Turn-by-turn BPM data analysis

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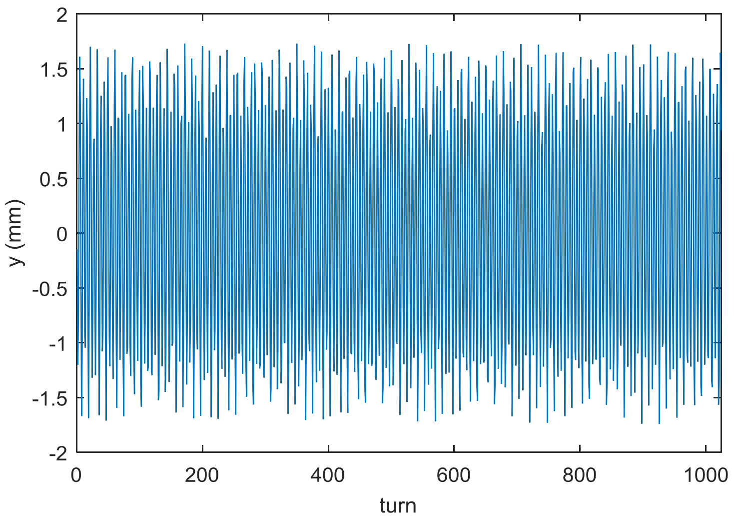
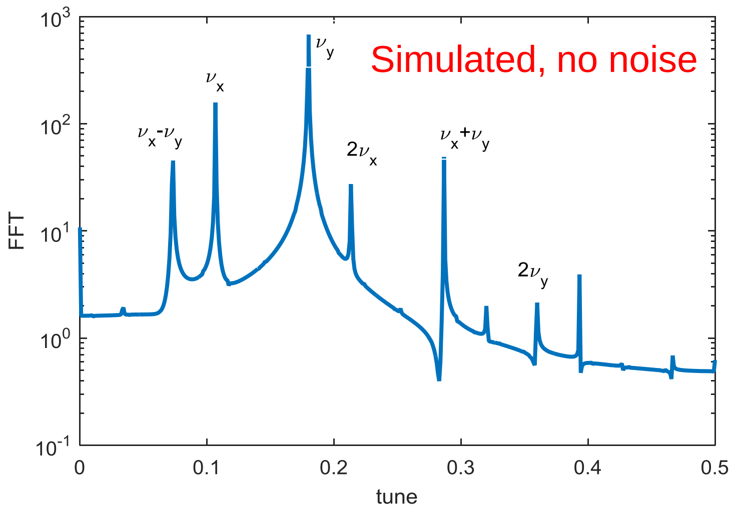
Outline

- Turn-by-turn BPM data example
- Precise tune measurements
- Model Independent Analysis (MIA) and Independent Component Analysis (ICA) for TbT BPM data
- Fitting ICA results for optics and coupling correction
- Fitting TbT data directly for optics and coupling correction
- Fitting trajectory scan data for linac/FEL optics correction

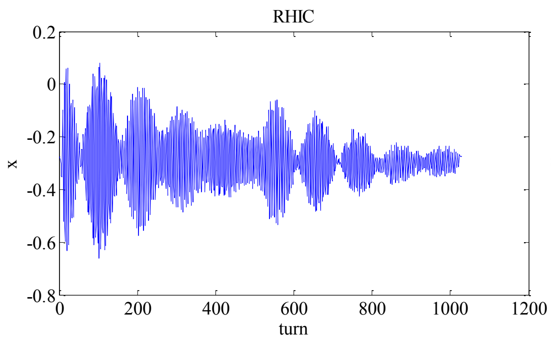
Turn-by-turn (TbT) BPM data

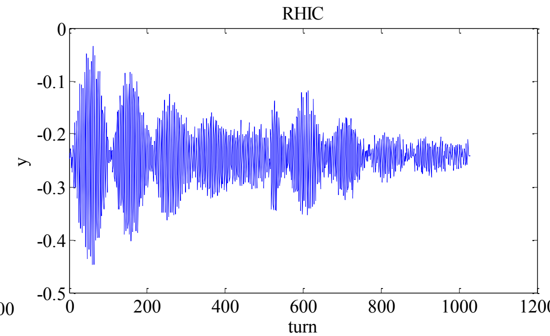
- In addition to the average orbit, TbT BPM data capture the dynamics of beam motion.
- TbT BPM data contain
  - Temporal information:
    - various frequency components from single particle dynamics
    - De-coherence/re-coherence – a collective effect
  - Spatial information: BPMs at different location see different amplitudes and phases of the frequency components.
  - Noise

