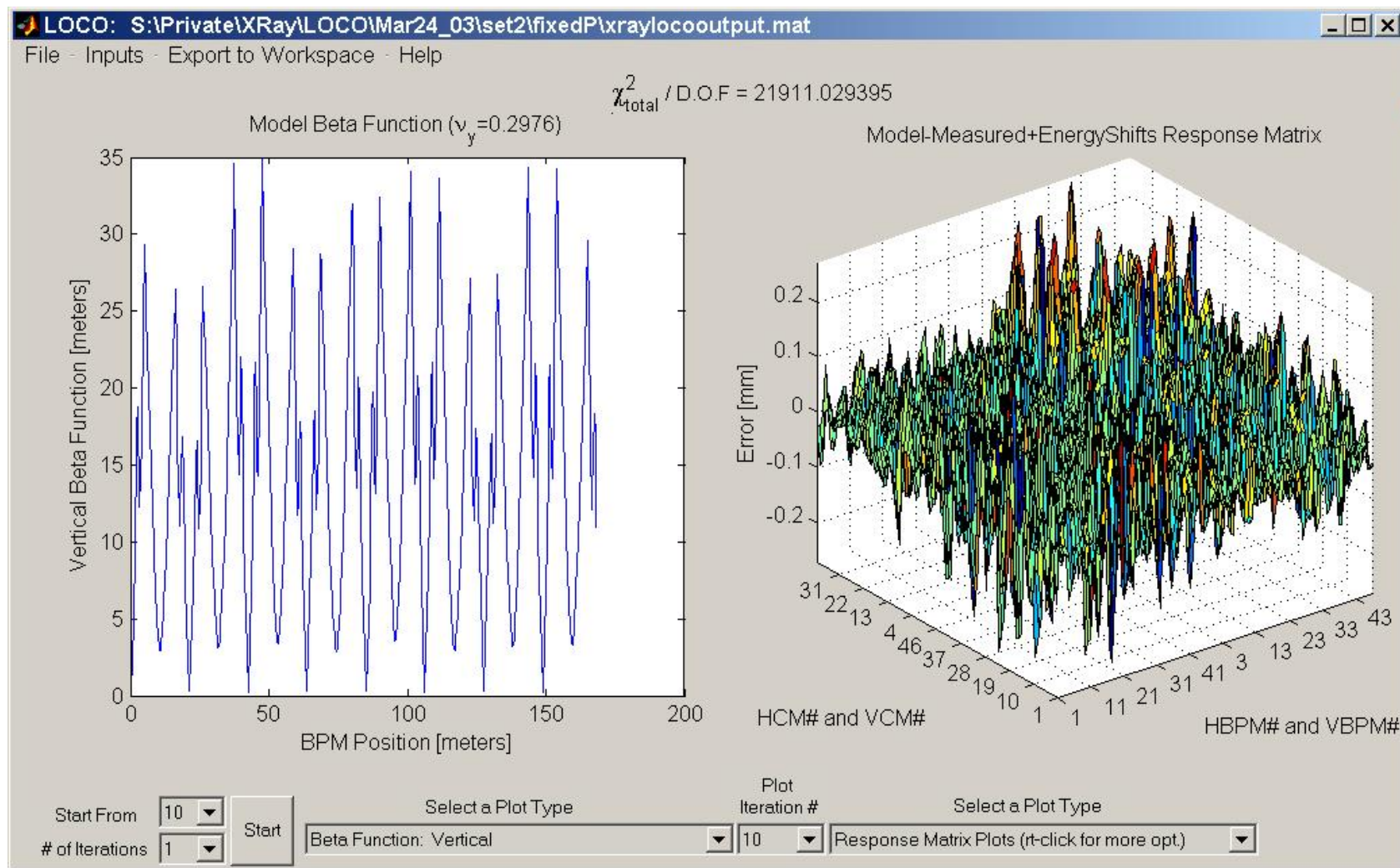
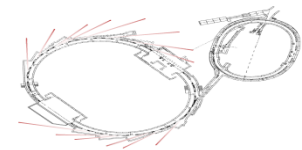
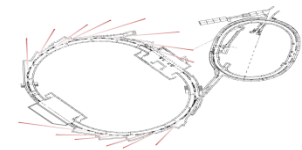


# Linear Optics from Closed Orbits (LOCO)



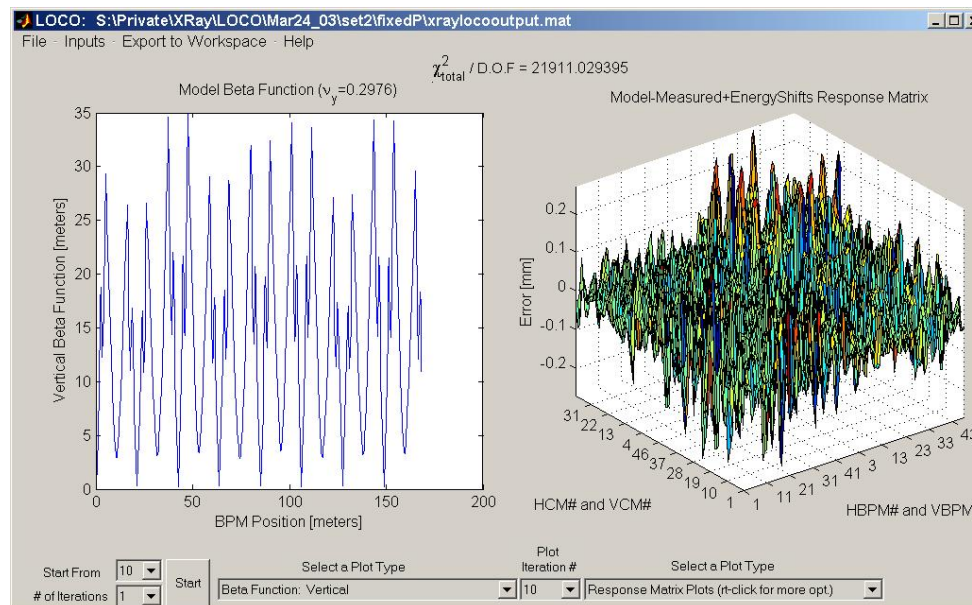
# Linear optics from closed orbits (LOCO)



- Given linear optics (quad. gradients), can calculate orbit response matrix.
- Reverse is possible – calculate gradients from measured response matrix.
- Orbit response matrix has thousands or tens of thousands of highly accurate data points giving a measure of linear optics.
- The LOCO code uses this data to calibrate and correct linear optics.

## Lecture outline

- LOCO method
- Error analysis
- Applications



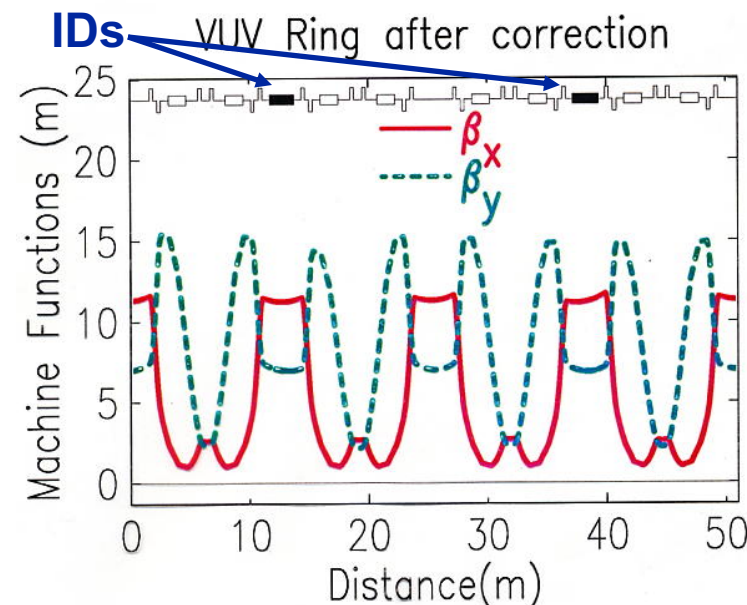
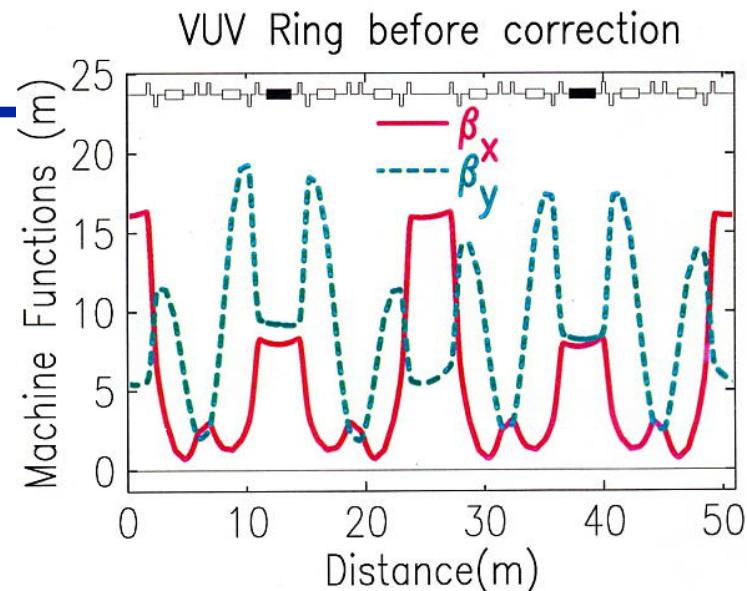
← LOCO GUI

# NSLS VUV ring example

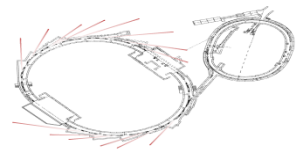
The VUV ring optics were not well controlled. There was a problem with incorrect compensation for insertion device (ID) focusing. LOCO was used to calibrate the strength of the ID focusing and to find the changes the current to the quadrupoles that best restored the optics.

The results were

- 20% increase in lifetime
- Few percent decrease in both  $\epsilon_x$  and  $\epsilon_y$
- Often correcting linear optics is critical for achieving design/optimized nonlinear dynamics



# Method



The orbit response matrix is defined as

$$\begin{bmatrix} \vec{X} \\ \vec{y} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \bar{\theta}_x \\ \bar{\theta}_y \end{bmatrix}$$

The parameters in a computer model of a storage ring are varied to minimize the  $\chi^2$  deviation between the model and measured orbit response matrices ( $M_{\text{mod}}$  and  $M_{\text{meas}}$ ).

$$\chi^2 = \sum_{i,j} \frac{(M_{ij}^{\text{meas}} - M_{ij}^{\text{model}})^2}{\sigma_i^2} \equiv \sum_{k=i,j} E_k^2$$

The  $\sigma_i$  are the measured noise levels for the BPMs;  $E$  is the error vector.

The  $\chi^2$  minimization is achieved by iteratively solving the linear equation

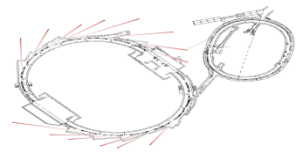
$$E_k^{\text{new}} = E_k + \frac{\partial E_k}{\partial K_l} \Delta K_l = 0$$

$$-E_k = \frac{\partial E_k}{\partial K_l} \Delta K_l \quad \frac{\partial E_k}{\partial K_l} = J_{kl} = \text{Jacobian}$$

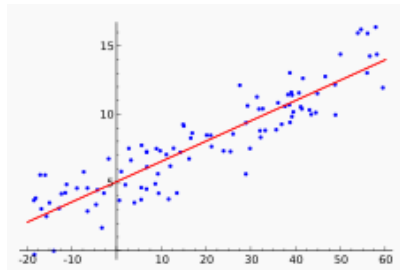
For the changes in the model parameters,  $K_l$ , that minimize  $\|E\|^2 = \chi^2$ .

Gauss-Newton minimization.

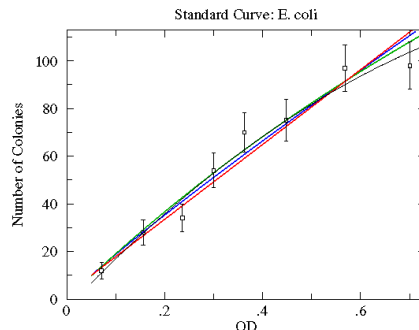
# Chi-square fitting



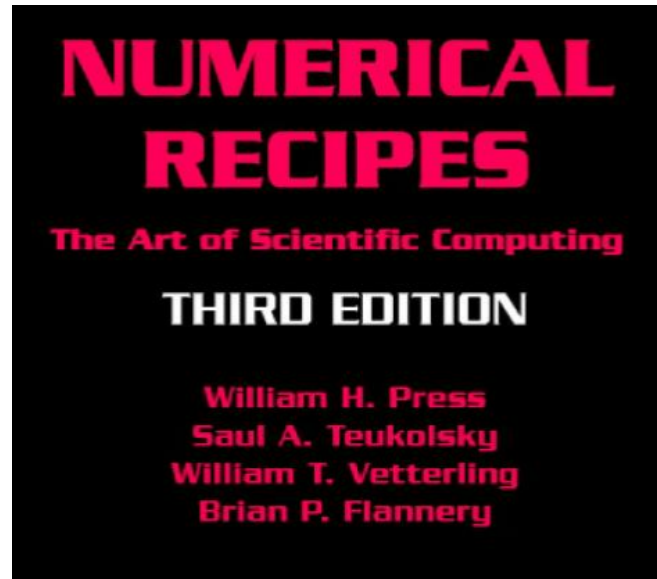
- A chi-squared fit gives the best fit model to data
- Two parameter model for linear fit (slope and intercept)



- More complicated data requires a more complicated model with more parameters, and usually more data to constrain them.

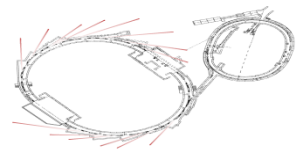


- An accelerator is a very complicated model, and an accelerator has many parameters – extreme chi-square problem



“Those who practice chi-by-eye get what they deserve.”

# Response matrix review



The response matrix is the shift in orbit at each BPM for a change in strength of each steering magnet.

Vertical response matrix, BPM  $i$ , steerer  $j$ :

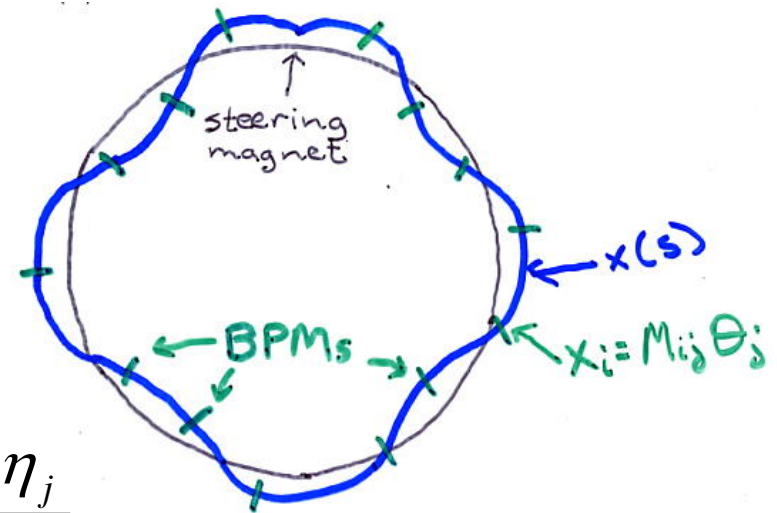
$$M_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin \pi \nu} \cos(|\phi_i - \phi_j| - \pi \nu)$$

Horizontal response matrix:

$$M_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin \pi \nu} \cos(|\phi_i - \phi_j| - \pi \nu) + \frac{\eta_i \eta_j}{\alpha_c L_0}$$

Additional  $\eta$  term keeps the path length constant (fixed rf frequency).

