Aperture



• Performance (Brightness) of undulators/wigglers (both permanent magnet and SC) depends on magnetic gap

•The vertical physical aperture at which the lifetime starts to get smaller depends strongly on how well global and local coupling is corrected!

Different integrated method: ESRF

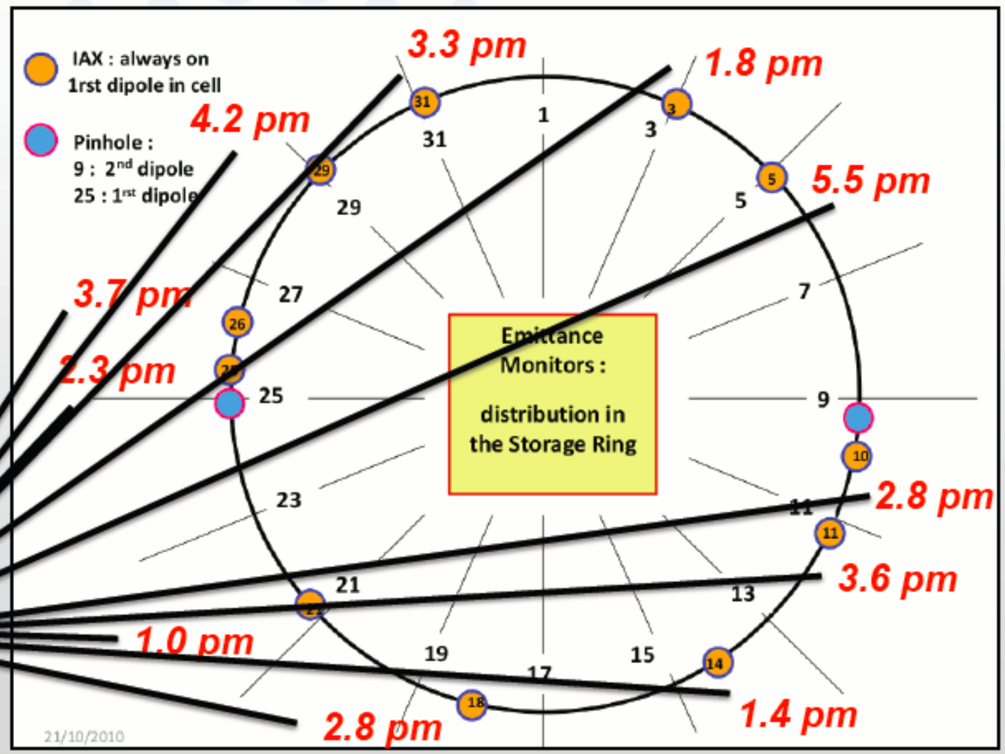
Meas. vertical emittance Ey from RMS beam size

Ex=4.2 nm

- Well corrected coupling
- Low beam current (20 mA)

$\overline{\text{Ey}}$=3.0 pm

±1.3 (STD)



(Plots courtesy of A. Franchi)

ESRF: Coupling RDT

Coupling correction via Resonance Driving Terms

$$f_{\substack{1001\\1010}} = \frac{\sum_{w}^{W} J_{w,1}\sqrt{\beta_x^w \beta_y^w}\, e^{i(\Delta\phi_{w,x} \mp \Delta\phi_{w,y})}}{4(1 - e^{2\pi i (Q_u \mp Q_v)})}$$

1. Build an error lattice model (quad tilts, etc. from Orbit Response Matrix or turn-by-turn BPM data) => RDTs and Dy

$\vec{F} = (a_1{*}f_{1001},\ a_1{*}f_{1010},\ a_2{*}Dy)\ ,\quad a_1+a_2=1$

1. Evaluate response matrix of the available skew correctors **M**
2. Find via SVD a corrector setting $\vec{J}$ that minimizes both RDTs and Dy

$\vec{J} = -M\vec{F}$ **to be pseudo-inverted**

(Plots courtesy of A. Franchi)

ESRF: Coupling RDT

Coupling correction via Resonance Driving Terms

$$\begin{pmatrix} a_1 \vec{f}_{1001} \\ a_1 \vec{f}_{1010} \\ a_2 \vec{D}_y \end{pmatrix}_{\text{meas}} = -\mathbf{M}\vec{J}_c,$$

a_2=0.7 (2010) , 0.4 (2011)

$a_1+a_2=1$

Different weights on f_{1001} and f_{1010} tried, best if equal.

1. Build an error lattice model (quad tilts, etc. from Orbit Response Matrix or turn-by-turn BPM data) => RDTs and Dy

$$\vec{F} = (a_1 {}^* f_{1001},\ a_1 {}^* f_{1010},\ a_2 {}^* Dy)\ ,\quad a_1+a_2=1$$