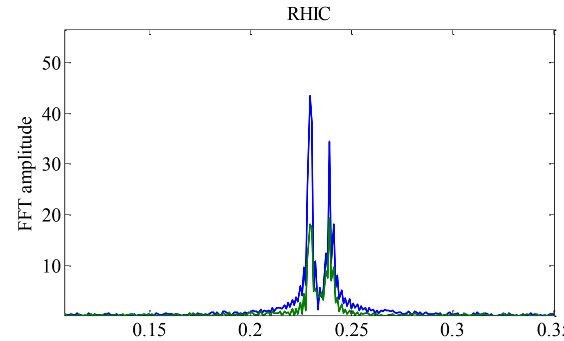


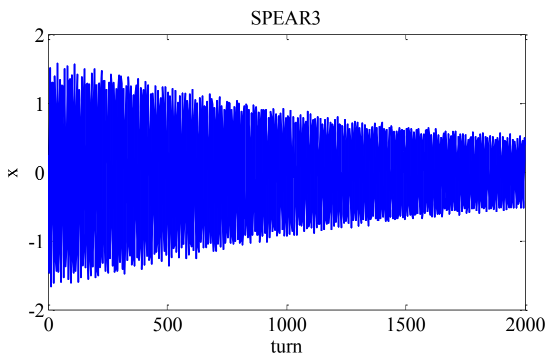


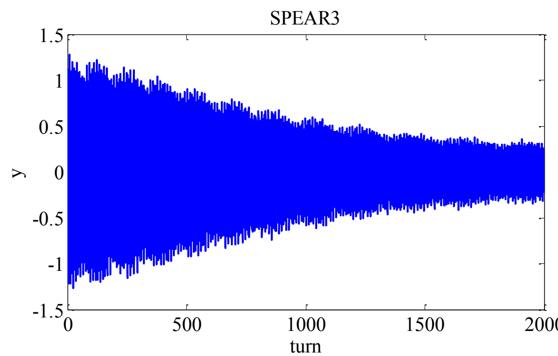
Examples of turn-by-turn BPM data

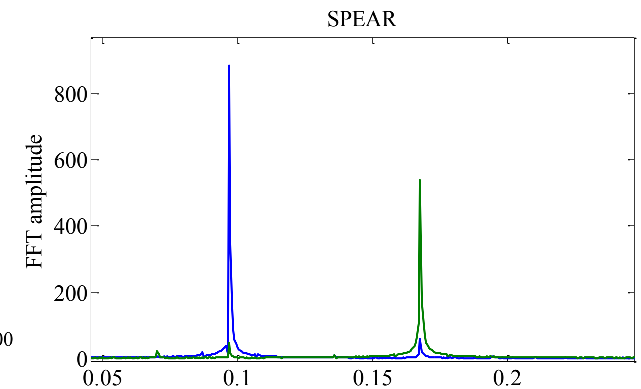


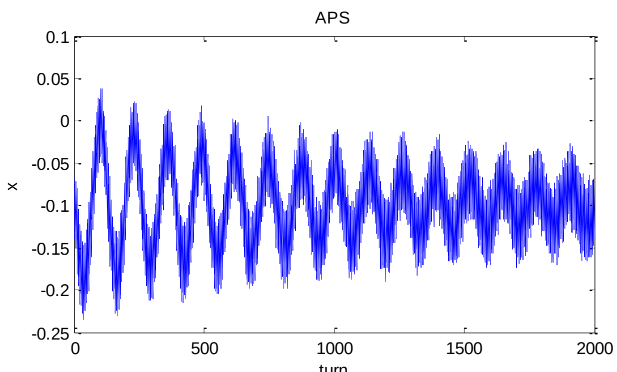


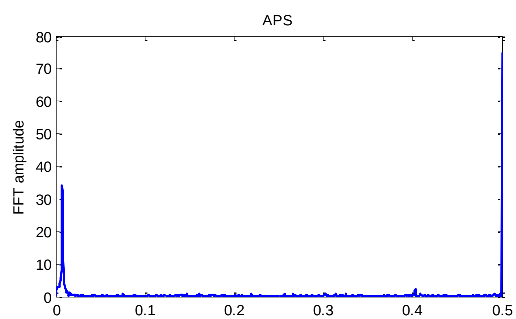












Precise tune determination from discrete data

- NAFF (numerical analysis of fundamental frequency)
  - Maximize the spectrum overlap between the sample and that of a pure sinusoidal signal. J. Laskar, et al,1990, Icarus 88, 266-291

$$F(\bar{\nu}) = \left| \sum_{n=1}^{N} s_n e^{-i2\pi\bar{\nu}n} W(n) \right|$$

- Interpolated FFT R. Bartolini, et al, EPAC 96
  - Interpolation with peak frequency and its highest neighbor.

$$\nu_{Fint} = \frac{k}{N} + \frac{1}{\pi}\text{atan}\left(\frac{|\phi(\nu_{k+1})|\sin(\frac{\pi}{N})}{|\phi(\nu_k)| + |\phi(\nu_{k+1})|\cos(\frac{\pi}{N})} \right)$$

These methods can achieve accuracy $\propto \frac{1}{N^2}$, or $\propto \frac{1}{N^4}$ when with data windowing.