LOCO option to use this linear form of the response matrix (faster) or can calculate response matrix including magnet nonlinearities and skew gradients (slower, more precise). First converge with linear response matrix, then use full response matrix.





# Parameters varied to fit the orbit response matrix

## NSLS XRay Ring fit parameters:

56 quadrupole gradients

48 BPM gains, horizontal

48 BPM gains, vertical

90 steering magnet kicks

=242 Total fit parameters

## NSLS XRay data:

(48 BPMs)\*(90 steering magnets)

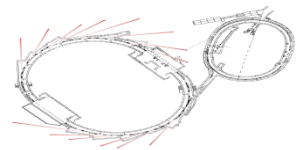
=4320 data points

$\chi^2$  fit becomes a minimization problem of a function of 242 variables.

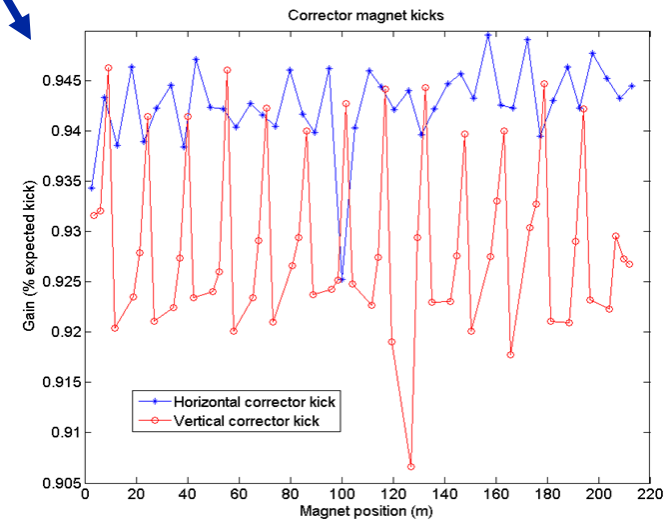
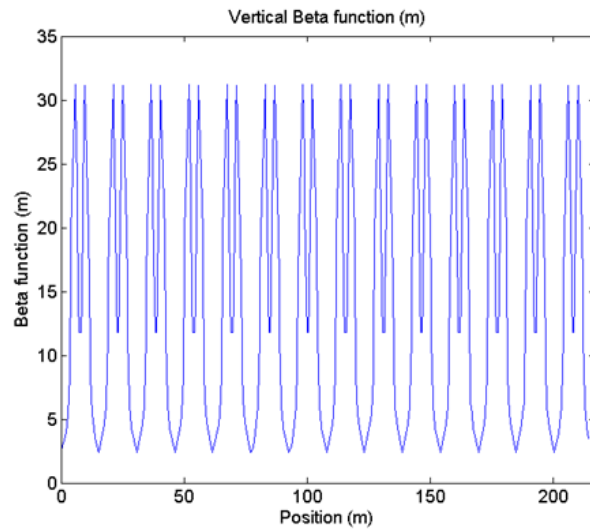
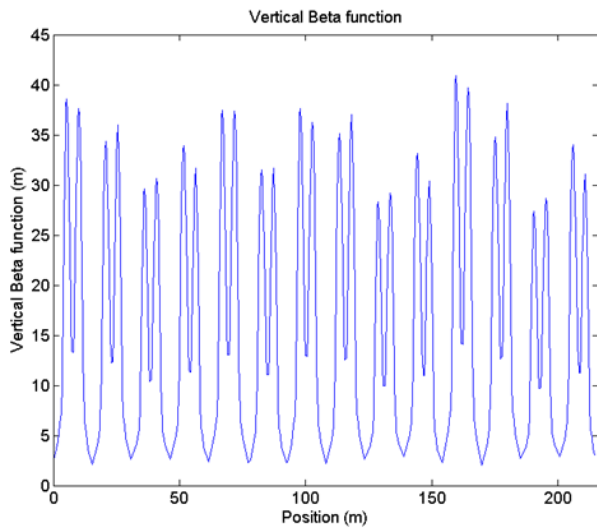
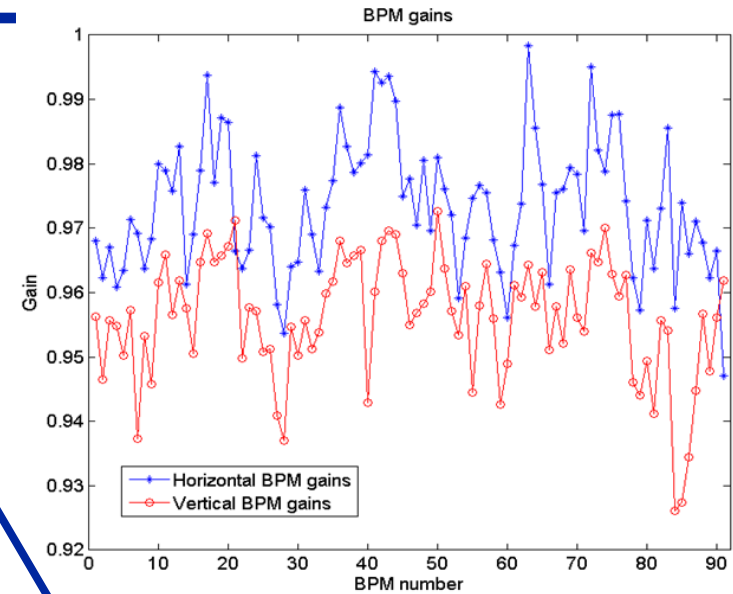
Fit converted to linear algebra problem, minimize  $\|E\|^2 = \chi^2$ .

For larger rings, fit thousands of parameters to tens of thousands of data points. For APS, full  $\partial \vec{E} / \partial \vec{K}$  matrix is ~9 Gbytes, so the size of the problem must be reduced by limiting the number of steering magnets in the response matrix. For rings the size of LEP, problem gets too large to solve all at once on existing computers. Need to divide ring into sections and analyze sections separately.

# LOCO in Australia



- **BPM gains**
- **Steering magnet calibration**
- $\beta_y$  **before & after correction**







# More fit parameters

## Why add BPM gains and steering magnet calibrations?

- Adding more fit parameters increases error bars on fit gradients due to propagation of random measurement noise on BPMs. If you knew that all the BPMs were perfectly calibrated, it would be better not to vary the BPM gains in the fit.
- More fit parameters decreases error on fit gradients from systematic modeling errors. Not varying BPM gains introduces systematic error.
- As a rule, vary parameters that introduce ‘significant’ systematic error. This usually includes BPM gains and steering magnet kicks.

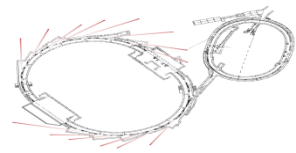
## Other parameters to vary:

- Quadrupole roll (skew gradient)
- Steering magnet roll
- BPM coupling
- Steering magnet energy shifts
- Steering magnet longitudinal centers

Parameters for coupled response matrix,

$$\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{bmatrix} \begin{bmatrix} \bar{\theta}_x \\ \bar{\theta}_y \end{bmatrix}$$

# Fitting energy shifts.



Horizontal response matrix: 
$$M_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin \pi \nu} \cos(|\phi_i - \phi_j| - \pi \nu) + \frac{\eta_i \eta_j}{\alpha_c L_0}$$

Betatron amplitudes and phases depend only on storage ring gradients:

$$\frac{1}{2} \beta \beta'' - \frac{1}{4} \beta'^2 + \beta^2 K = 1 \quad d\phi = \frac{ds}{\beta}$$

Dispersion depends both on gradients and dipole field distribution:

$$\eta'' + K\eta = \frac{1}{\rho}$$

If the goal is to find the gradient errors, then fitting the full response matrix, including the term with  $\eta$ , will be subject to systematic errors associated with dipole errors in the real ring not included in the model. This problem can be circumvented by using a “fixed momentum” model,

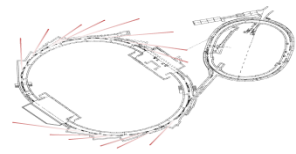
$$M_{ij}^{\text{fixed p}} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin \pi \nu} \cos(|\phi_i - \phi_j| - \pi \nu),$$

and adding a term to the model proportional to the measured dispersion

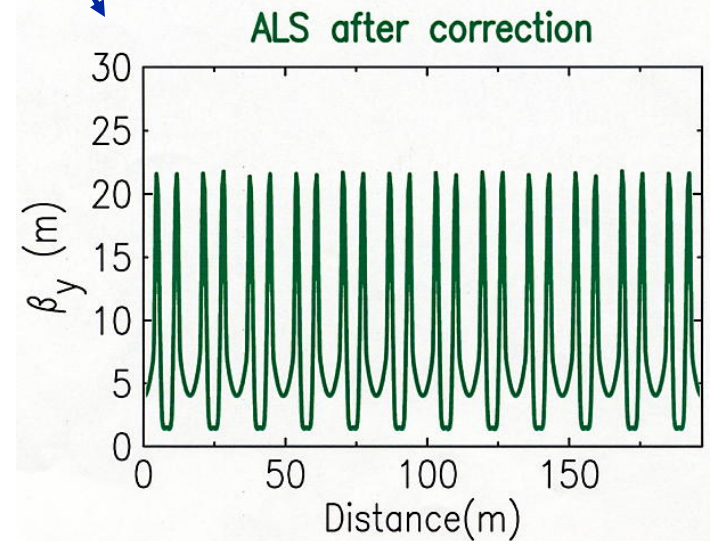
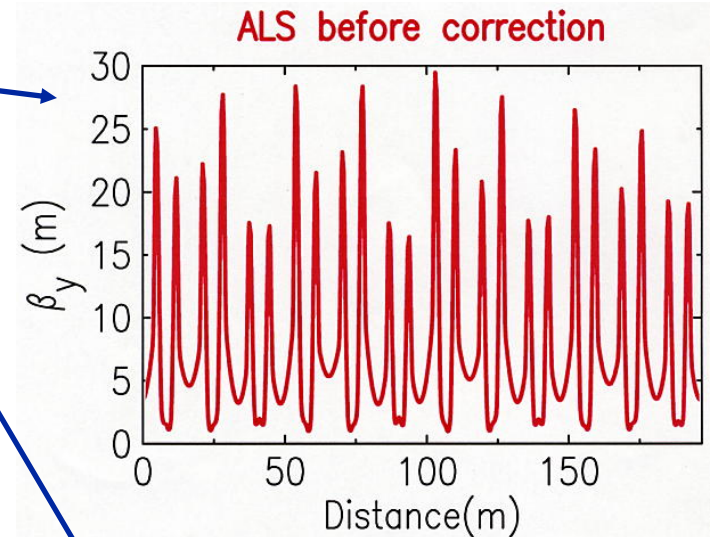
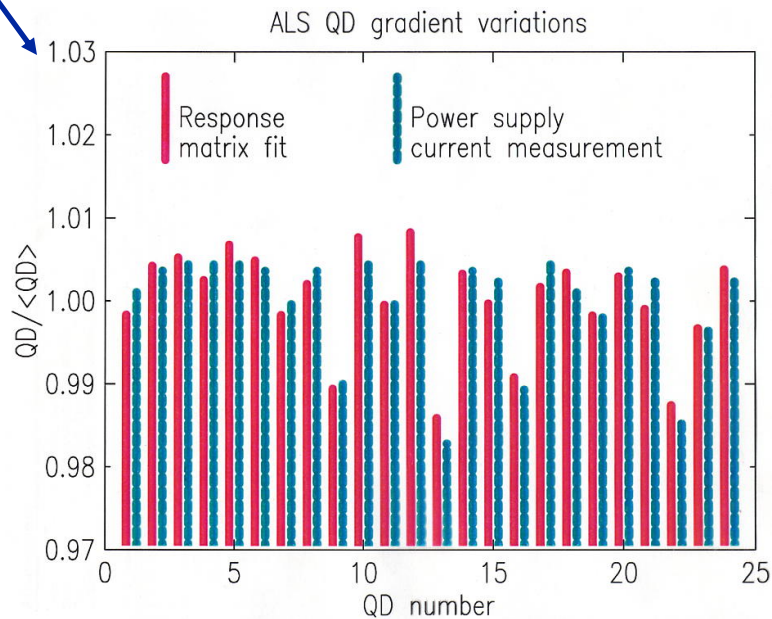
$$M_{ij}^{\text{mod}} = M_{ij}^{\text{fixed p}} + \frac{\Delta p_i}{p} \eta_j^{\text{meas}}$$

$\Delta p_i / p$  is a fit parameter for each steering magnet. In this way the  $\eta^{\text{model}}$  is eliminated from the fit, along with systematic error from differences between  $\eta^{\text{model}}$  and  $\eta^{\text{meas}}$ .

# Finding gradient errors at ALS

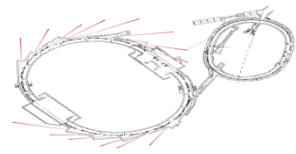


- LOCO fit indicated gradient errors in ALS QD magnets making  $\beta_y$  distortion.
- Gradient errors subsequently confirmed with current measurements.
- LOCO used to fix  $\beta_y$  periodicity.
- Operational improvement (Thursday lecture).



# Different goals when applying LOCO

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There are a variety of results that can be achieved with LOCO:

1. Finding actual gradient errors.
2. Finding changes in gradients to correct betas.
3. Finding changes in gradients to correct betas and dispersion.
4. Finding changes in local gradients to correct ID focusing.
5. Finding changes in skew gradients to correct coupling and  $\eta_y$ .
6. Finding transverse impedance.

The details of how to set up LOCO and the way the response matrix is measured differs depending on the goal.

In the previous example for the ALS, the goals were 1 and 2. LOCO is set up differently for each.



# Correcting betas and dispersion

Measure response matrix with ring in configuration for delivered beam.

- Sextupoles on
- Correct to golden orbit
- IDs closed (depending on how you want to deal with ID focusing)

Fit only gradients that can be adjusted in real ring.

- Do not fit gradients in sextupoles or ID gradients
- If a family of quadrupoles is in a string with a single power supply, constrain the gradients of the family to be the same.

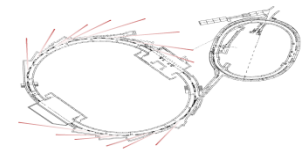
To correct betas only, use fixed-momentum model matrix and fit energy shifts, so dispersion is excluded from fit.

To correct betas and dispersion, use fixed-path length matrix and can use option of including  $\eta$  as an additional column in response matrix.

To implement correction, change quadrupole current of  $n^{\text{th}}$  quadrupole or quad family:

$$\frac{\Delta I_n}{I_n} = - \frac{K_{\text{fit},n} - K_{\text{ideal},n}}{K_{\text{ideal},n}}$$

# Finding gradient errors



If possible, measure two response matrices – one with sextupoles off and one with sextupoles on.

- Fit the first to find individual quadrupole gradients.
- Fit the second to find gradients in sextupoles.
- Fewer gradients are fit to each response matrix, increasing the accuracy.
- ... Measure a 3<sup>rd</sup> response matrix with IDs closed.

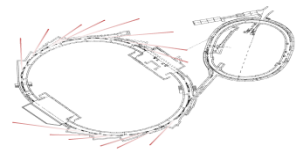
Vary all quadrupole gradients individually (maybe leave dipole gradient as a family).

Use either 1.) fixed-momentum response matrix and fit energy shifts or 2.) fixed-path-length depending on how well  $1/\rho$  in the model agrees with  $1/\rho$  in the ring (i.e. how well is the orbit known and controlled).