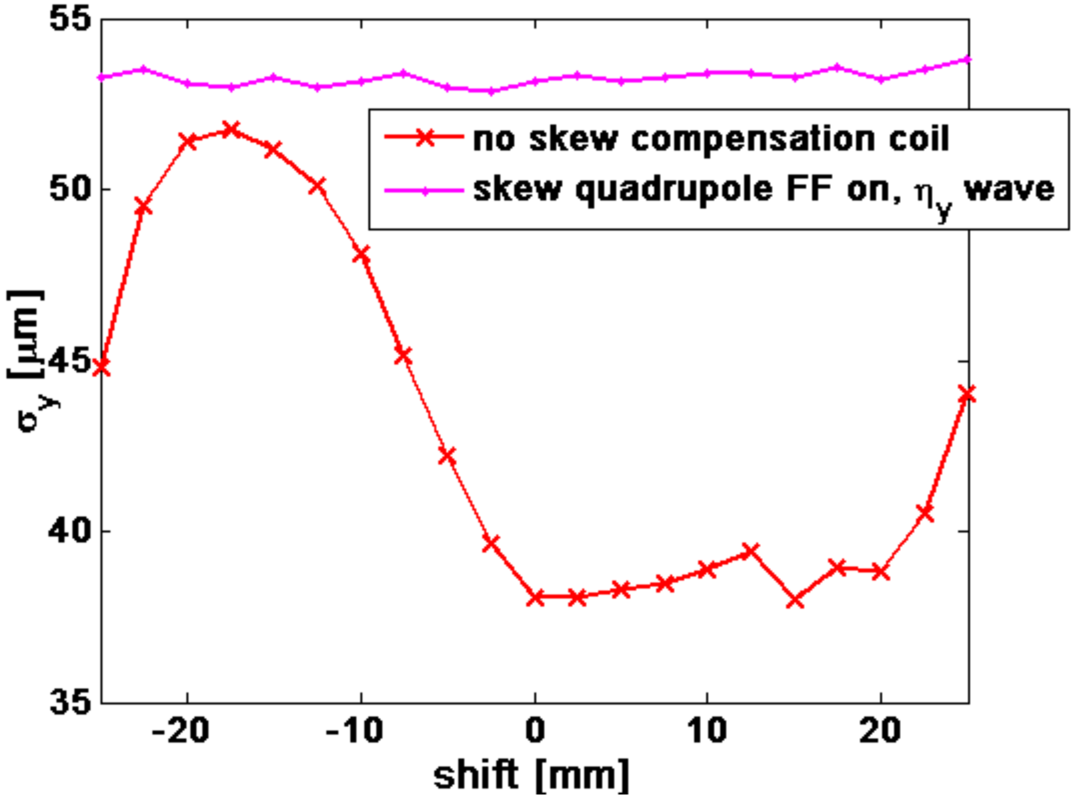
1. Evaluate response matrix of the available skew correctors **M**
2. Find via SVD a corrector setting $\vec{J}$ that minimizes both RDTs and Dy

$\vec{J}$ =-M $\vec{F}$ to be pseudo-inverted

(Plots courtesy of A. Franchi)

Skew quadrupole compensation for EPUs

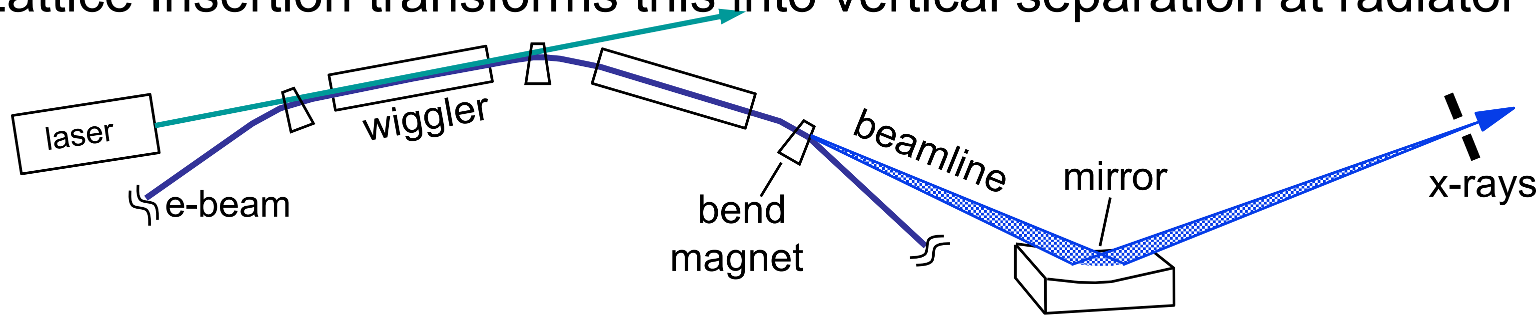
- Beamsize variation was solved by installing skew correction coils for feedforward based compensation
- Feed-forward tables were generated analyzing multiple orbit response matrix measurements (and fits). Result is an excellent compensation.
- Also identified root cause: 2-3 micron correlated motion of magnet modules due to magnetic forces. Newer devices have modified design to reduce effect.

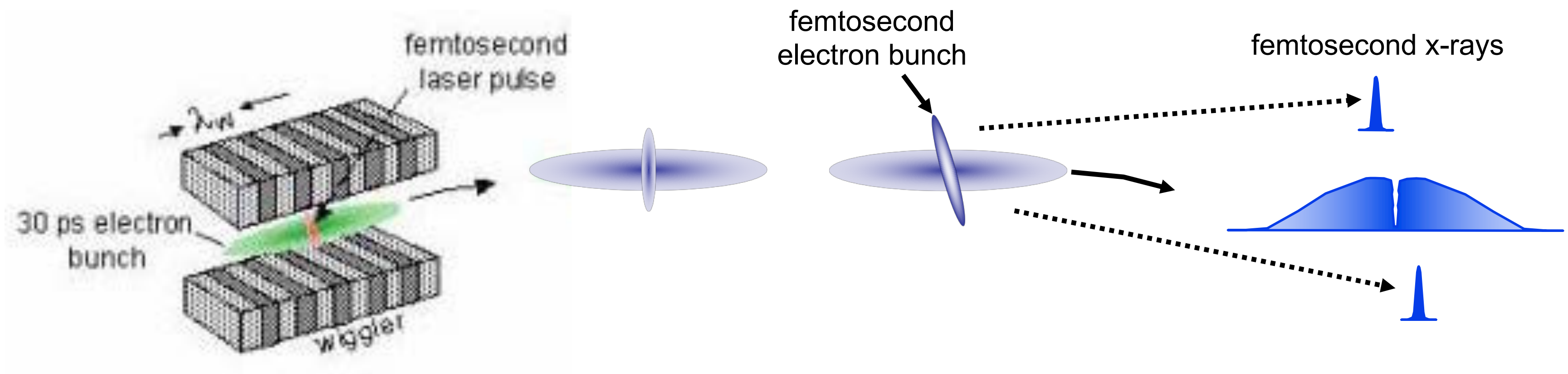


For reference: Whenever an undulator moves, about 120-150 magnets are changed to compensate for the effect (slow+fast feed-forward, slow+fast feedback)