Extracting frequency components

- With the tune determined, amplitude and phase of the frequency component can be obtained by

$C = \sum_n x(n) \cos 2\pi n\nu, \qquad S = \sum_n x(n) \sin 2\pi n\nu,$

Then amplitude and phase are

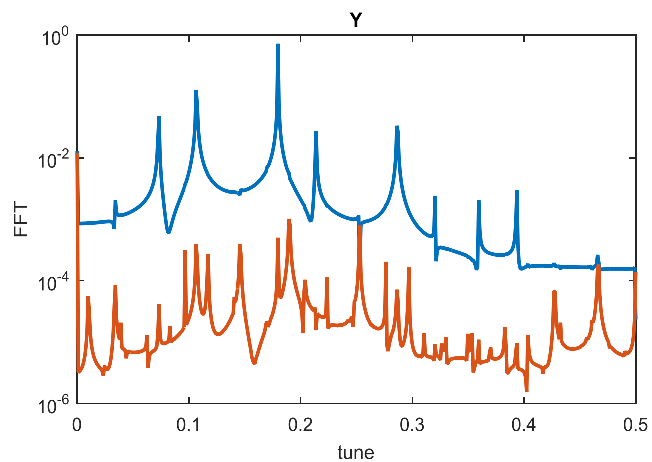
$A = \frac{2\sqrt{C^2+S^2}}{N}$, and $\psi = -\cot^{-1}\frac{S}{C}$,

where N is the number of turns. Error of phase

$$\sigma_\psi = \frac{1}{A}\sqrt{\frac{2}{N}}\sigma_x$$

P. Castro, et al, PAC'93

- The frequency components can be iteratively extracted.



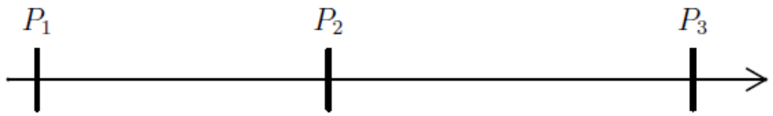
J. Laskar, et al,1990, Physica D, 67, 257, (1993)

The amplitudes and phases of the various frequency components from BPMs at different locations can be used for optics, coupling, and nonlinear dynamics correction. (See Christoph's lecture)

A. Franchi, et al, PRSTAB 14, 034002 (2011)
A. Franchi, et al, PRSTAB 17, 074001 (2014)

The three-BPM method

- The measured phase advances on three BPMs can be used to determine the beta function (aided by a model)



Denote the transfer matrix from P_1 to P_2, $\mathbf{A} = \mathbf{M}(P_2|P_1)$, from P_1 to P_3, $\mathbf{B} = \mathbf{M}(P_3|P_1)$. We have

$$\frac{A_{11}}{A_{12}} = \frac{\cot\psi_{21} + \alpha_1}{\beta_1}, \quad \frac{B_{11}}{A_{12}} = \frac{\cot\psi_{31} + \alpha_1}{\beta_1},$$

where ψ_{21} and ψ_{31} are phase advances from P_1 to P_2 and P_3, respectively, and α_1 and β_1 are C-S parameters at P_1.

$$\frac{B_{11}}{B_{12}} - \frac{A_{11}}{A_{12}} = \frac{\cot\psi_{31} - \cot\psi_{21}}{\beta_1},$$

$$\rightarrow \quad \beta_1|_{\mathrm{meas}} = \beta_1|_{\mathrm{model}} \frac{\cot\psi_{31} - \cot\psi_{21}|_{\mathrm{meas}}}{\cot\psi_{31} - \cot\psi_{21}|_{\mathrm{model}}}$$

P. Castro, et al, PAC'93

R. Tomas, et al, PRSTAB 13, 121004 (2010)

Measured phase advance and beta function can be used to fit model and correct the linear optics.

Model independent components and independent component analysis for TbT data processing

- In the previous method data processing is performed for each individual plane (x or y) of each BPM
- A better approach is to treat all BPM data collectively and coherently, because
  - All BPMs observe the same signals.
  - Number of BPMs is typically much bigger than the number of signals.
  - Statistical advantages can be achieved when proper methods are used to extract the signals from the large number of samples.
- Methods taking the coherent approach include
  - Model independent analysis (principal component analysis)
  - Independent component analysis.

J. Irwin, et al, PRL 82, 1684 (1999)

X. Huang, PRSTAB, 8, 064001, 2005

A model of BPM turn-by-turn data

- The turn-by-turn beam position signal is a combination of various source signals.

$$x_i(t) = \sum_j a_{ij} s_j(t) + n_j(t)$$

For the i'th BPM

or $$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$$

$\mathbf{A}$ is the mixing matrix

There are only a few meaningful source signals, such as betatron oscillation and synchrotron oscillation.