Get the model parameters to agree as best as possible with the real ring: model dipole field roll-off; check longitudinal positions of BPMs and steering magnets; compensate for known nonlinearities in BPMs.

Add more fitting parameters if necessary to reduce systematic error (for example, fit steering magnet longitudinal centers in X-Ray Ring.)

# Correcting betas in PEP-II

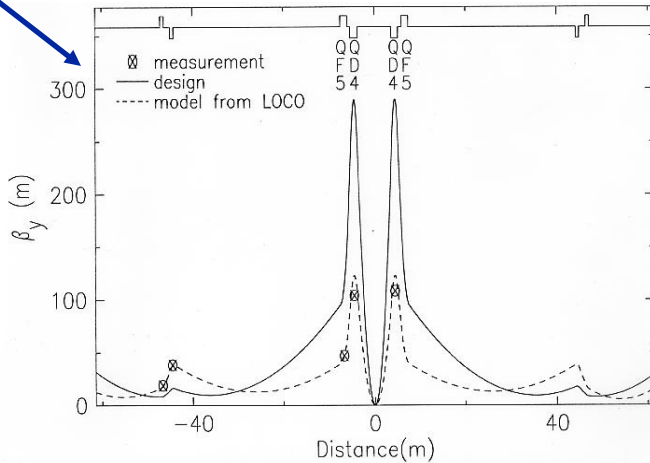


Often times, finding the quad changes required to correct the optics is easier than finding the exact source of all the gradient errors.

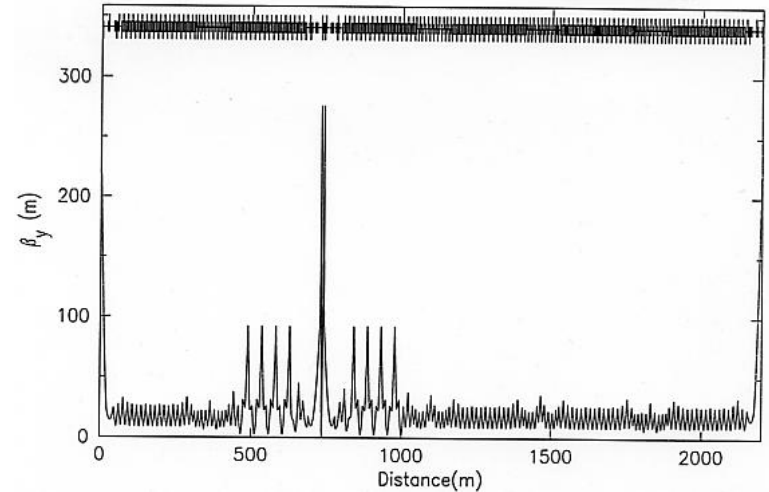
For example, in PEP-II there are not enough BPMs to constrain a fit for each individual quadrupole gradient. The optics still could be corrected by fitting quadrupole families.

Independent  $\beta$  measurements confirmed that LOCO had found the real  $\beta$ 's (x2.5 error!)

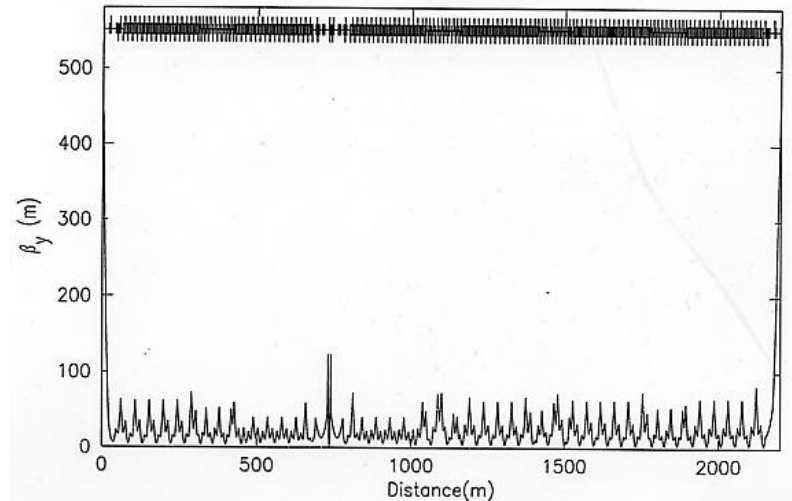
Quadrupole current changes according to fit gradients restored ring optics to the design.



PEP-II HER  $\beta_y$ , design



PEP-II HER  $\beta_y$ , LOCO fit



# Correcting betas in PEP-II



Optics distortion from a single gradient error:

The shift in tune from an integrated gradient error of  $\Delta KL$  is

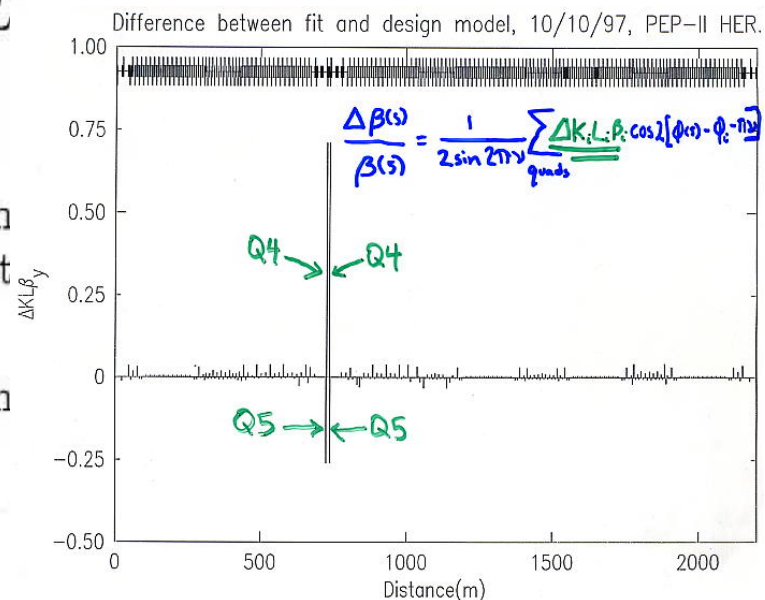
$$\Delta\nu = \frac{1}{4\pi} \beta_q \Delta KL.$$

(Recall that measuring the change in tune for a change in quadrupole gradient gives a measure of the  $\beta$ -function at the quadrupole,  $\beta_q$ .)

A gradient error also creates a distortion of the  $\beta$ -function around the ring.

$$\frac{\Delta\beta}{\beta} = \frac{1}{2\sin(2\pi\nu)} \beta_q \Delta KL \cos[2(\phi(s) - \phi_q) - 2\pi\nu],$$

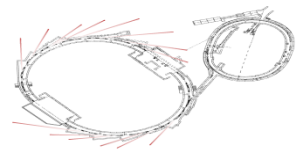
so the distortion of the  $\beta$ -function from a gradient error scales as  $\beta_q \Delta KL$ . Looking at the fit value of  $\beta_q \Delta KL$  for the different quadrupoles in the ring shows the source of the optics distortion.





# Insertion device linear optics correction

---



The code LOCO can be used in a beam-based algorithm for correcting the linear optics distortion from IDs with the following procedure:

1. Measure the response matrix with the ID gap open.
2. Then the response matrix is measured with the gap closed.
3. Fit the first response matrix to find a model of the optics without the ID distortion.
4. Starting from this model, LOCO is used to fit a model of the optics including the ID. In this second fit, only a select set of quadrupoles in the vicinity of the ID are varied. The change in the quadrupole gradients between the 1<sup>st</sup> and 2<sup>nd</sup> fit models gives a good correction for the ID optics distortion.
5. Alternatively, LOCO can be used to accurately fit the gradient perturbation from the ID, and the best correction can be calculated in an optics modeling code.

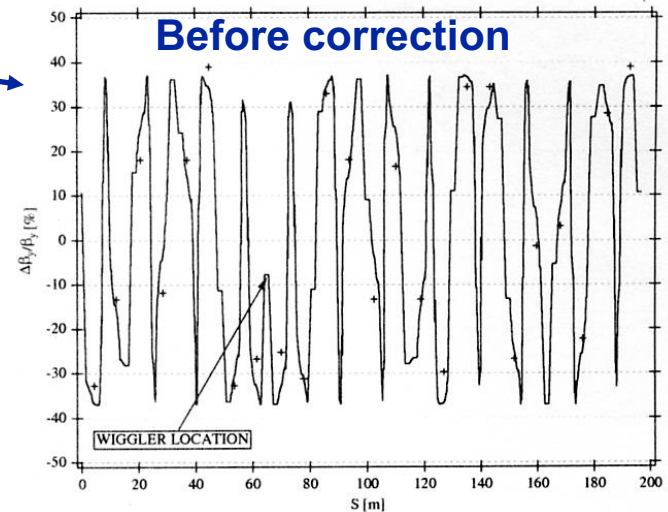
1.) L. Smith, LBNL, ESG Technical Note No. 24, 1986.

# Linear optics correction at ALS

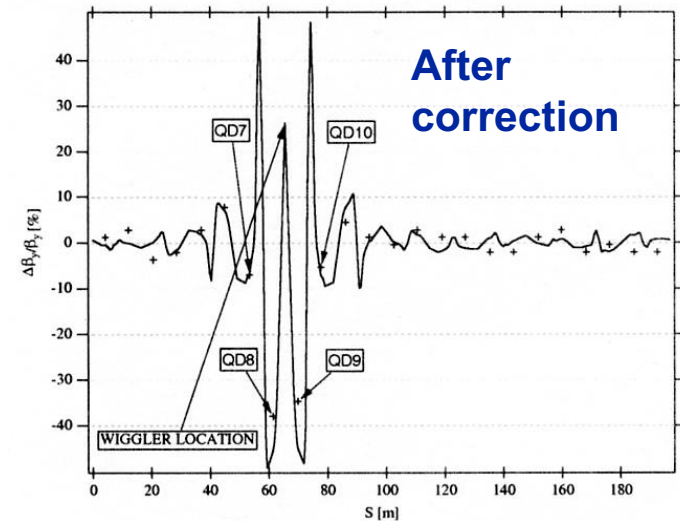
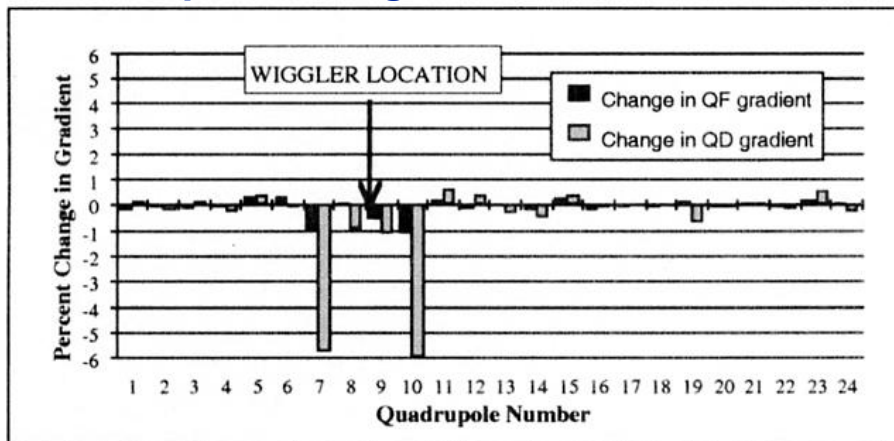


Beta function distortion from wiggler.

At ALS the quadrupoles closest to the IDs are not at the proper phase to correct optics distortions, so the optics correction cannot be made entirely local.



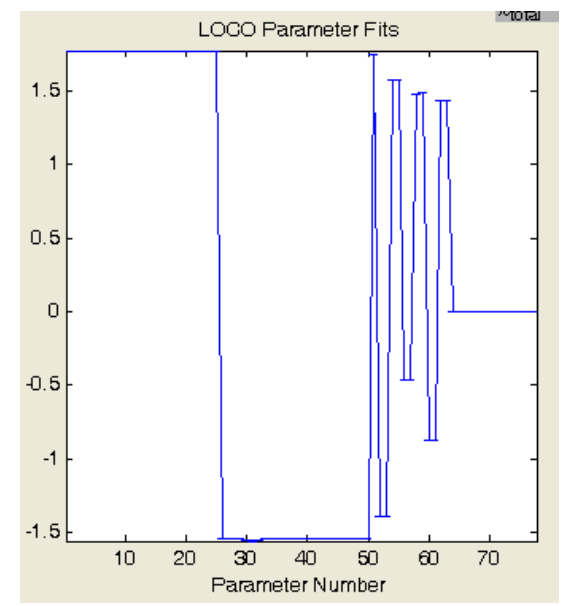
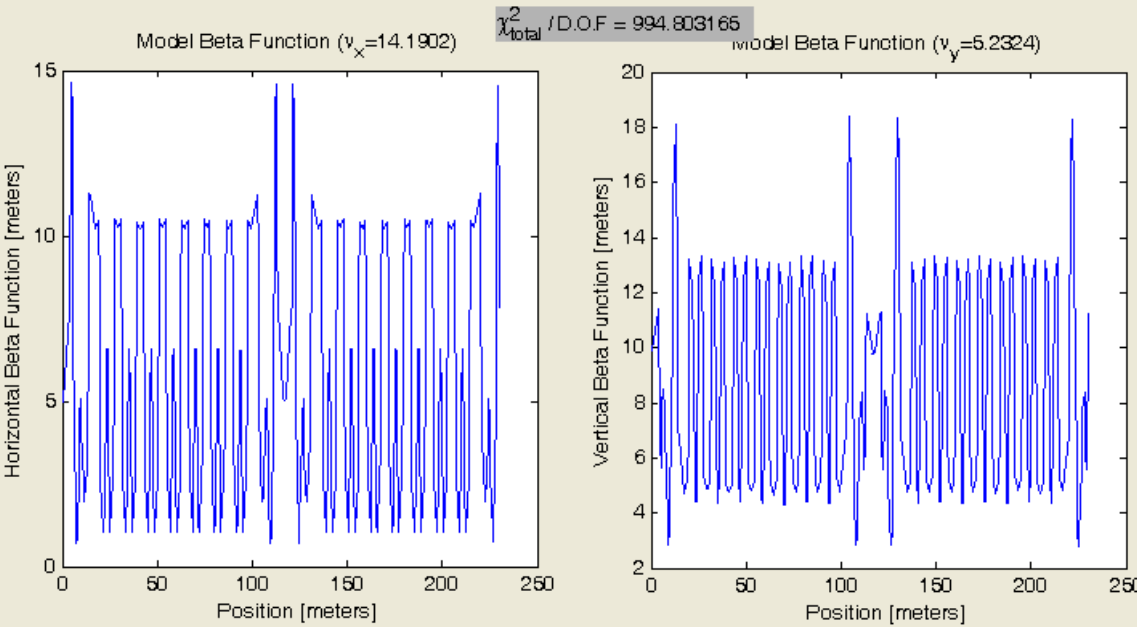
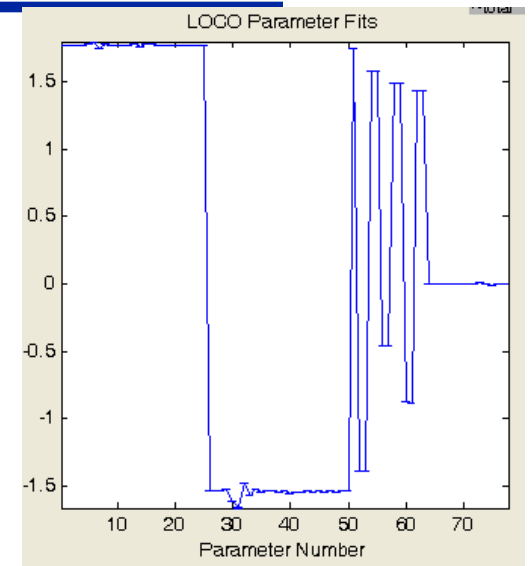
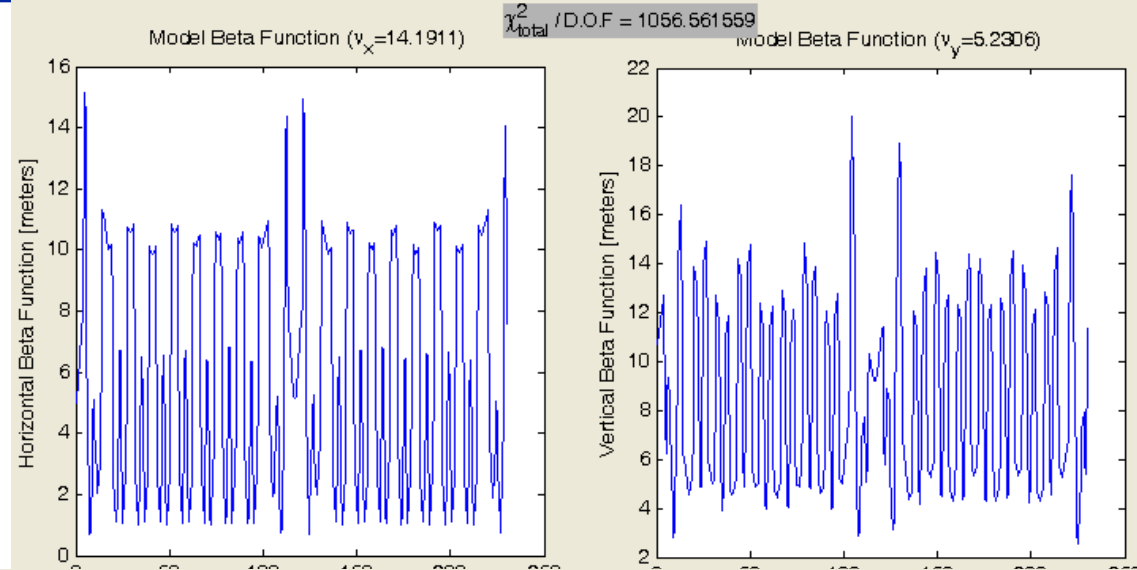
Quadrupole changes used for correction