Unusual Example: Dispersion/Coupling Insertion

- Inverse FEL interaction in Wiggler/Undulator used to impose big energy spread on small slice of bunch
- Subsequent arc provides horizontal dispersion
- Lattice Insertion transforms this into vertical separation at radiator





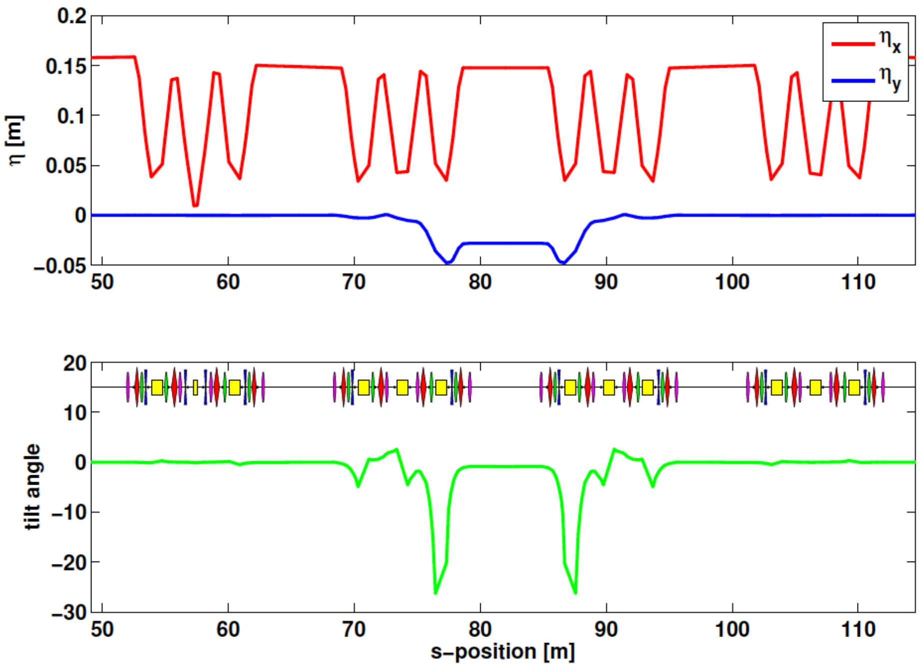
electron-photon interaction in wiggler

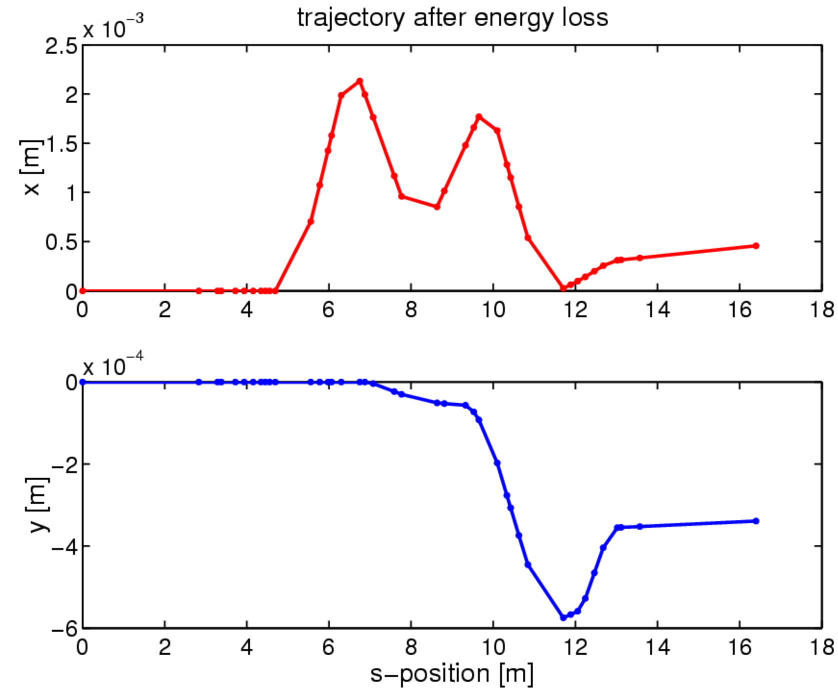
Vertical spatial separation skew quad dispersion bump

Undulator beamline

Zholents and Zolotorev, *Phys. Rev. Lett.*, **76**, 916,1996.