Form a matrix of the BPM data

$$\mathbf{x} = \begin{pmatrix} x_1(1) & x_1(2) & \cdots & x_1(T) \\ x_2(1) & x_2(2) & \cdots & x_2(T) \\ \vdots & \vdots) & \ddots & \vdots \\ x_m(1) & x_m(2) & \cdots & x_m(T) \end{pmatrix}$$

m BPMs and T turns

X. Huang, PRSTAB, 8, 064001, 2005

Betatron modes via singular value decomposition

It has been proven* that when the BPM reading contains only one betatron mode, i.e.

$$x_m(t) = \sqrt{2J(t)\beta_m}\cos(\phi(t)+\psi_m)$$

Note the constant orbit offsets are always removed for each BPM. This is called "centering".

then there are only two non-trivial SVD eigen-modes

$$\mathbf{x} = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathrm{T}} = s_+\mathbf{u}_+\mathbf{v}_+^{\mathrm{T}} + s_-\mathbf{u}_-\mathbf{v}_-^{\mathrm{T}}$$

U,V are orthogonal matrices,
S is a block-diagonal matrix.

$$u_{+,m} = \frac{1}{s_+}\sqrt{<J>\beta_m}\cos(\phi_0+\psi_m),$$

$$u_{-,m} = \frac{1}{s_-}\sqrt{<J>\beta_m}\sin(\phi_0+\psi_m)$$

$$v_+(t) = \sqrt{\frac{2J(t)}{T<J>}}\cos(\phi(t)-\phi_0),$$

$$v_-(t) = -\sqrt{\frac{2J(t)}{T<J>}}\sin(\phi(t)-\phi_0)$$

u: spatial vector

v: temporal vector

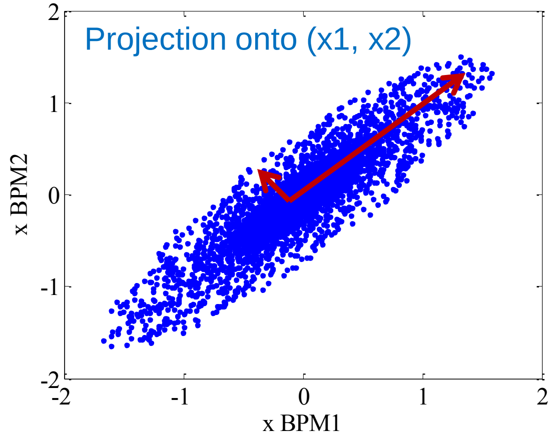
Beta function and betatron phase advance can be calculated from the spatial vector.

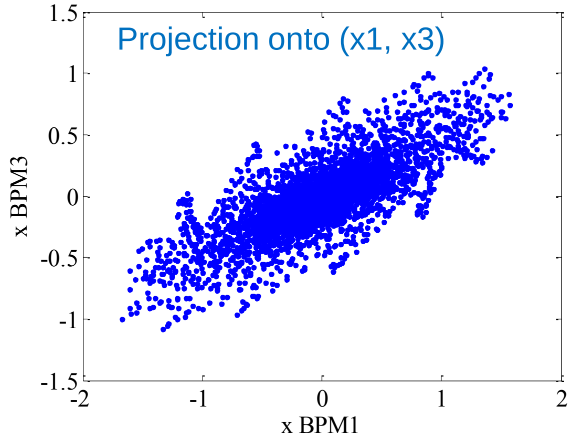
$$\psi_m = \tan^{-1}\left(\frac{s_-u_{-,m}}{s_+u_{+,m}}\right)$$

$$\beta_m = \frac{1}{<J>}[(s_+u_{+,m})^2 + (s_-u_{-,m})^2]$$

* Chun-xi Wang, et al. PR-STAB 6, 104001 (2003).

What does SVD do?





The BPM data can be viewed as T points in the m-dimensional space.

$$P(t) = (x_1(t), x_2(t), \cdots, x_m(t))$$

These points form an hyper-ellipsoid. What SVD does is to identify its principal-axes. This is called principal component analysis (PCA).
PCA: with a linear orthogonal transformation to obtain a set of linearly un-correlated components (variables) which holds (successively) the largest variances.

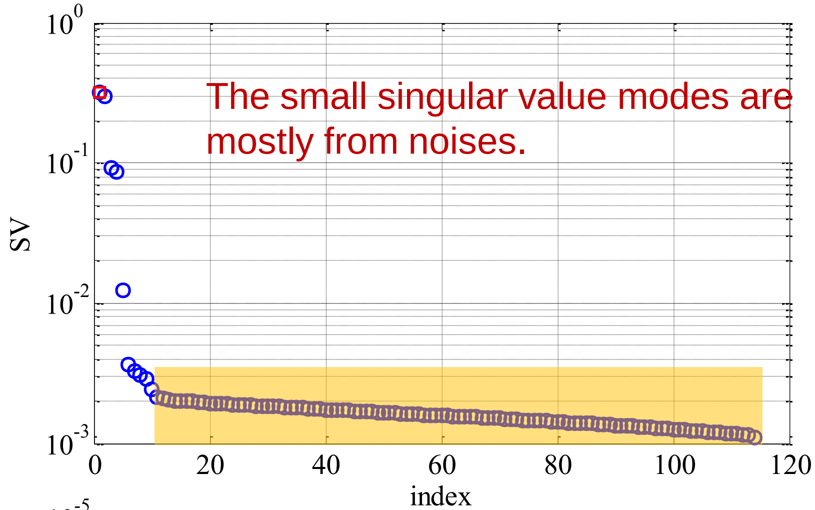
$$\mathbf{x}\mathbf{x}^T = \mathbf{U}\Sigma\mathbf{U}^T, \quad \Sigma = \mathbf{S}\mathbf{S}^T$$