D. Robin et al. PAC97



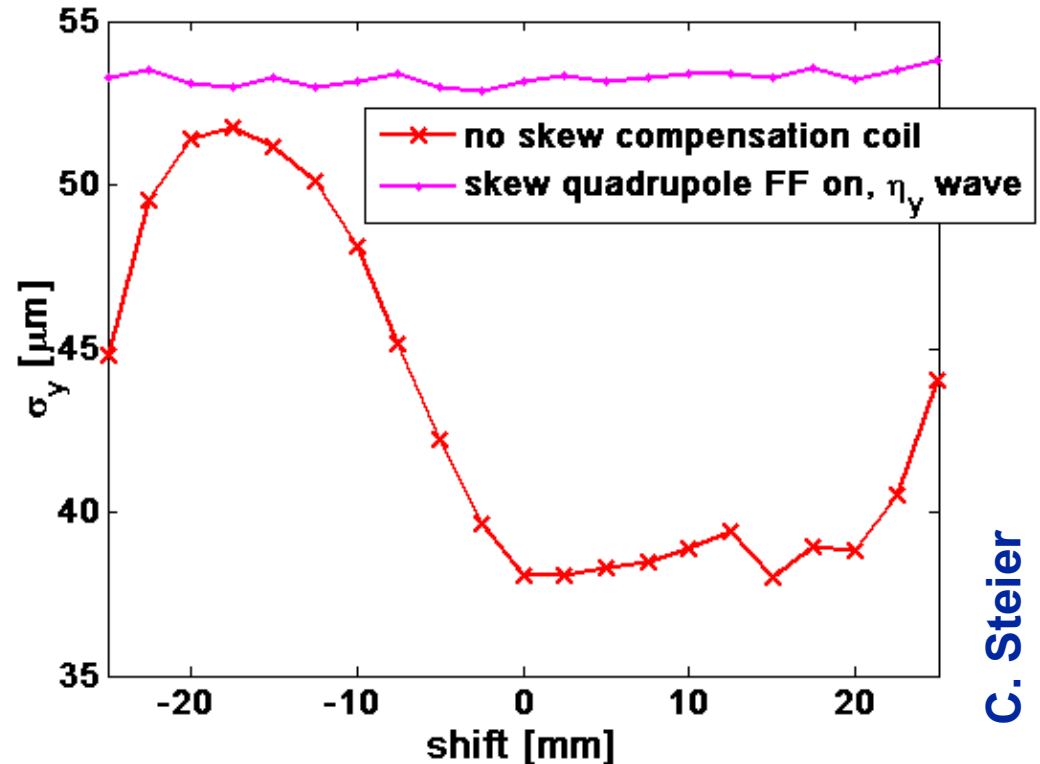
# Optics correction at SPEAR3



# Skew quadrupole compensation for ALS EPU's



- Beamsizes variation was solved in 2004: Installed correction coils for feedforward based compensation – routine use since June/September
- Early 2005 we identified the root cause: 2-3 micron correlated motion of magnet modules due to magnetic forces
- Will be able to modify design of future device such that active correction will not be necessary!



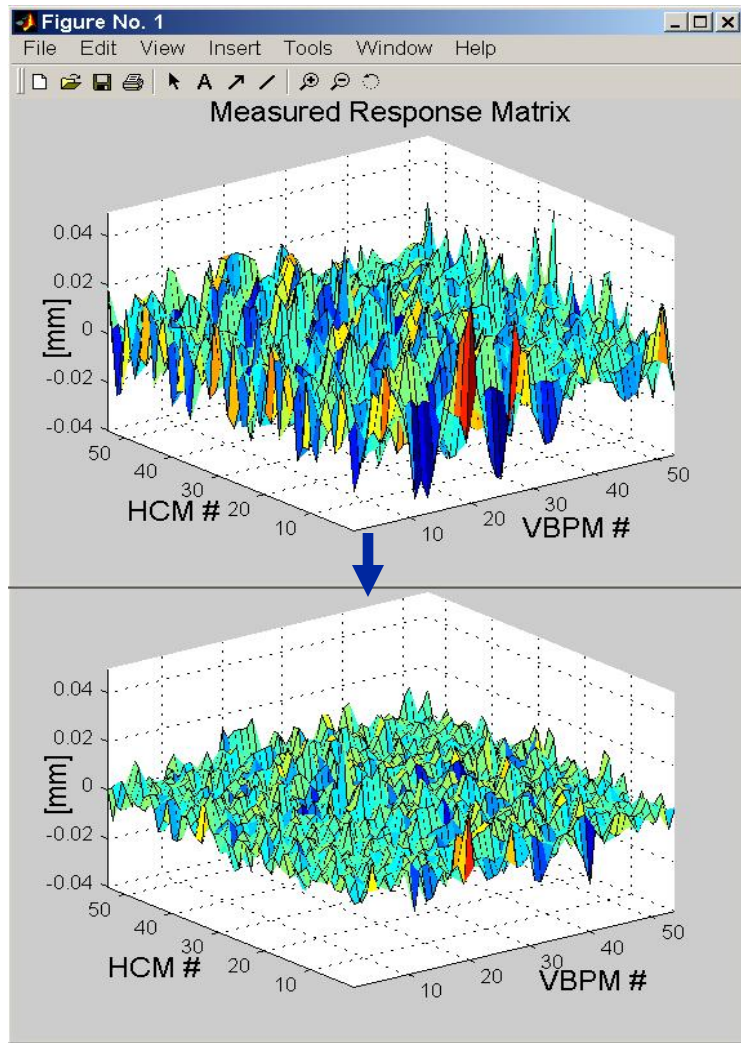
C. Steier

- Just for reference: Whenever an undulator moves, about 120-150 magnets are changed to compensate for the effect (slow+fast feed-forward, slow+fast feedback)

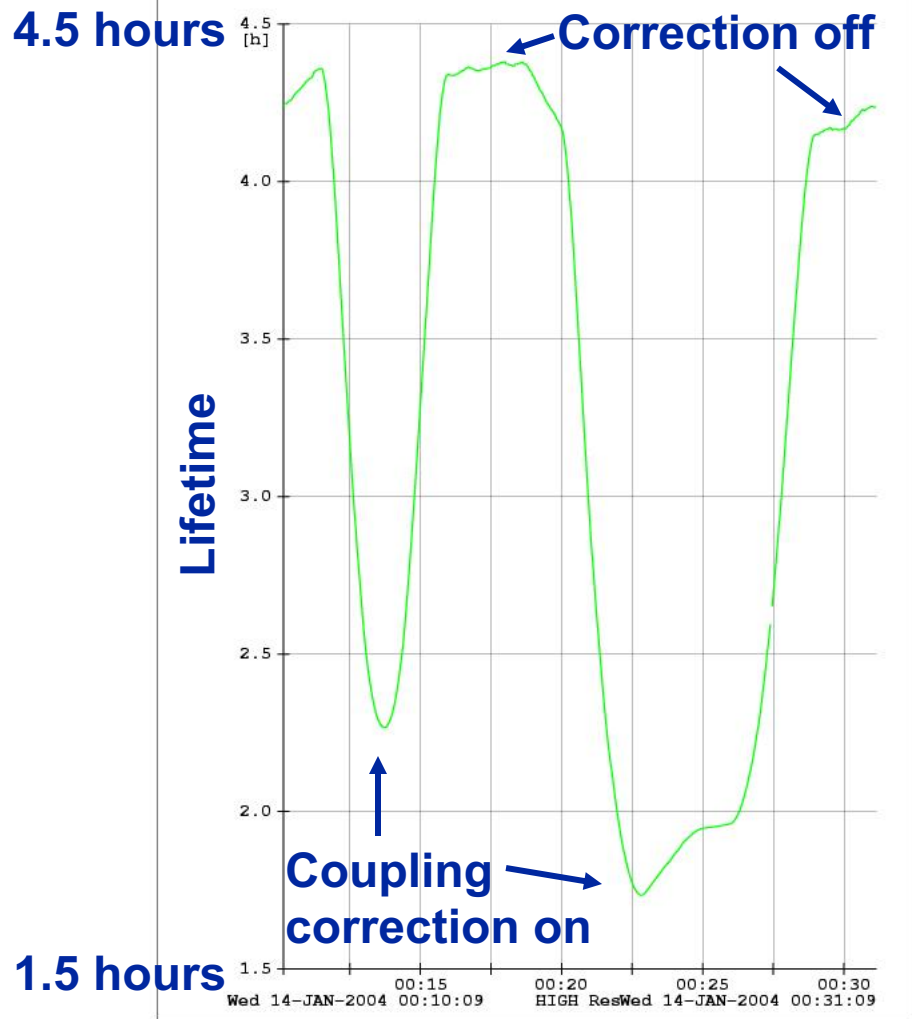


# Coupling & $\eta_y$ correction, LOCO

Minimize  $\eta_y$  and off-diagonal response matrix:



Lifetime, 19 mA, single bunch



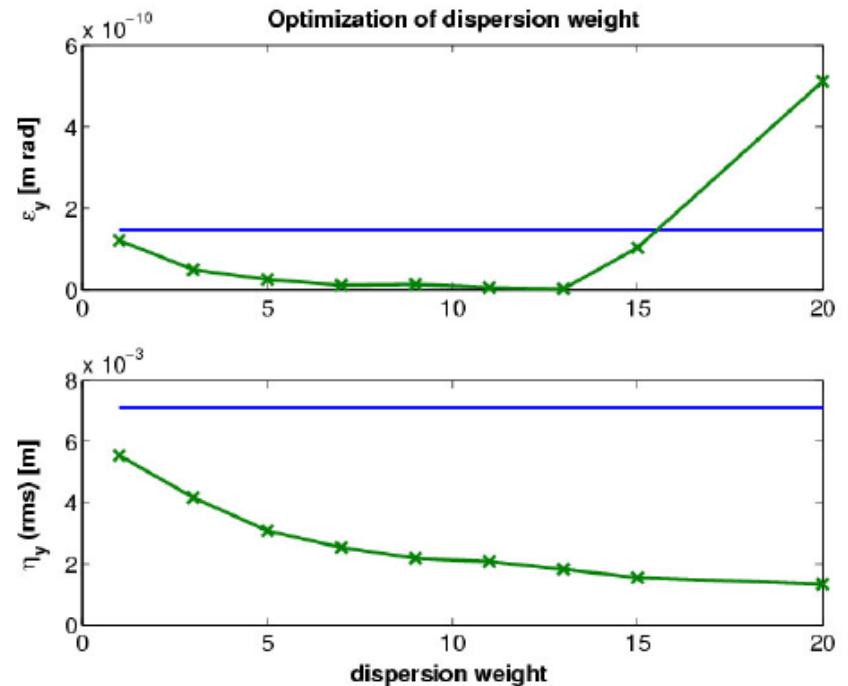


# Simulation of coupling correction with LOCO

- ❖ Use accelerator toolbox (Andrei Terebilo), Matlab and LOCO (James Safranek, Greg Portman) for simulations
- ❖ Use random skew error seeds
- ❖ Try to find effective skew corrector distributions and to optimize correction technique in simulation
- ❖ Used two correction approaches:
  1. Response Matrix fitting – ‘deterministic’, small number of iterations
  2. Direct minimization (nelder-simplex, ...) – easy to do on the model, difficult on real machine
- Surprisingly both approaches gave about the same performance in the model calculations
- For response matrix analysis you have to optimize several parameters of the code as well (weight of dispersion, number of SVs, use of effective model/full model ...)

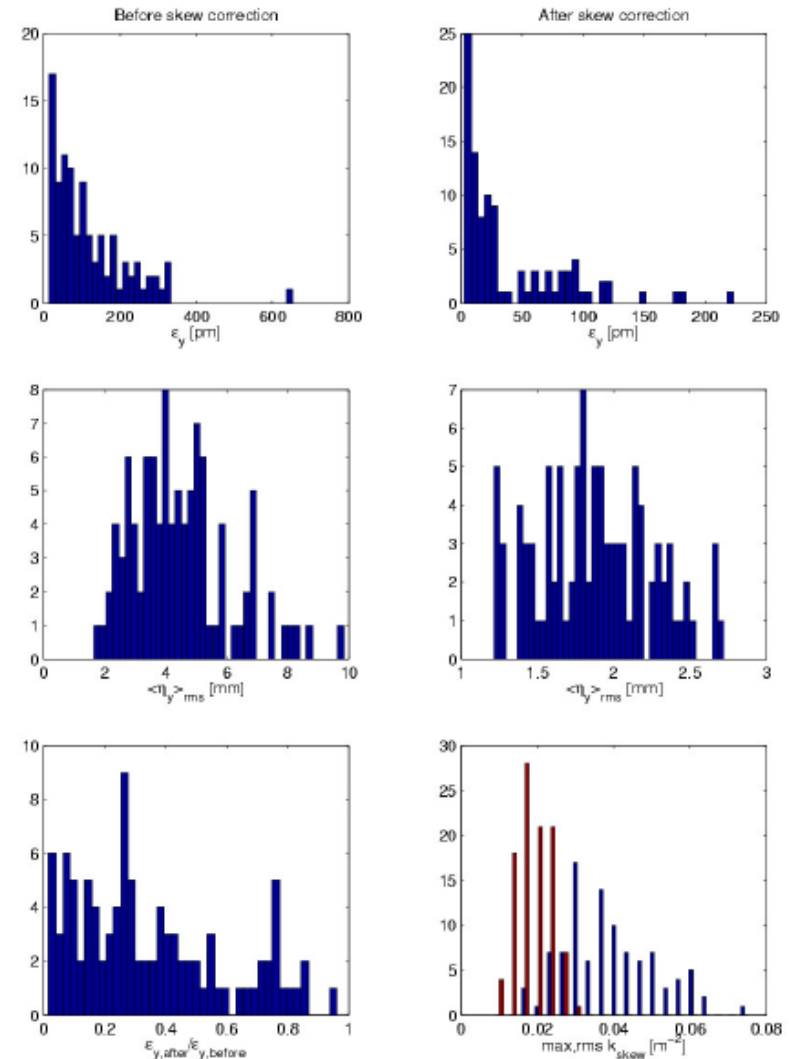
# Weight of dispersion in LOCO fit

- ❖ The relative contribution of vertical dispersion and coupling to the vertical emittance depends on the particular lattice (and the particular error distribution).
- ❖ Therefore the optimum weight for the dispersion in the LOCO fit has to be determined (experimentally or in simulations).
- ❖ The larger the weight factor, the better the vertical dispersion gets corrected, but eventually the coupling ‘explodes’.
- ❖ Set weight to optimum somewhat below that point.
- ❖ Outlier rejection tolerance might be important parameter as well.