ALS: Dispersion/Coupling Insertion





- Design uses 12 skew quadrupoles to generate closed vertical dispersion bump with negligible global coupling and small local coupling at radiation
- Some sextupoles are already operated deep in saturation – sextupole field gets suppressed by reasonably strong skew quadrupole (>1% effect) → nonlinear dynamics could be important! → No problem.
- Horizontal dispersion in straights automatically generates horizontal separation in addition to vertical separation of dispersion bump
- Separation shown on the right is for 4 cm η_y, 6 cm η_x and 9 σ energy kick

Summary

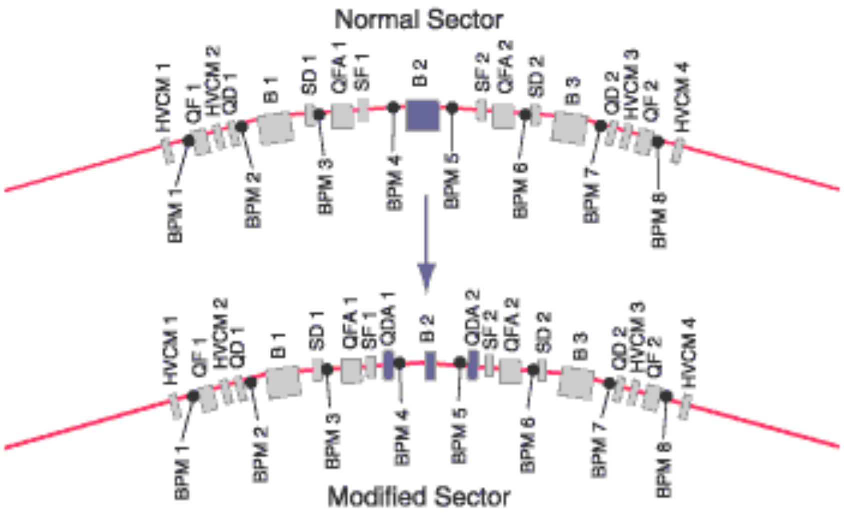
- Coupling correction is important to optimize the performance of an accelerator.
  - Direct benefits are increased brightness or increased luminosity.
  - More indirect improvements are dynamic (momentum) aperture and therefore injection efficiency and lifetime.
- There are several correction methods:
  - At light sources a combined approach targeting local coupling, global coupling and vertical dispersion simultaneously has been most successful.
- Using orbit response matrix analysis (LOCO), emittance ratios below 0.1% have been achieved.
  - For the ALS that corresponds to a vertical emittance of below 1 pm rad, which is within a factor of a few of the theoretical limit due to the finite opening angle ($1/\gamma$) of the synchrotron radiation!

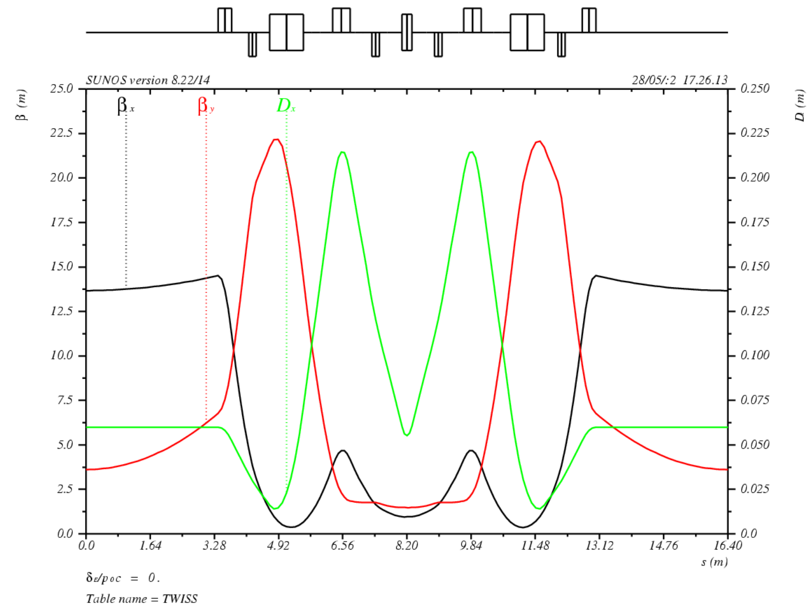
Further Reading

- Guignard, CERN 76-06 1976
- De Ninno & Fanelli, PRST-AB, Vol 3, 2000
- K. Ohmi et al., PRE 49, No 1, 1994
- D. Sagan and D. Rubin, PRST-AB, Vol 2, 1999; D. Sagan et al. PRST-AB, Vol. 3, 2000.
- J. Safranek, and S. Krinsky, PAC 1993 and AIP Proc. 315, 1993.; J. Safranek, NIM A 388, p 27, 1997.
- C. Steier, and D. Robin, EPAC 2000.
- P. Nghiem, and Tordeux, Coupling correction for the ESRF, 1999.
- R. Nagaoka, EPAC 2000; R. Nagaoka, and L. Farvacque, PAC 2001.
- K. Kubo, et al., Phys. Rev. Lett. 88:194891 (2002)
- C. Steier, et al., 'Coupling Correction …', 'fs-slicing', PAC 2003
- J. Shanks, 'Low Emittance Tuning in CesrTA', LER 2011
- A. Streun, 'Low Emittance in SLS', ESLS XIX, 2011
- A. Franchi, 'Coupling Correction in ESRF', FLS 2012