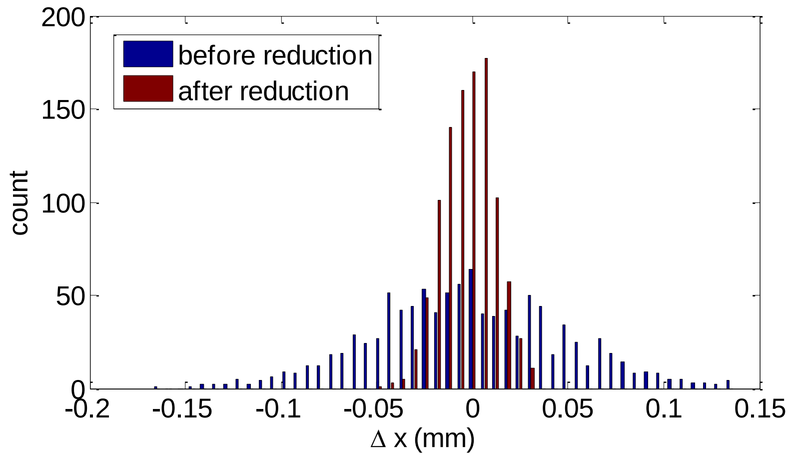
The U matrix diagonalize the covariance matrix.

The results in the previous slide states: with only one betatron mode in the BPM data, the hyper-ellipsoid degenerates to an ellipse (2D).

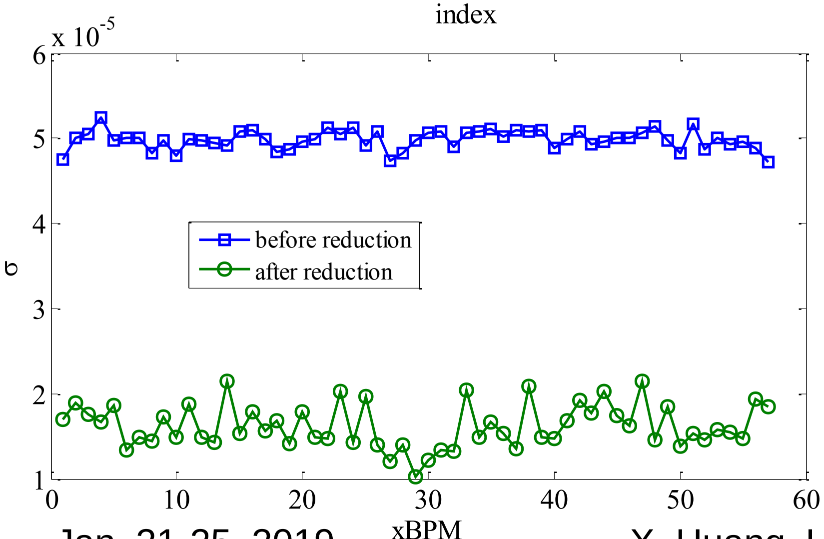
Noise reduction with SVD

As the random noises are distributed in all eigen-modes while the signals are concentrated in the leading eigen-modes, noise can be reduced by re-constructing the data after removing the noise-only (with small singular values) modes.