# Finding an Effective Skew Quadrupole Set

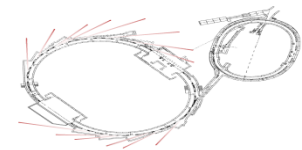
- ❖ To find an effective skew quadrupole distribution, we used several correction methods, first in simulations – best method was **orbit response matrix fitting (using LOCO)**
- ❖ **Predictive method, can be easily used on real machine**
- ❖ Issues are:
  - Cover set of phases relative to dominant coupling resonance(s)
  - Magnets should be distributed around the ring in order to avoid excessive local coupling/vertical dispersion
  - Need different values of dispersion/beta function to be effective both for coupling and vertical dispersion correction
- ❖ Set of **12 skew quadrupoles** was **reasonably efficient**



Advanced Light Source



# Skew quadrupole corrector distribution



- Distribute in difference coupling resonance phase

$$\kappa = \frac{1}{4} \pi \int ds K_s \sqrt{\beta_x \beta_y} e^{i\Phi_D} \quad \frac{\Phi_D(s)}{2\pi} = (\mu_x(s) - \mu_y(s)) - \frac{s}{C} (\nu_x - \nu_y - N)$$

- In sum coupling resonance phase

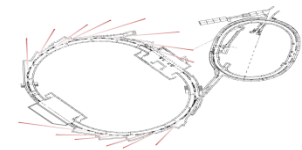
$$\frac{1}{4} \pi \int ds K_s \sqrt{\beta_x \beta_y} e^{i\Phi_S} \quad \frac{\Phi_S(s)}{2\pi} = (\mu_x(s) + \mu_y(s)) - \frac{s}{C} (\nu_x + \nu_y - M)$$

- And in  $\eta_y$  phase

$$\eta_x \sqrt{\beta_y} e^{i\Phi_{\eta_y}} \quad \frac{\Phi_{\eta_y}(s)}{2\pi} = \mu_y(s) - \frac{s}{C} (\nu_y - N)$$

↪ Need some skew quadrupoles at non-zero  $\eta_x$

# LOCO GUI fitting options menu



Remove bad BPMs or steerers from fit.

Include coupling terms ( $M_{xy}$ ,  $M_{yx}$ )

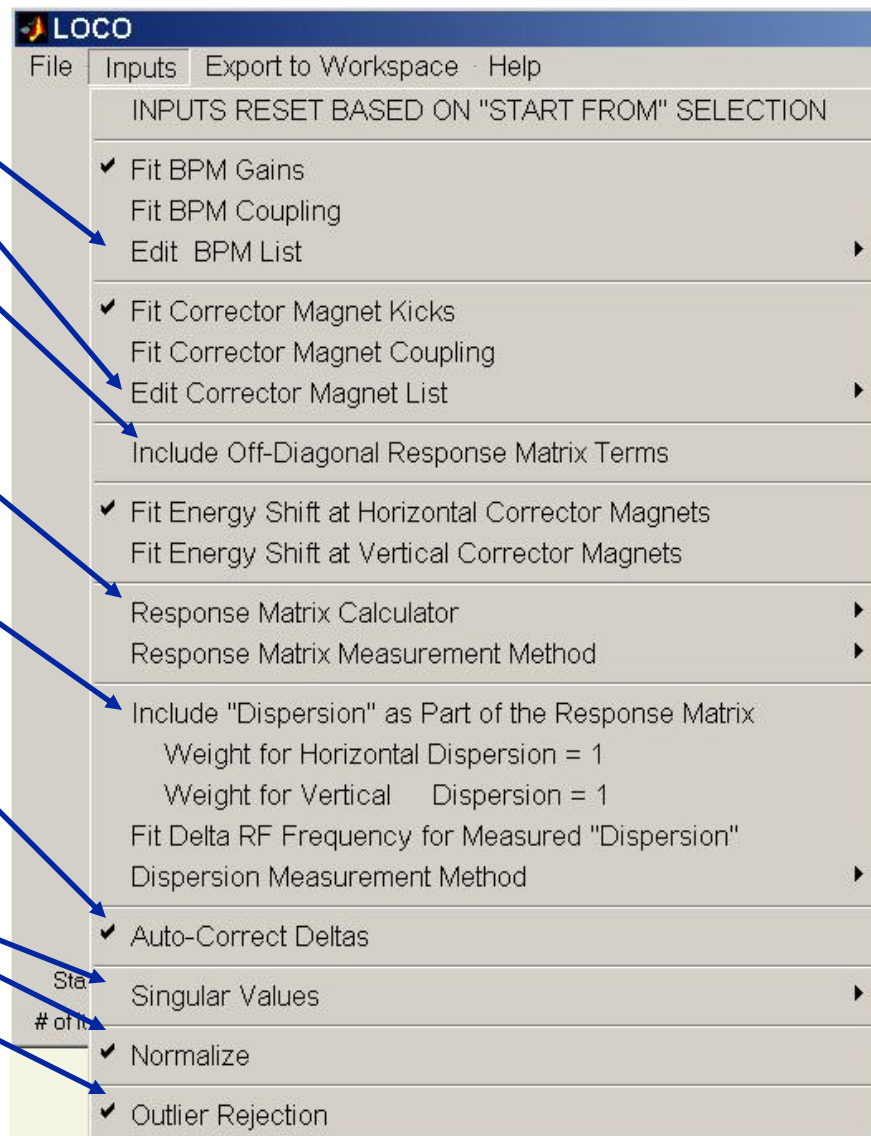
Model response matrix: linear or full non-linear; fixed-momentum or fixed-path-length

Include  $\eta$  as extra column of  $M$

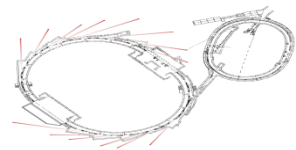
Let program choose  $\Delta$ s when calculating numerical derivatives of  $M$  with quadrupole gradients.

More on these coming.

Reject outlier data points.



# Error bars from BPM measurement noise



LOCO calculates the error bars on the fit parameters according to the measured noise levels of the BPMs. LOCO uses singular value decomposition (SVD) to invert  $\partial E_k / \partial K_l$  and solve for fit parameters.

$$-E_k = \frac{\partial E_k}{\partial K_l} \Delta K_l \quad E_k = \frac{M_{ij}^{\text{meas}} - M_{ij}^{\text{model}}}{\sigma_i}$$

The results from SVD are useful in calculating and understanding the error bars.

$$\mathbf{J} = \frac{\partial \vec{E}}{\partial \vec{K}} = \mathbf{U} \mathbf{S} \mathbf{V}^T = \sum \vec{u}_l w_l \vec{v}_l^T$$

SVD reduces the matrix to a sum of a product of eigenvectors of parameter changes,  $\mathbf{v}$ , times eigenvectors,  $\mathbf{u}$ , which give the changes in the error vector,  $\mathbf{E}$ , corresponding to  $\mathbf{v}$ . The singular values,  $w_l$ , give a measure of how much a change of parameters in the direction of  $\mathbf{v}$  in the multidimensional parameter space changes the error vector.

(For a more detailed discussion see Numerical Recipes, Cambridge Press.)



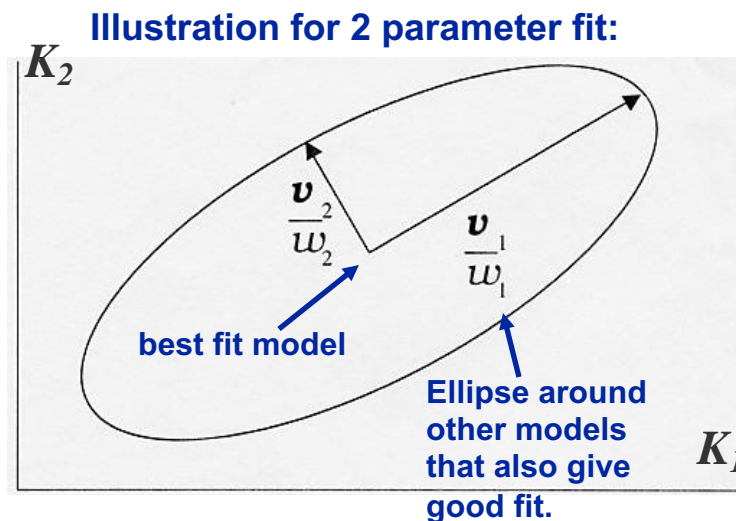
# SVD and error bars

$$\frac{\partial \bar{E}}{\partial \bar{K}} = \mathbf{U} \mathbf{S} \mathbf{V}^T = \sum \bar{u}_l w_l \bar{v}_l^T$$

A small singular value,  $w_l$ , means changes of fit parameters in the direction  $v_l$  make very little change in the error vector. The measured data does not constrain the fit parameters well in the direction of  $v_l$ ; there is relatively large uncertainty in the fit parameters in the direction of  $v_l$ . The uncertainty in fit parameter  $K_l$  is given by

$$\sigma^2(K_m) = \sum_l \frac{1}{\sigma_l^2} [\bar{v}_l]_m^2$$

Together the  $v_l$  and  $w_l$  pairs define an ellipse of variances and covariances in parameter space. LOCO converges to the center of the ellipse. Any model within the ellipse fits the data as well, within the BPM noise error bars.



# SVD and error bars, II

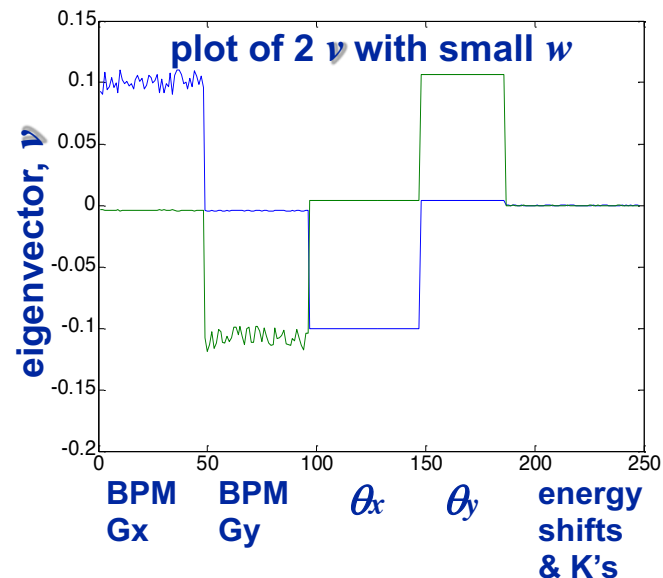
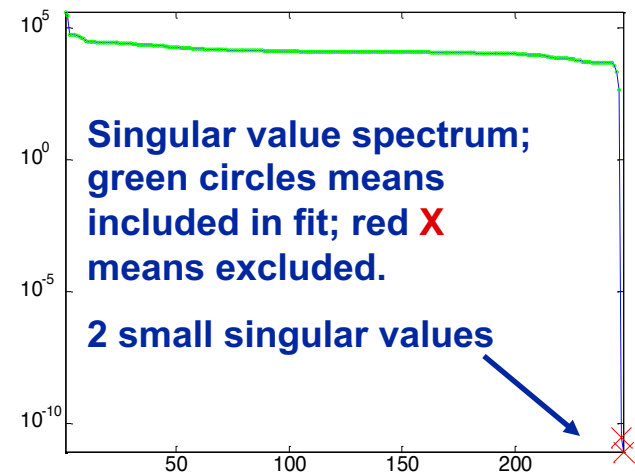


Eigenvectors with small singular values indicate a direction in parameter space for which the measured data does not constrain well the fit parameters.

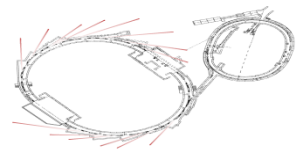
The two small singular values in this example are associated with a degeneracy between fit BPM gains and steering magnet kicks. If all BPM gains are increased and kicks decreased by a single factor, the response matrix does not change.

There two small singular values – horizontal and vertical plane.

This problem can be eliminated by including coupling terms in the fit and including the dispersion as a column of the response matrix (without fitting the rf frequency change).

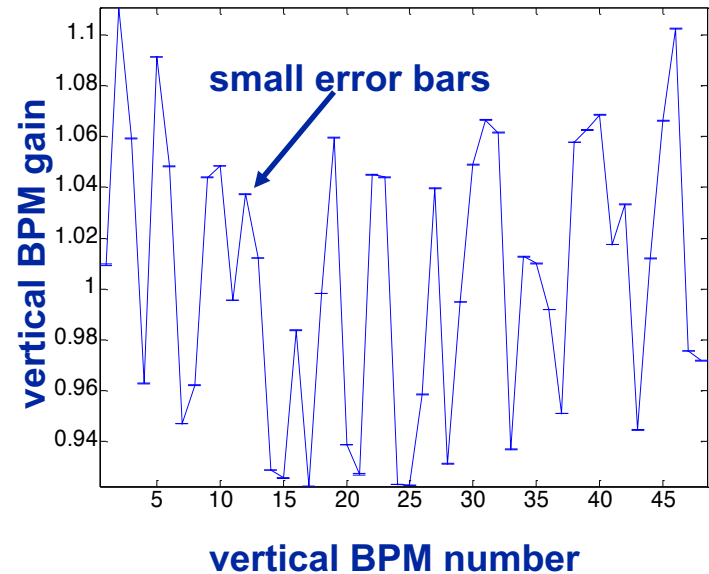


# SVD and error bars, III

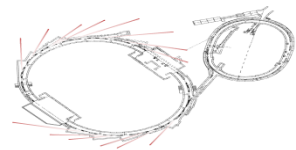


LOCO throws out the small singular values when inverting  $\partial\vec{E}/\partial\vec{K}$  and when calculating error bars. This results in small error bars calculated for BPM gains and steering magnet kicks. The error bars should be interpreted as the error in the relative gain of one BPM compared to the next. The error in absolute gain is much greater.

If other small singular values arise in a fit, they need to be understood.



# Analyzing multiple data sets



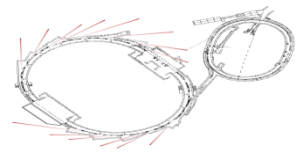
Analyzing multiple data sets provides a second method for investigating the variation in fit parameters from measurement noise. The results shown here are for the NSLS X-Ray ring, and are in agreement with the error bars calculated from analytical propagation of errors.