Backup Slides

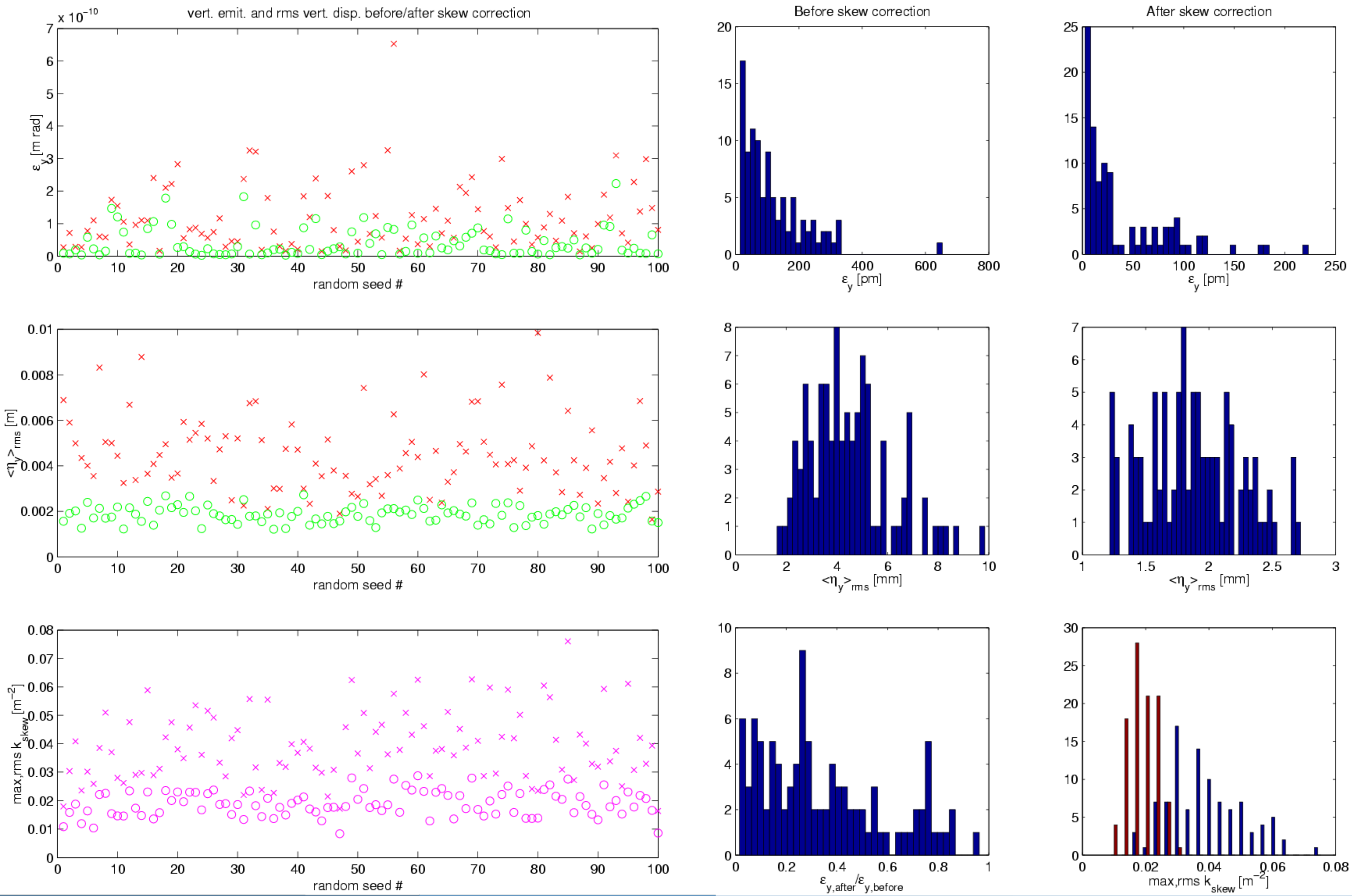
ALS Lattice – Location of Skew Quadrupoles





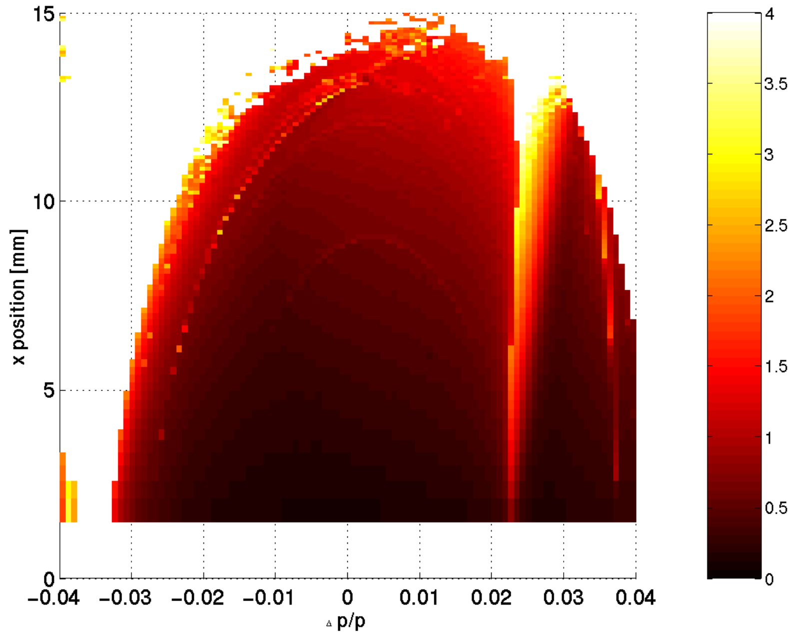
- 24 (by now has grown to 56) individual skew quadrupoles (integrated in sextupoles) serve several purposes:
  - global vertical emittance/dispersion control
    - including minimizing local coupling angles everywhere
  - local vertical dispersion bump
  - correction of skew errors induced by undulators (EPUs)

Finding an effective skew set (2)