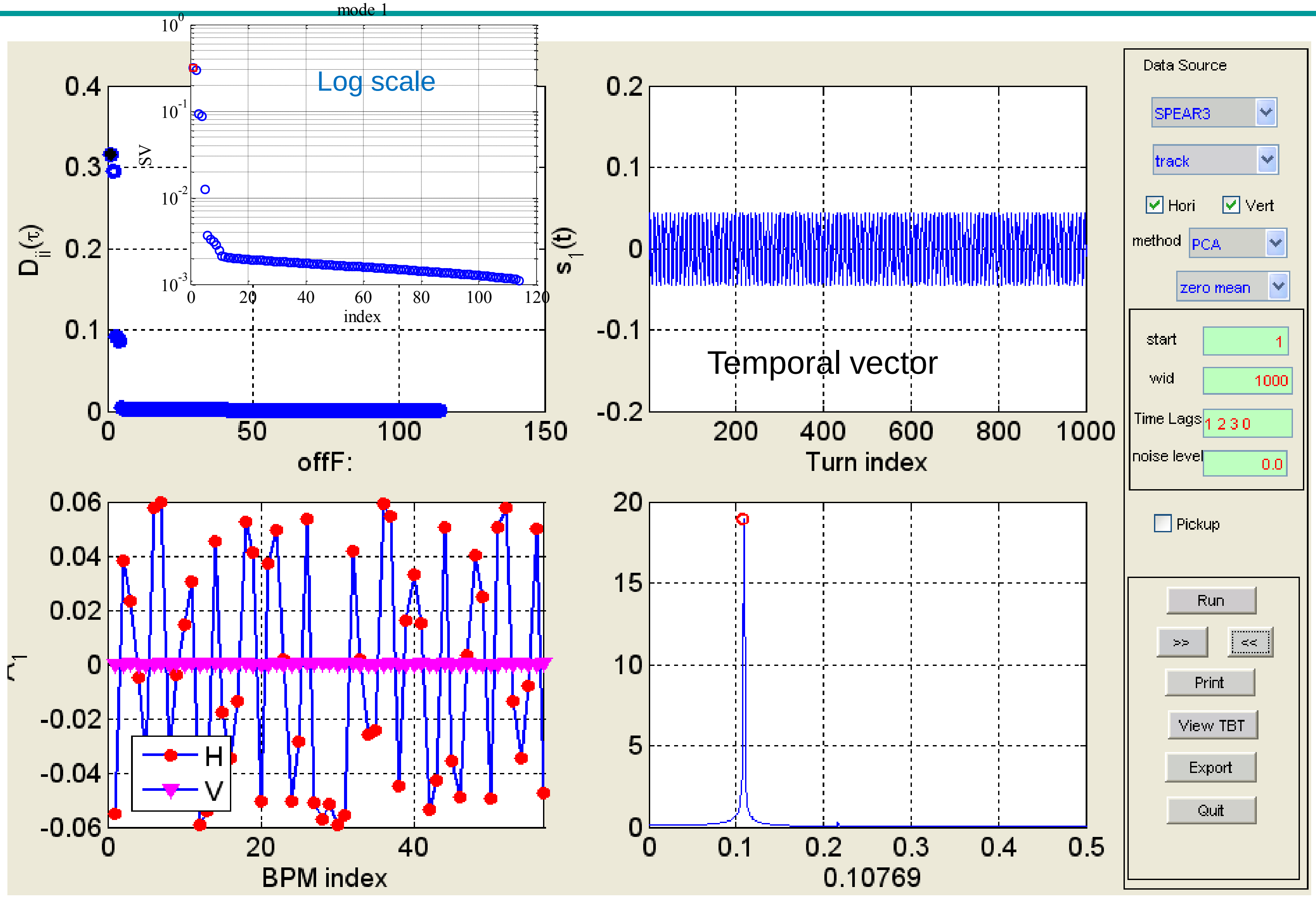
Keep 10 out of 114 modes.



The noise level (sigma) is reduced (keeping p out of 2m modes) to

$$\sigma_n = \sigma\sqrt{\frac{p}{2m}}$$

Example of SVD analysis



Spatial vector

This data set is from tracking the SPEAR3 lattice with added random noise (sigma=0.05 mm). You will play with this program (and the data sets) in the computer-lab class.

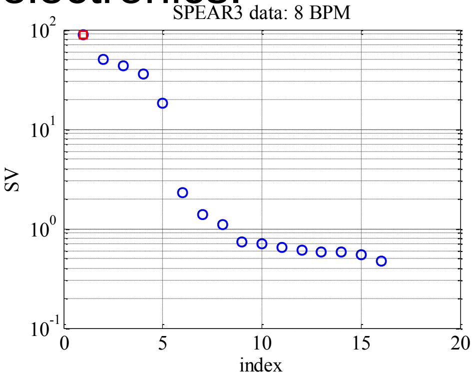
Limitation of the PCA method

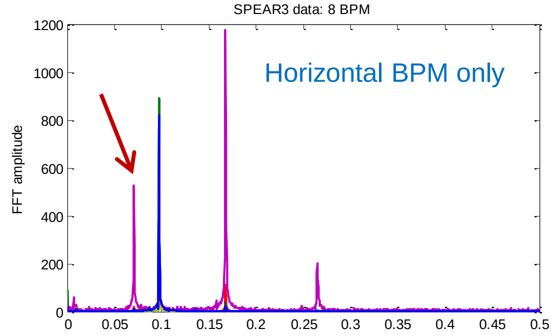
The eigen-modes are determined by the orthogonality and variances (strengths) of the components. If two signals have nearly the same strengths, they will be mixed in the eigen-modes (degeneracy in eigen-analysis). In reality this is common:

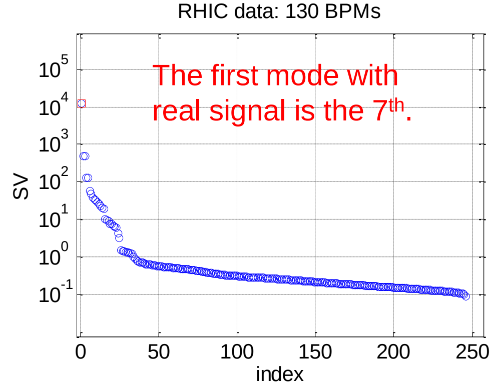
(1) Horizontal and vertical betatron modes can be mixed.