## ERROR BARS ON THE FIT PARAMETERS DUE TO RANDOM ERROR IN THE MEASURED ORBIT.

The variations given in this table are the rms error bars on the fit parameters due to random orbit measurement errors. We measured the response matrix ten times, and fit a model to each response matrix. Then, for each of the parameters we took the average over the ten data sets and calculated the rms variation from the average.

Parameter	rms variation
quadrupole gradients	.04 %
quadrupole rolls	.4 mrad
BPM gain	.05 %
BPM rolls	.5 mrad
BPM C-parameter	.0004
steering magnet calibration	.05 %
steering magnet rolls	.8 mrad
steering magnet longitudinal center	2 mm
steering magnet fractional energy shift	3.4E-7
$\beta$ functions	.08 %

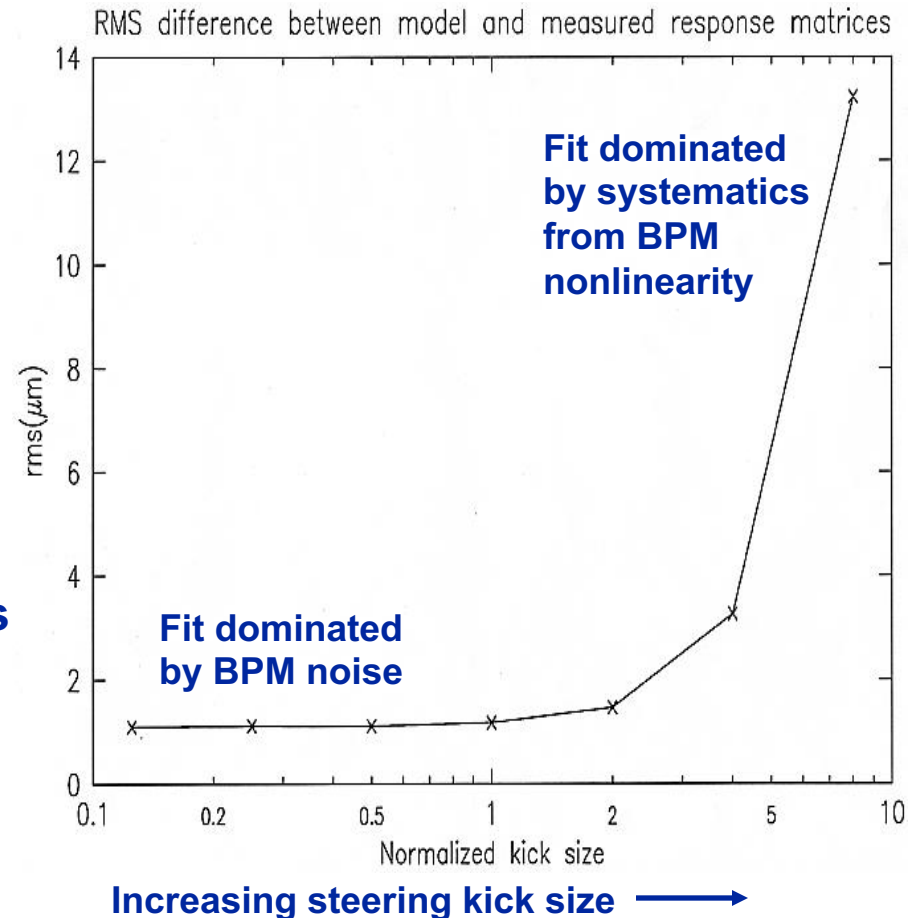
# Systematic error



The error in fit parameters from systematic differences between the model and real rings is difficult to quantify.

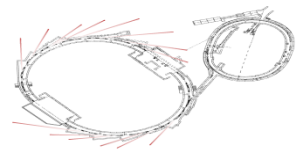
Typical sources of systematic error are:

- Magnet model limitations – unknown multipoles; end field effects.
- Errors in the longitudinal positions of BPMs and steering magnets.
- Nonlinearities in BPMs.
  - electronic and mechanical
  - avoid by keeping kick size small.





# Systematic error, II



With no systematic errors, the fit should converge to

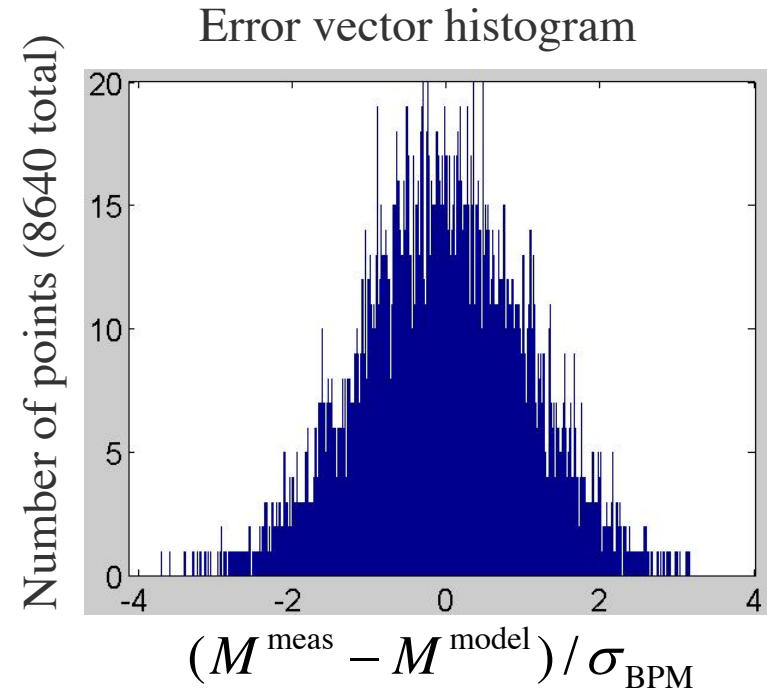
$$\chi^2/\text{D.O.F.} = 1 \pm \sqrt{2/\text{D.O.F.}}$$

D.O.F. = N - M = degrees of freedom

N = # of data points

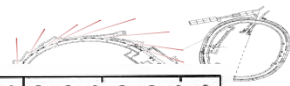
M = # of fit parameters

This plot shows results with simulated data with  $\chi^2/\text{D.O.F.} = 1.01$ . With real data the best fit I've had is  $\chi^2/\text{D.O.F.} \approx 1.2$  fitting NSLS XRay ring data to  $1.2 \mu\text{m}$  for  $1.0 \mu\text{m}$  noise levels. Usually  $\chi^2/\text{D.O.F.}$  is considerably larger.

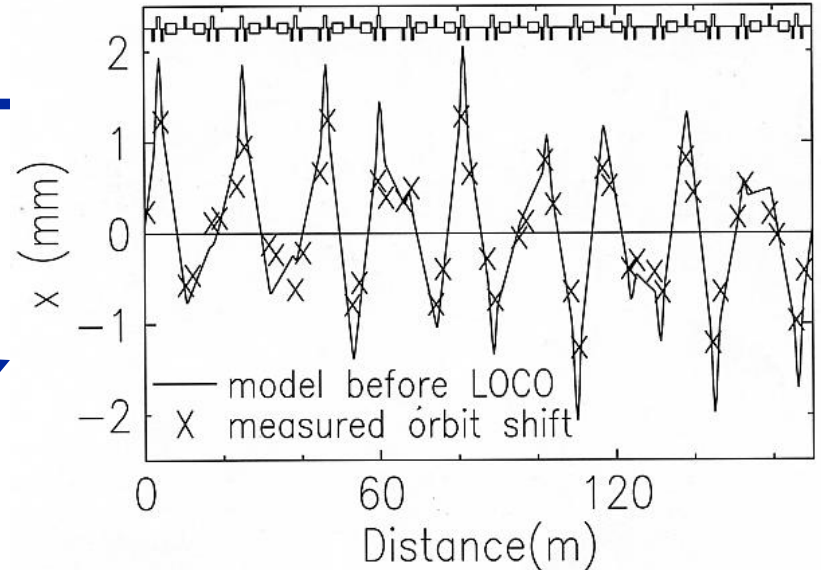


**The conclusion:** In a system as complicated as an accelerator it is impossible to eliminate systematic errors. The error bars calculated by LOCO are only a lower bound. The real errors include systematics and are unknown. The results are still not useless, but they must be compared to independent measurements for confirmation.

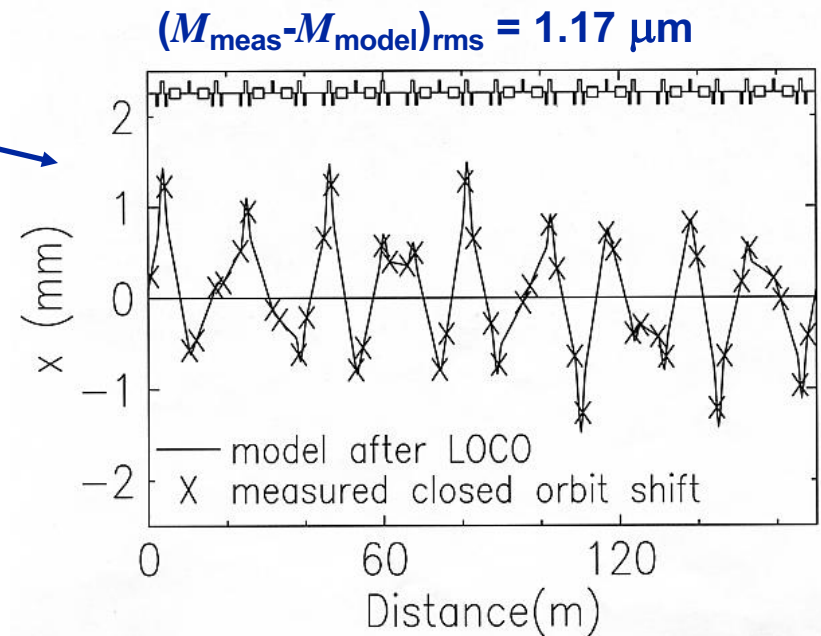
# LOCO fit for NSLS X-Ray Ring



Before fit, measured and model response matrices agree to within ~20%.



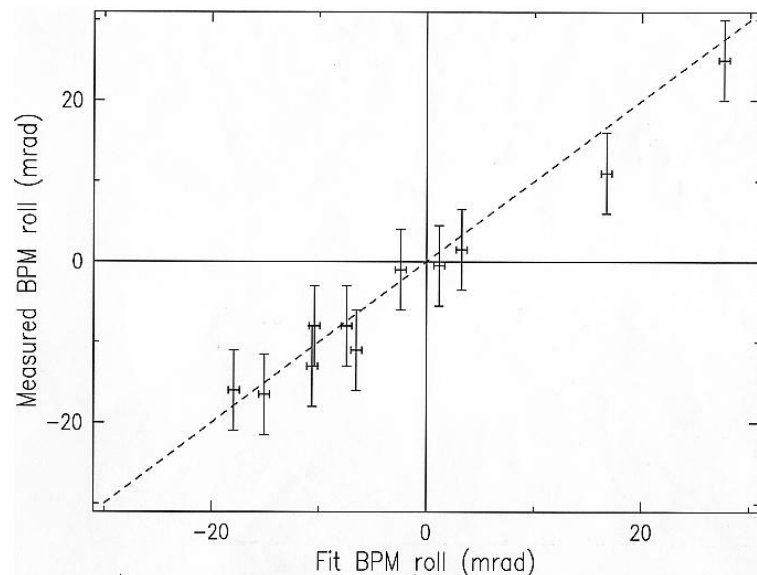
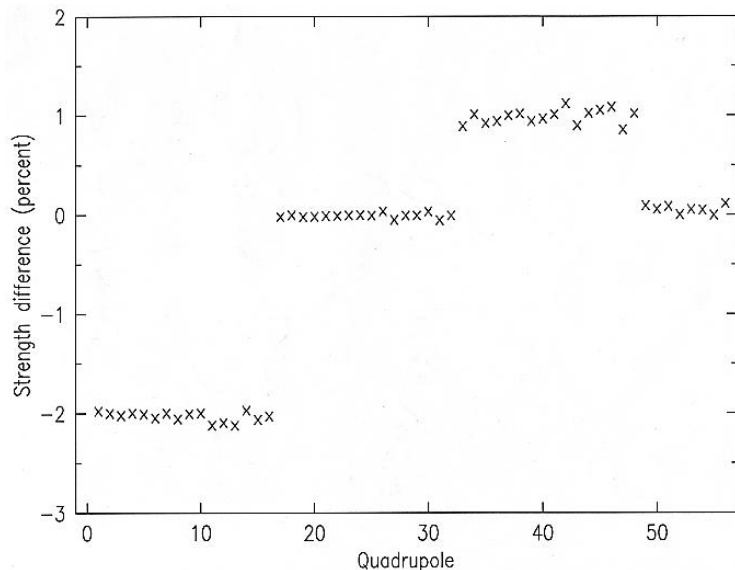
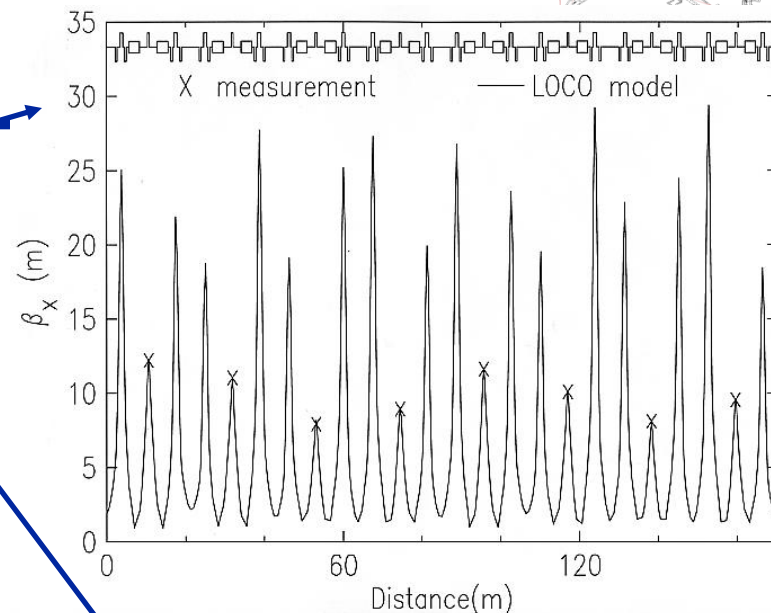
After fit, response matrices agree to  $10^{-3}$ .