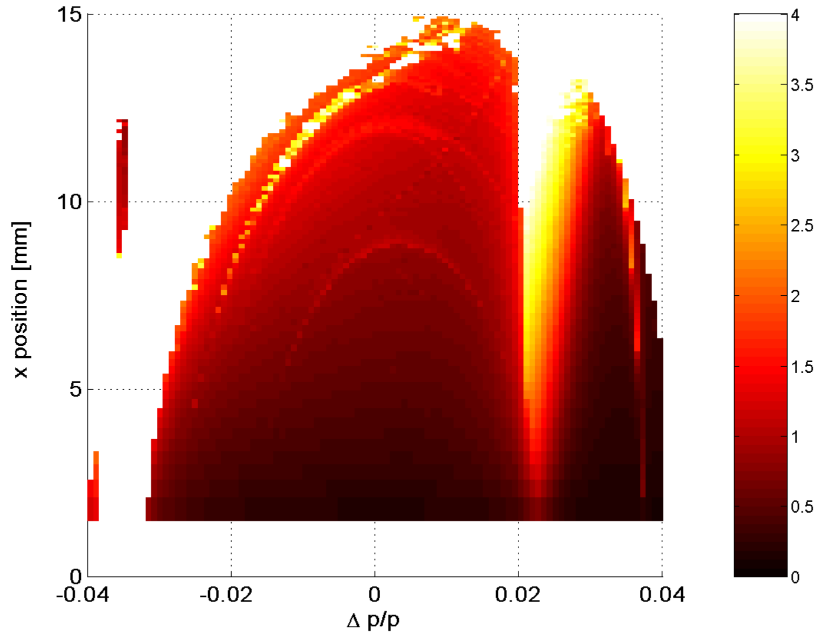


Simulation Results (Momentum Aperture – Gap)

Emittance increased using vertical dispersion wave …



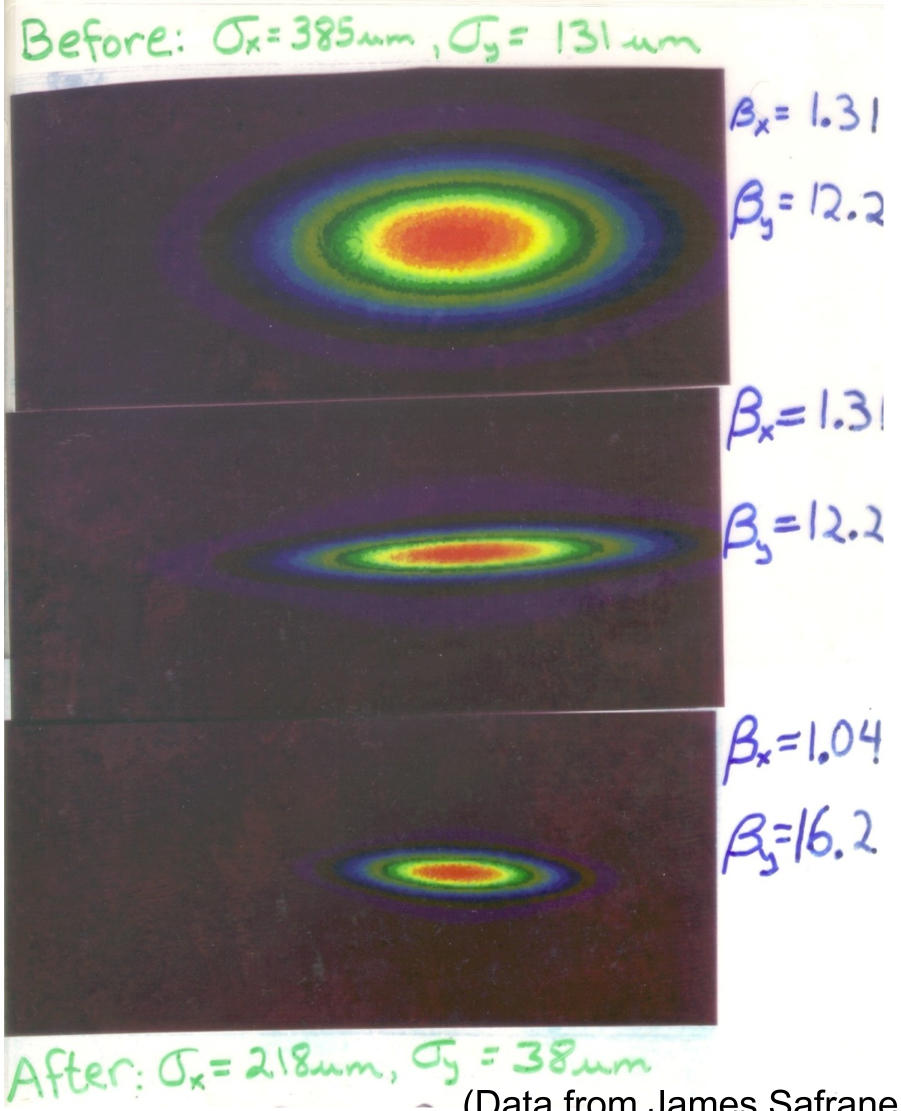
using excitation of coupling resonance



- Tracking results are in good agreement with measured effects, i.e. case with dispersion wave has less yellow and orange areas than the one with excited coupling resonance, indicating less sensitivity to reduced vertical aperture

Other Examples: NSLS

- James was (to my knowledge) the first to use response matrix based fitting to correct coupling (>20 Years ago).
- Applied it very successful at the NSLS, achieving less than 0.1% emittance ratio. Still close to the best emittance ratio reached anywhere, though the absolute vertical emittance was somewhat large, because of much larger natural emittance of X-ray ring.