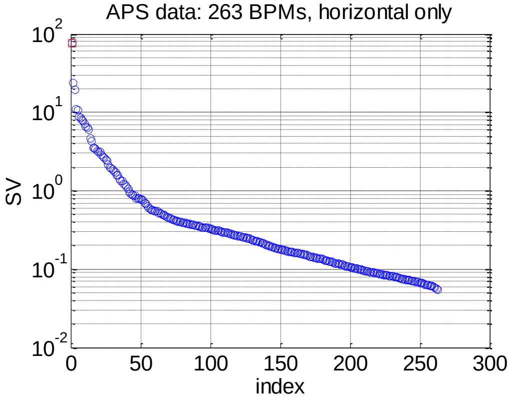
(2) Betatron modes can be mixed with the synchrotron mode.

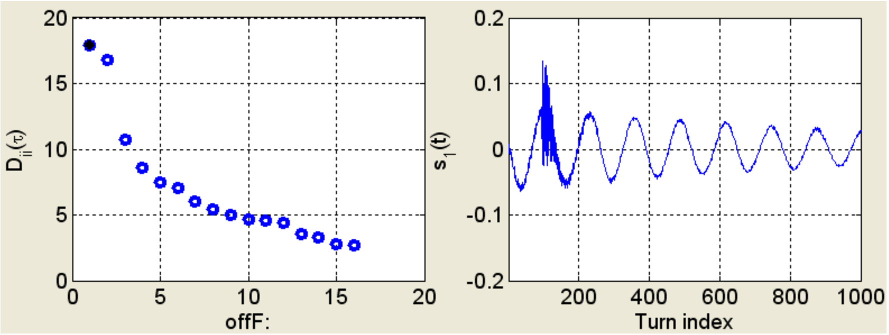
(3) Actual BPM data are often plagued by signal contamination or failing electronics.











The independent component analysis (ICA)

- The source signals are assumed statistically independent.

$$p(x_1, x_2) = p(x_1)p(x_2)$$

This is a strong condition that the PCA analysis does not make full use of.

$$E\{h_1(x_1)h_2(x_2)\} = E\{h_1(x_1)\}E\{h_2(x_2)\}$$ For any function h_1,h_2.

PCA only requires the components to be linearly uncorrelated, i.e., the covariance between two variables is zero.

$$E\{x_1 x_2\} - E\{x_1\}E\{x_2\} = 0$$

For two Gaussian variables, uncorrelatedness is equivalent to independence. Many ICA algorithms exploit the non-gaussianity of the signals, such as fastICA.

It is possible to use non-gaussianity based methods for BPM data analysis. But we will focus on an algorithm that relies on the time-spectrum of the source signals.

The Principle

- The source signals are assumed to be narrow-band with non-overlapping spectra, so their un-equal time covariance matrices are diagonal.

$$\langle \mathbf{s}(t)\mathbf{s}(t+\tau)^T \rangle = \mathrm{diag}[\rho_1(\tau), \rho_2(\tau), \ldots, \rho_n(\tau)]$$

Since $\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$

$$\mathbf{C}_x(0) \equiv \langle \mathbf{x}(t)\mathbf{x}(t)^T \rangle = \mathbf{A}\mathbf{C}_s(0)\mathbf{A}^T + \sigma^2\mathbf{I}$$

$$\mathbf{C}_x(\tau) \equiv \langle \mathbf{x}(t)\mathbf{x}(t+\tau)^T \rangle = \mathbf{A}\mathbf{C}_s(\tau)\mathbf{A}^T, \tau \neq 0$$

The mixing matrix *A* diagonalizes the un-equal time sample covariance matrices simultaneously.

The Algorithm* - 1

- Diagonalize the equal-time covariance matrix (data whitening)

D_1,D_2 are diagonal

Set to remove noise

$$\mathbf{C}_x(0) = [\mathbf{U}_1, \mathbf{U}_2]\begin{bmatrix} \mathbf{D}_1 & \\ & \mathbf{D}_2 \end{bmatrix}[\mathbf{U}_1, \mathbf{U}_2]^T \quad \text{with} \quad 0 \le \max(\mathbf{D}_2) < \lambda_c \le \min(\mathbf{D}_1)$$

Construct an intermediate "whitened" data matrix

$$\mathbf{z} = \mathbf{D}_1^{-\frac{1}{2}}\mathbf{U}_1^T\mathbf{x} = \mathbf{V}\mathbf{x} \quad \text{which satisfies} \quad <\mathbf{z}\mathbf{z}^T> = \mathbf{I}$$

This pre-processing step is just PCA. Matrix z contains the temporal vectors.

* The second order blind identification (SOBI) algorithm of A. Belouchrani, et al. in IEEE Trans. Signal Processing, 48, 900, (2003).