# Confirming LOCO fit for X-Ray Ring

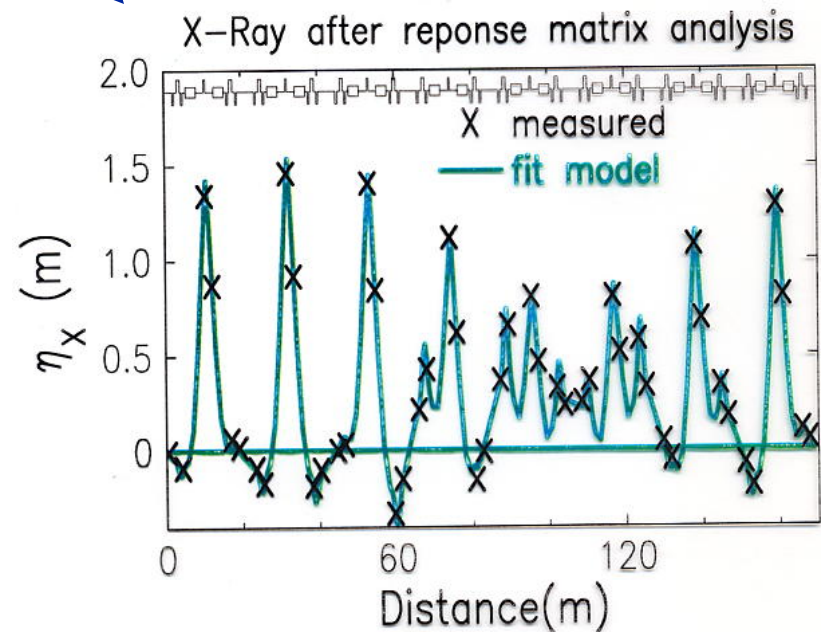
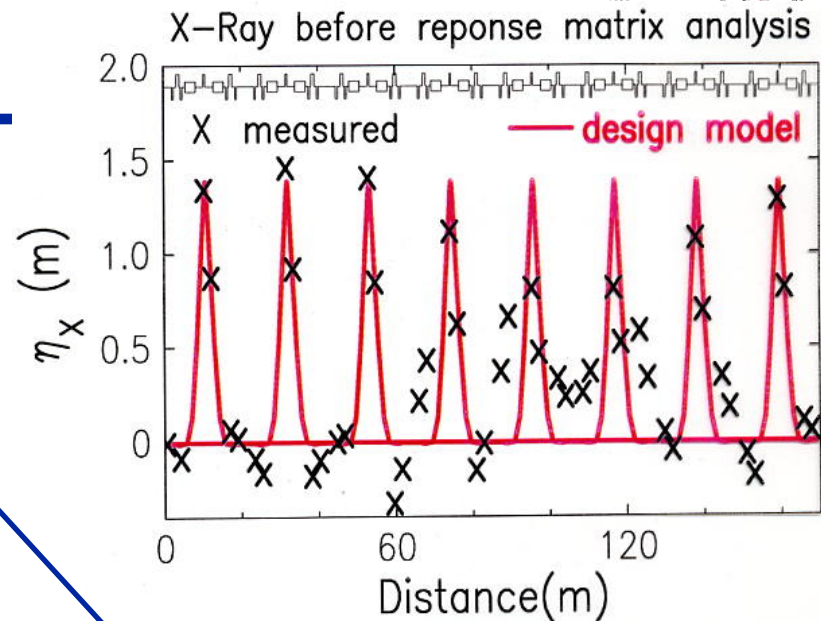
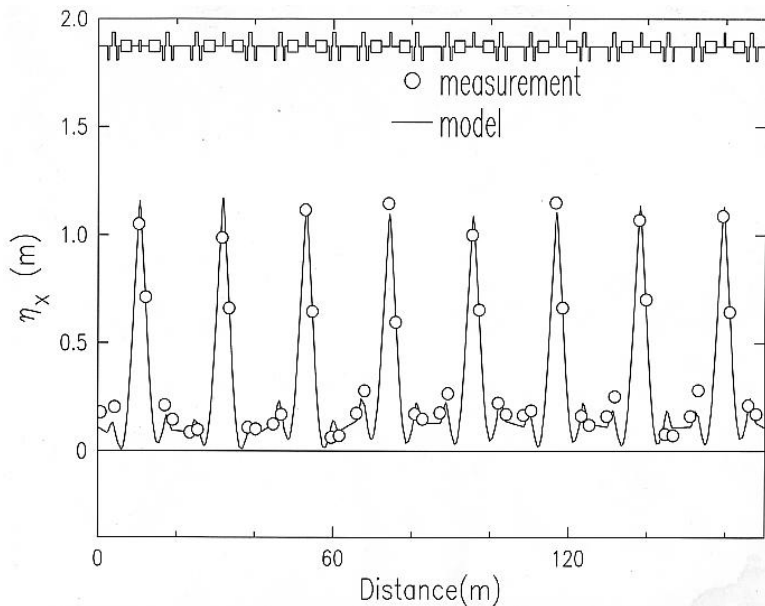
LOCO predicts measured  $\beta$ 's, BPM roll.

LOCO confirms known quadrupole changes, when response matrices are measured before and after changing optics.

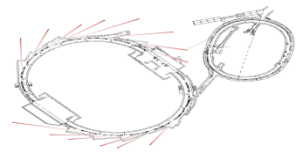


# Correcting X-Ray Ring $\eta_x$

LOCO predicts measured  $\eta_x$ ,  
and is used to find gradient  
changes that best restore  
design periodicity.



# NSLS X-Ray Ring Beamsizes

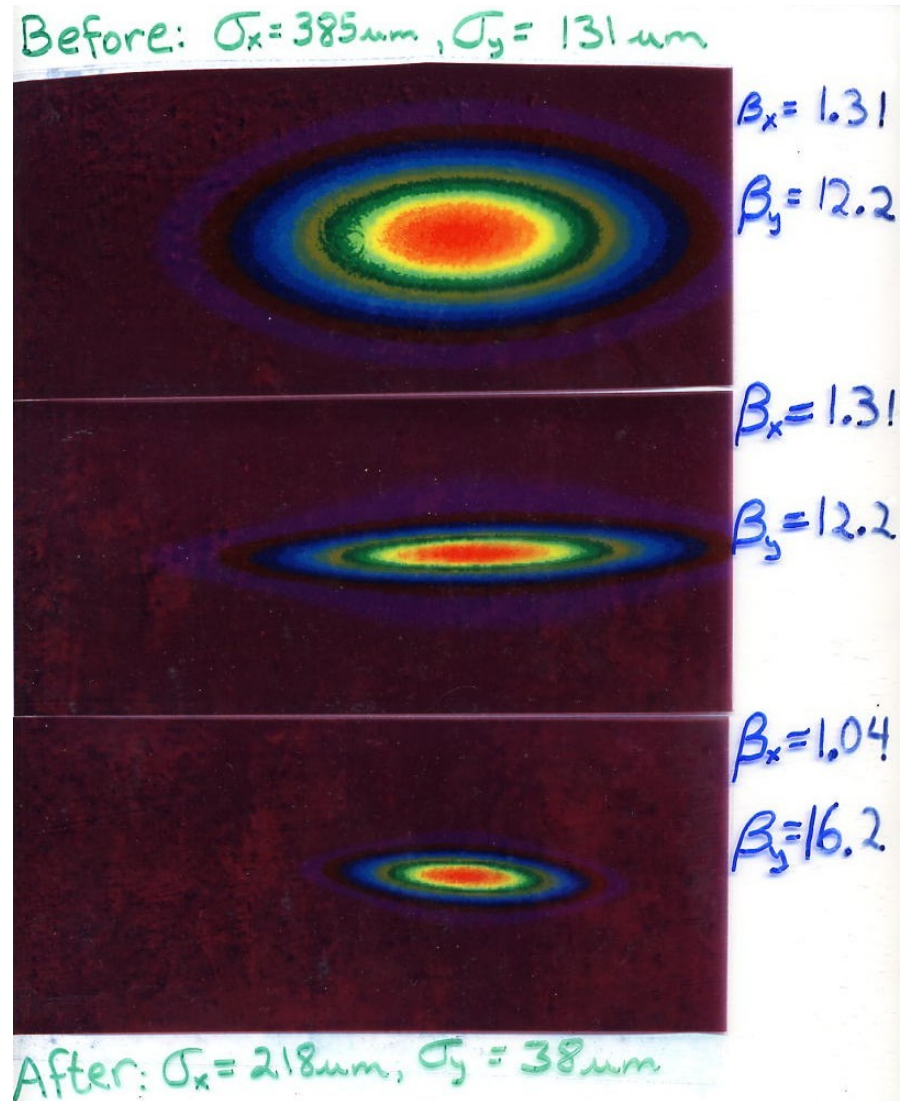


The improved optics control in led to reduction in the measured electron beam size.

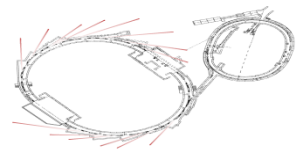
The fit optics gave a good prediction of the measured emittances. The vertical emittance is with coupling correction off.



	model	measurement
horizontal emittance	93.3 nm*rad	94.2 nm*rad
vertical emittance	6.6 nm*rad	8.6 nm*rad



# Original algorithm sometimes failed



- Original LOCO algorithm worked well for NSLS rings, ALS, PEP-II, SLC damping rings, Taiwan Light Source, ... SPEAR3 (at first)
- Problem correcting SPEAR3 optics after adding three quadrupoles

Correlations between quadrupole changes:

$$\rho_{12} = \frac{\mathbf{J}_1^T \mathbf{J}_2}{\|\mathbf{J}_1\| \|\mathbf{J}_2\|},$$

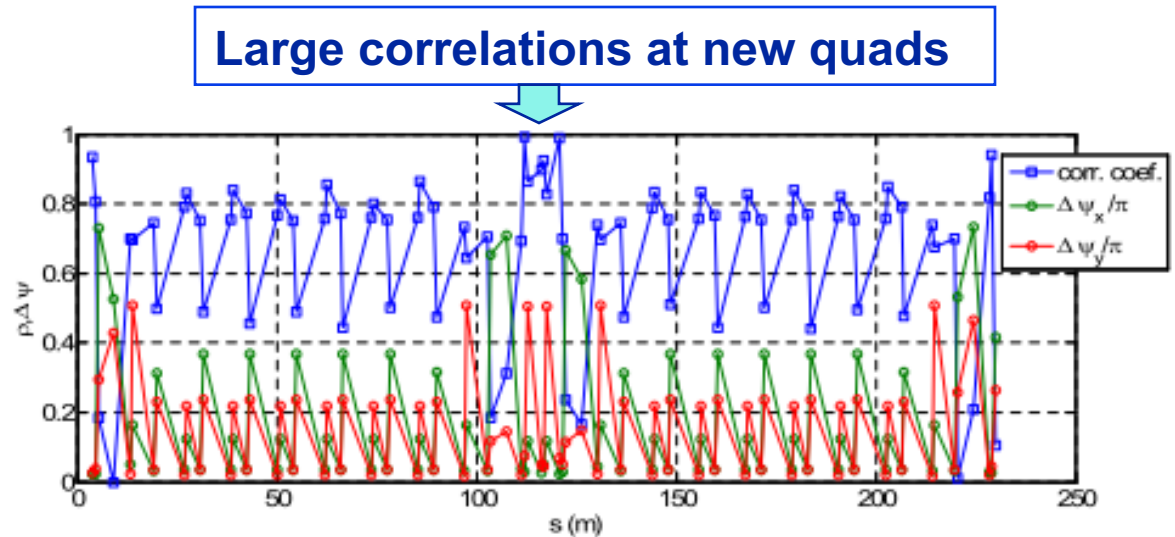
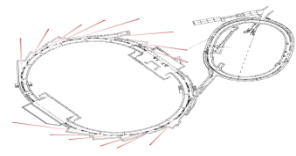


Figure 1: the correlation coefficients and betatron phase advances between neighboring quadrupoles (excluding QFC) in SPEAR3.

- LOCO fits were unreliable and produced large, unrealistic quadrupole changes
- Similar LOCO problems at SOLEIL, DIAMOND, DAFNE, ...

# Constrained fitting (Xiaobiao Huang)



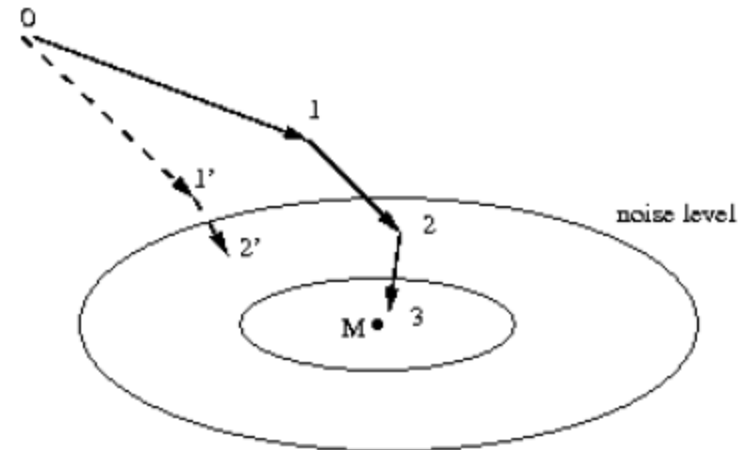
- **Constrained fitting to limit quadrupole gradient changes in fit.**

↪ Instead, add constraints to restrict magnitude of gradient changes:

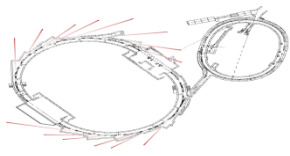
$$\chi^2 = \sum_{i,j} \frac{(M_{\text{mod},ij} - M_{\text{meas},ij})^2}{\sigma_i^2} + \frac{1}{\sigma_{\Delta K}^2} \sum_k w_k^2 \Delta K_k^2$$

$$\mathbf{J} \begin{bmatrix} \Delta K_1 \\ \Delta K_2 \\ \vdots \end{bmatrix} = \mathbf{R}^{\text{meas}} - \mathbf{R}^{\text{mod}} \longrightarrow \begin{bmatrix} \mathbf{J} \\ \text{-----} \\ w_1 \ 0 \ 0 \ \dots \\ 0 \ w_2 \ 0 \ \dots \\ \vdots \end{bmatrix} \begin{bmatrix} \Delta K_1 \\ \Delta K_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} \mathbf{R}^{\text{meas}} - \mathbf{R}^{\text{mod}} \\ \text{-----} \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

↪ **Gauss-Newton with cost function**



# Levenberg-Marquardt fitting algorithm



- More robust convergence for LOCO fitting when starting with large errors

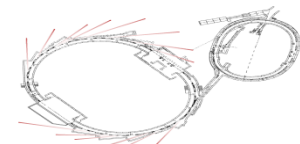
$$\mathbf{J} \Delta \mathbf{K} = \mathbf{R}^{\text{meas}} - \mathbf{R}^{\text{mod}} \longrightarrow (\mathbf{J}^T \mathbf{J} + \lambda \text{diag}(\text{diag}(\mathbf{J}^T \mathbf{J}))) \Delta \mathbf{K} = \mathbf{J}^T (\mathbf{R}^{\text{meas}} - \mathbf{R}^{\text{mod}})$$