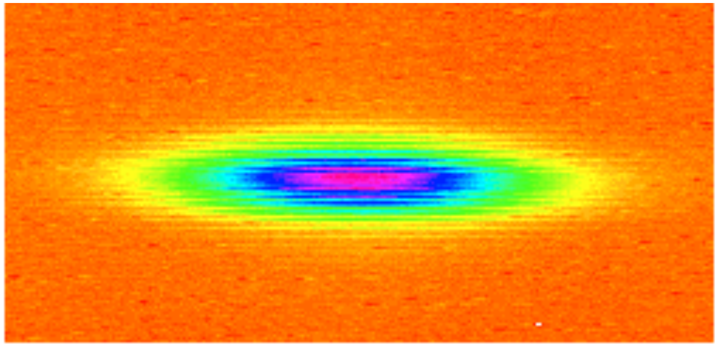
(Data from James Safranek)

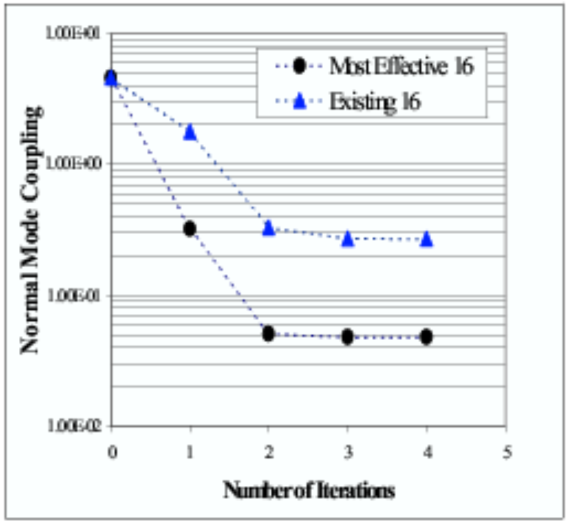
Other Examples: ESRF

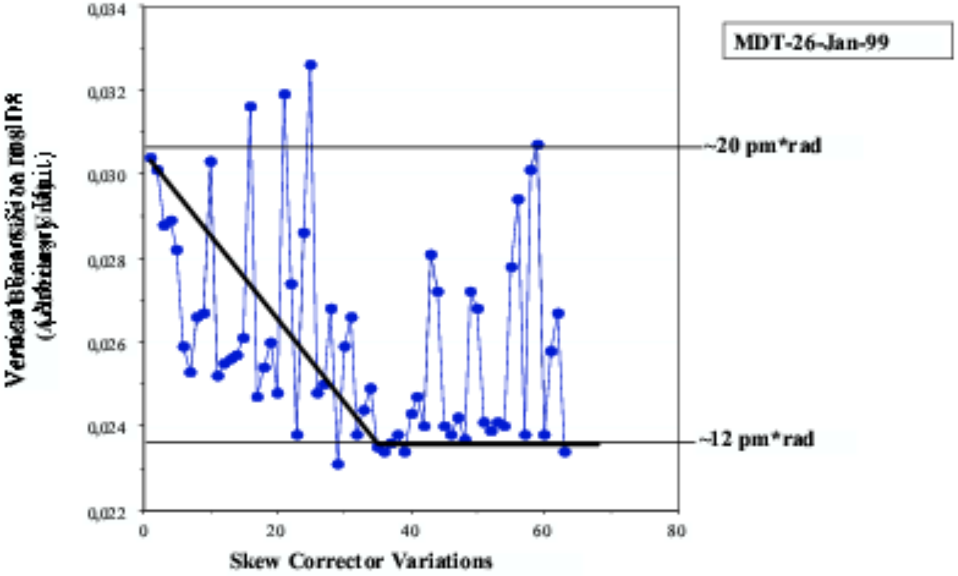
- Nghiem, Nagaoka, and Tordeux carried out work at ESRF using a method similar to LOCO (back in 1999).
  - Challenge was large number of elements in ESRF, order of magnitude is 400 correctors and 400 BPMs and similar number of quadrupoles, sextupoles.
  - Back then, could only use partial response matrix in analysis. Averaged over several of those matrices.
- Did not fit tilt errors of individual magnets, but effective skew distribution (enough to describe the local coupling structure, but few enough to not get strong degeneracies)
  - It was important to study precisely what singular values to keep in inversion and which ones to neglect.
  - Had to iterate with empirical correction on top of the LOCO predicted correction – reason seems to be relatively small number of skew quadrupoles (16).

Other Examples: ESRF

- Reached about 10 pm emittance.
- Predictions from model (tune scan, …) agree very well with independent measurements.
- (All plots courtesy of R. Nagaoka ESRF/Soleil)

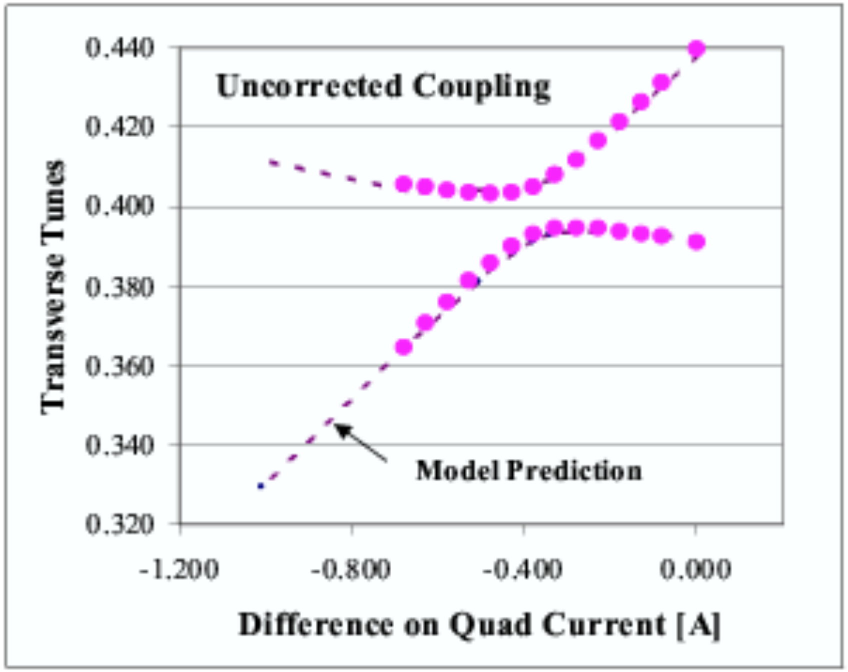


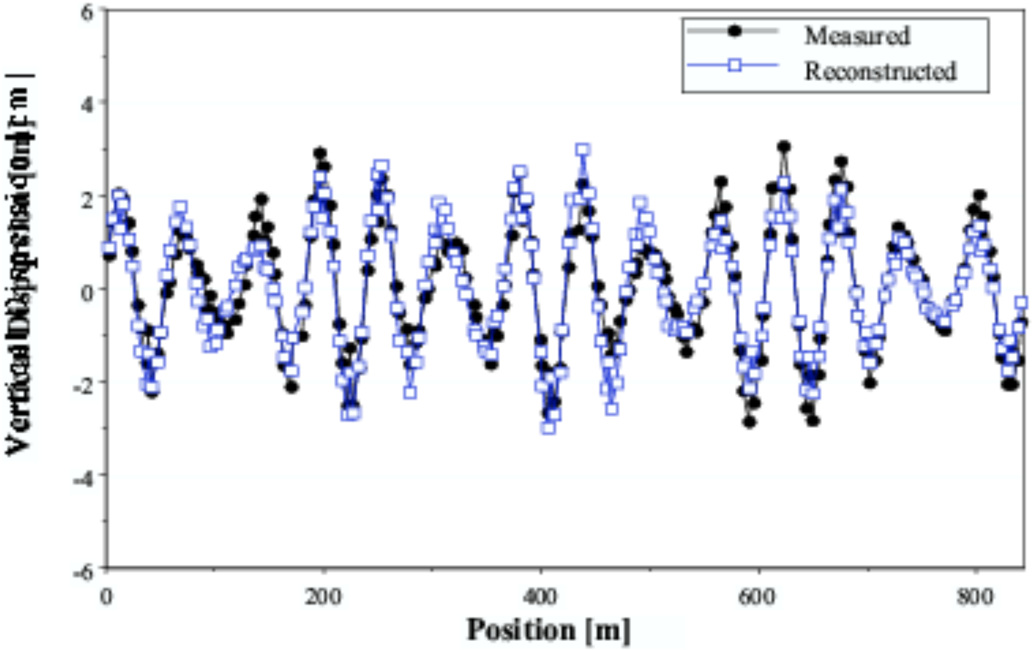




Other Examples: ESRF

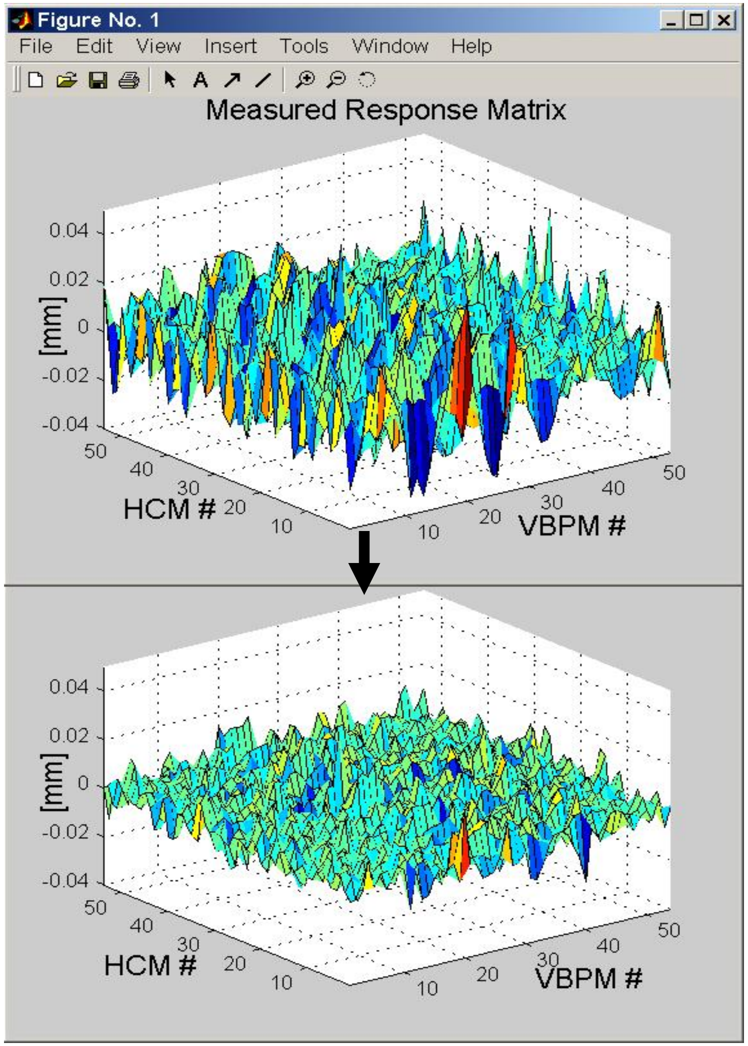
- (Plots courtesy of R. Nagaoka)





Other Examples: Spear 3

Minimize η_y and off-diagonal response matrix:



Lifetime, 19 mA, single bunch