Gauss-Newton Gauss-Newton/steepest-descent, depending on  $\lambda$

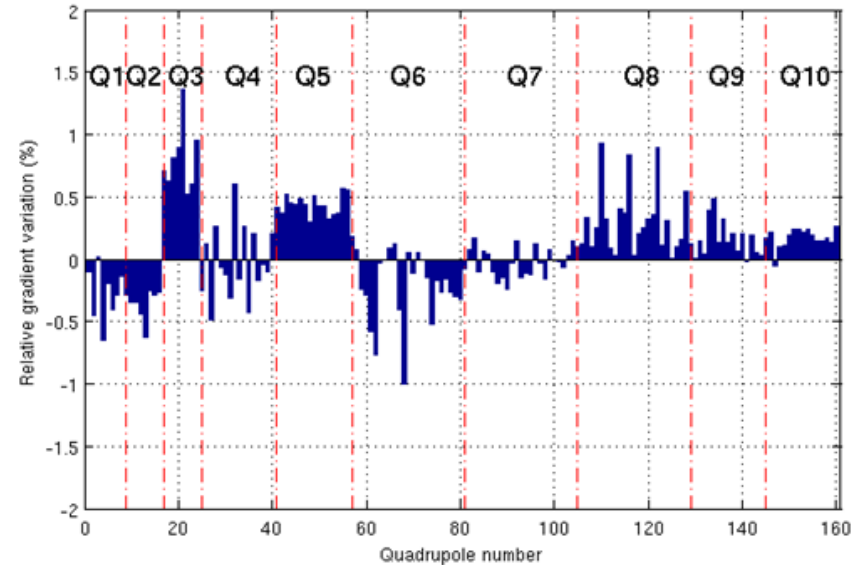
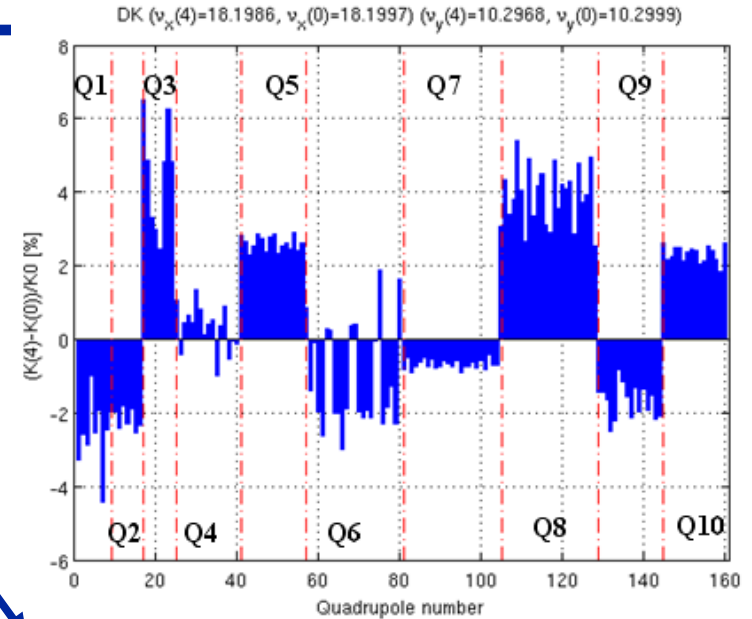
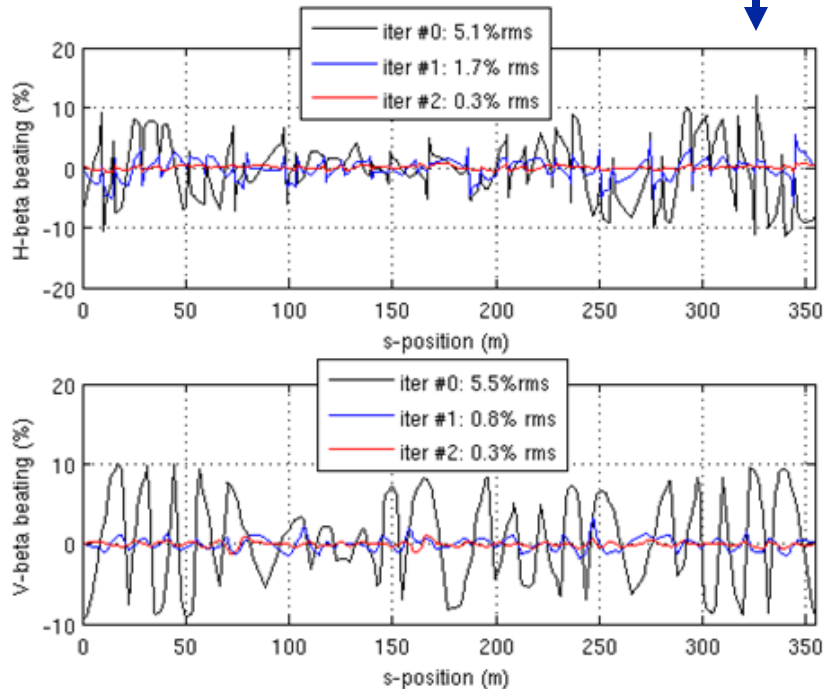
- Algorithm automatically adjusts  $\lambda$ .
  - ↪ Large  $\lambda > 1$  at first when far from  $\chi^2$  minimum; steepest decent
  - ↪ Small  $\lambda \ll 1$  once close to minimum; Gauss-Newton method.
- Useful when correcting optics for the first time, when errors are particularly large.



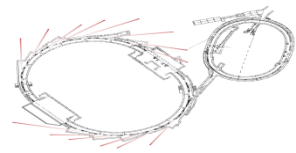
# Optics correction at SOLEIL



- Fit gradients w/out constraints (>6% corrections)
- Fit gradients with constraints (<1.5% corrections)
- Successfully corrected optics errors

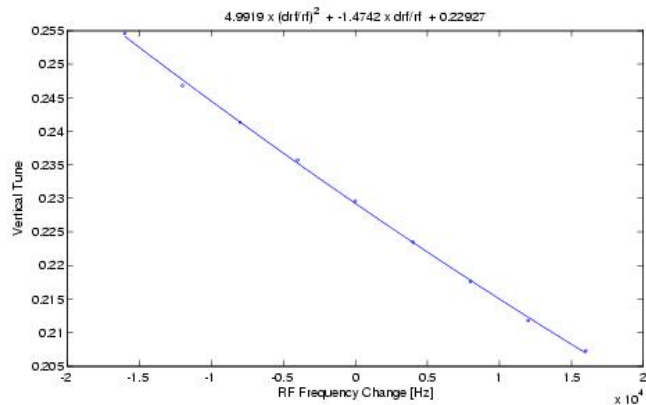
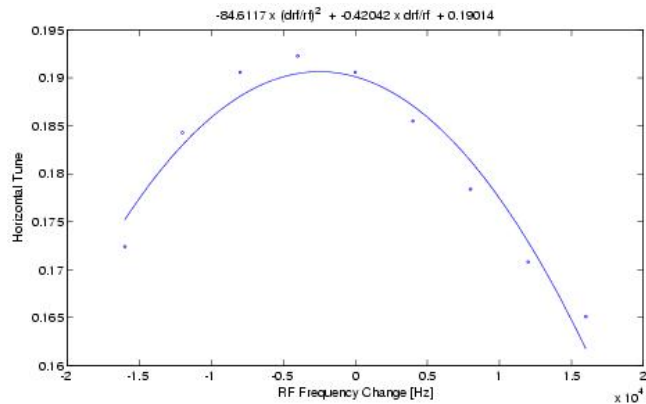


# Chromaticity

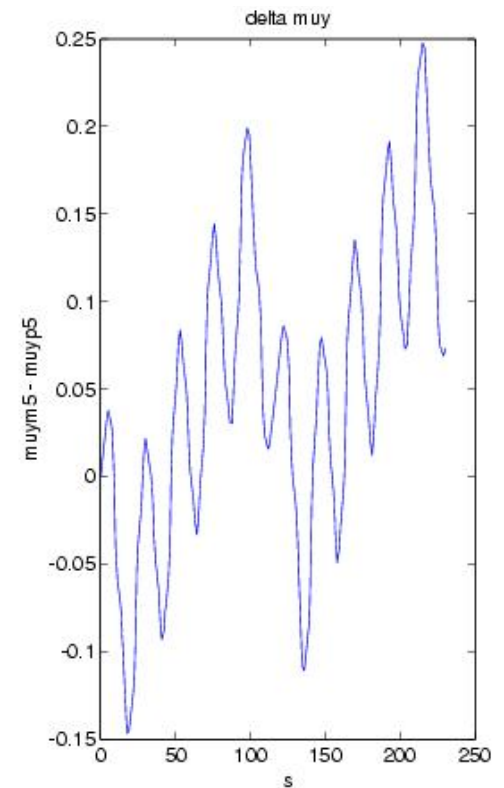
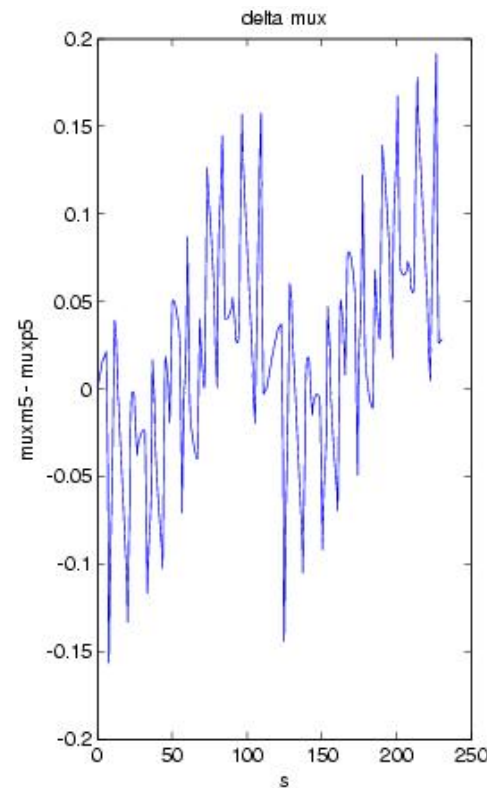


Nonlinear  $\xi$ :

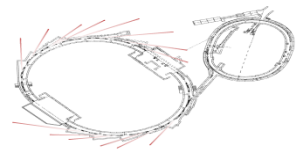
$(v_x, v_y)$  vs.  $f_{rf}$  agrees with model.



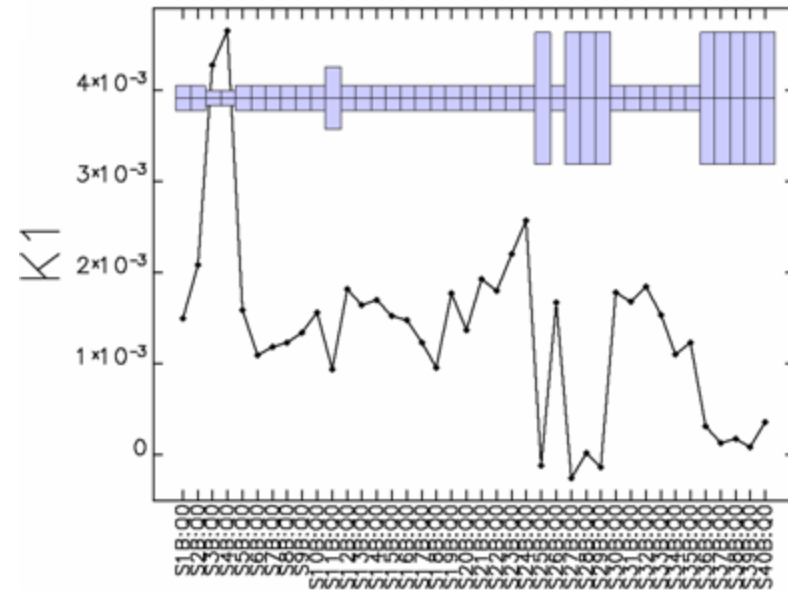
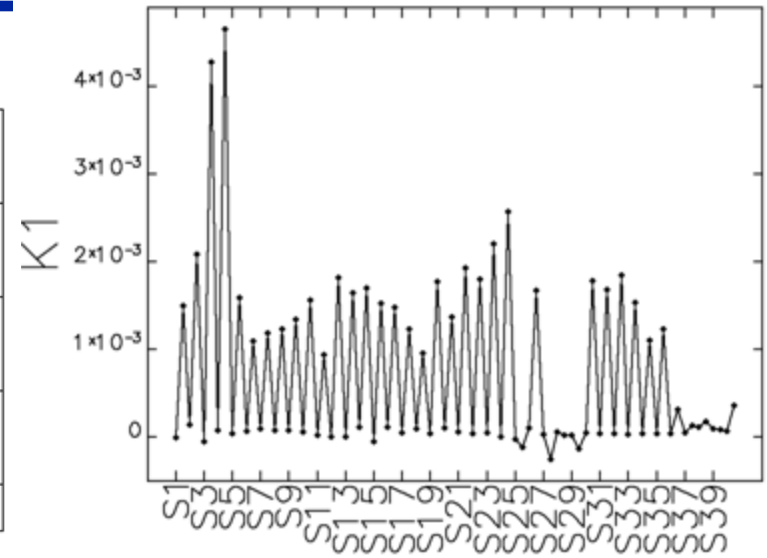
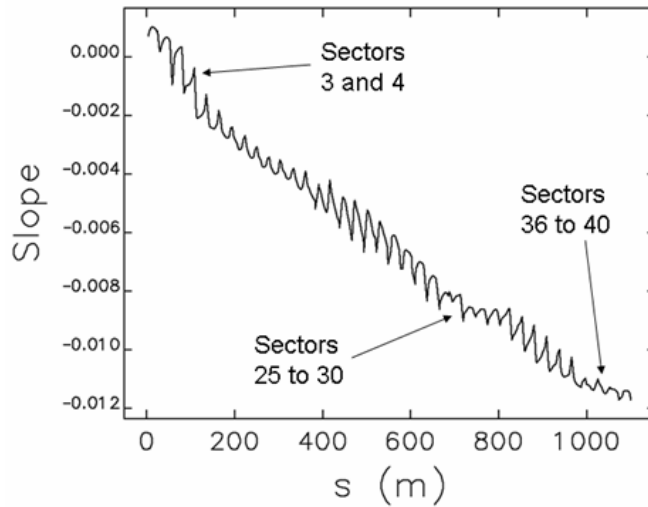
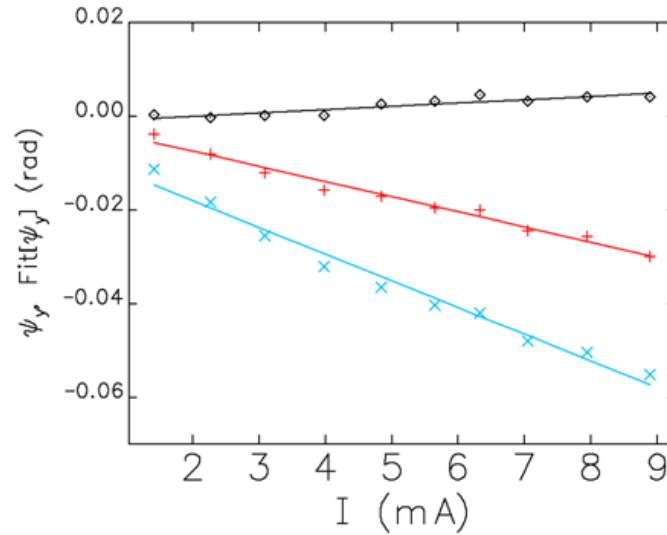
Local chromaticity calibrated with LOCO shows no sextupole errors:



# Transverse impedance at APS (V. Sajaev)

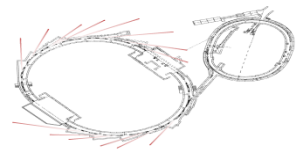


$$Z_{eff}^i = \frac{1}{2} \frac{E \sigma_s \beta_q d(K_1 L)}{R \beta_i} dl$$



# Further reading

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See December, 2007 ICFA Beam Dynamics Newsletter #44, [http://icfa-usa.jlab.org/archive/newsletter/icfa\\_bd\\_nl\\_44.pdf](http://icfa-usa.jlab.org/archive/newsletter/icfa_bd_nl_44.pdf). Includes X. Huang article on constrained fitting.

Numerical Recipes, Cambridge University Press, is an excellent reference for SVD,  $\chi^2$  model fitting, and error bars, as well many other numerical techniques for analyzing data.

J. Safranek, “Experimental determination of storage ring optics using orbit response measurements”, Nucl. Inst. and Meth. A388, (1997), pg. 27.

D. Robin, J. Safranek, W. Decking, “Realizing the benefits of restored periodicity in the advanced light source”, Phys. Rev. Special Topics-AB, v. 2 (1999).

Search <http://accelconf.web.cern.ch/AccelConf/> “Text of paper” for LOCO.

The LOCO code is available at

[http://als.lbl.gov/als\\_physics/portmann/MiddleLayer/applications](http://als.lbl.gov/als_physics/portmann/MiddleLayer/applications)

LOCO uses Andrei Terebilo’s AT accelerator modeling code to calculate response matrices. AT is available at <http://ssrl.slac.stanford.edu/at/>

The idea for LOCO came from previous work:

W.J. Corbett, M.J. Lee, and V. Ziemann, “A fast model calibration procedure for storage rings,” SLAC-PUB-6111, May, 1993.