The Algorithm - 2

- Jointly diagonalize* the un-equal time covariance matrices of matrix z of selected time-lag constants.

$$\mathbf{C}_z(\tau) = \mathbf{W}\mathbf{C}_s(\tau)\mathbf{W}^T \quad \text{for} \quad \tau = \{\tau_i \mid i = 1, 2, \cdots, k\}$$

Then

$$\mathbf{s} = \mathbf{W}^T\mathbf{V}\mathbf{x} \quad \text{and} \quad \mathbf{A} = (\mathbf{U}_1\mathbf{D}_1^{\frac{1}{2}})\mathbf{W}$$

The columns of A (spatial vectors) and corresponding rows (temporal vectors) of s are the resulting modes.

*Algorithm for joint diagonalization can be found in J.F. Cardoso and A. Souloumiac, SIAM J. Matrix Anal. Appl. 17, 161 (1996)

Linear Lattice Functions Measurements

- There are two betatron modes because each BPM sees different phase.

The betatron component $x = A_{b1}s_1 + A_{b2}s_2$

Beta function and phase advance

$$\beta = a(A_{b1}^2 + A_{b2}^2) \qquad \psi = \tan^{-1}\left(\frac{A_{b1}}{A_{b2}}\right)$$

- There is one synchrotron mode.

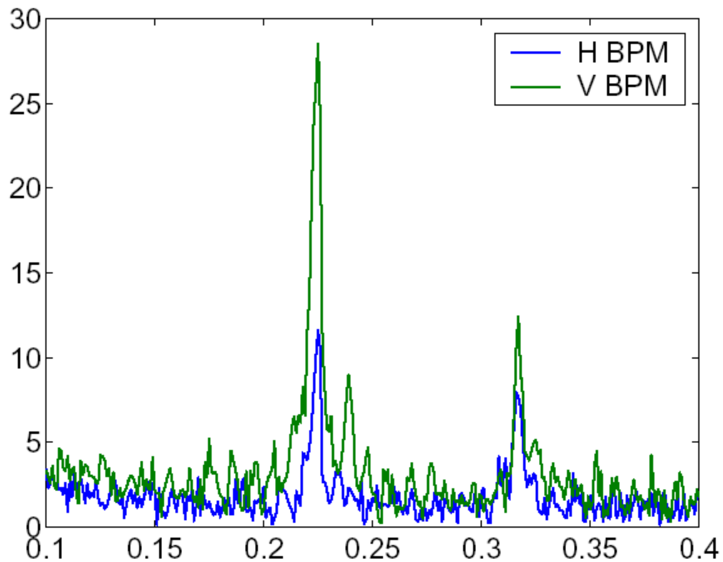
The synchrotron component $x = A_l s_l$

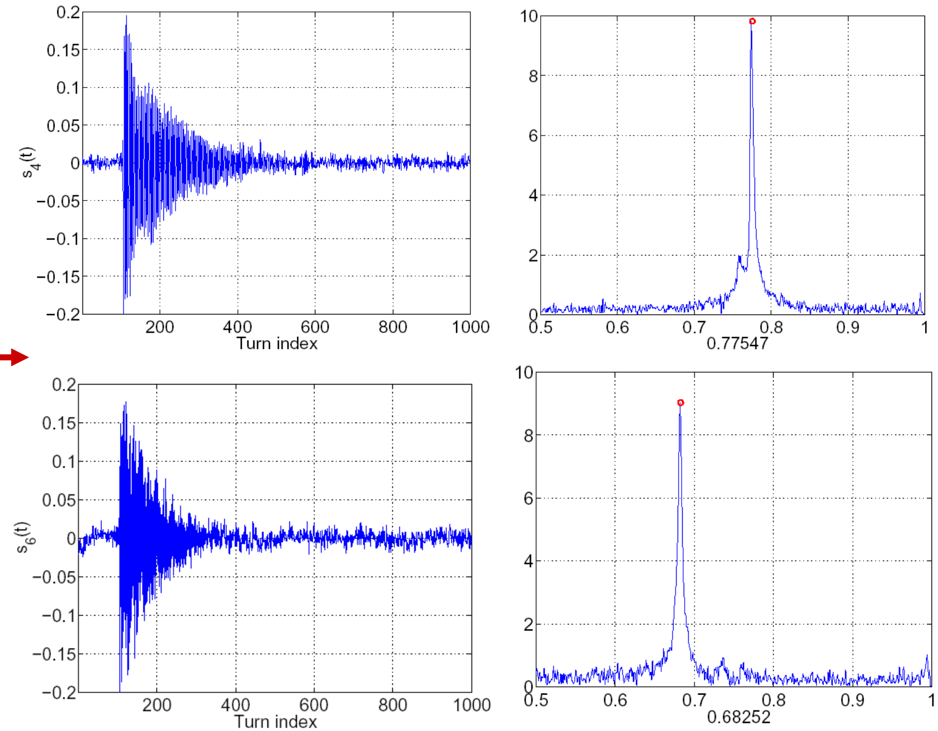
Dispersion function and momentum deviation

$$D_x = bA_l \qquad \delta = \frac{s_l}{b}$$

Example: de-coupling

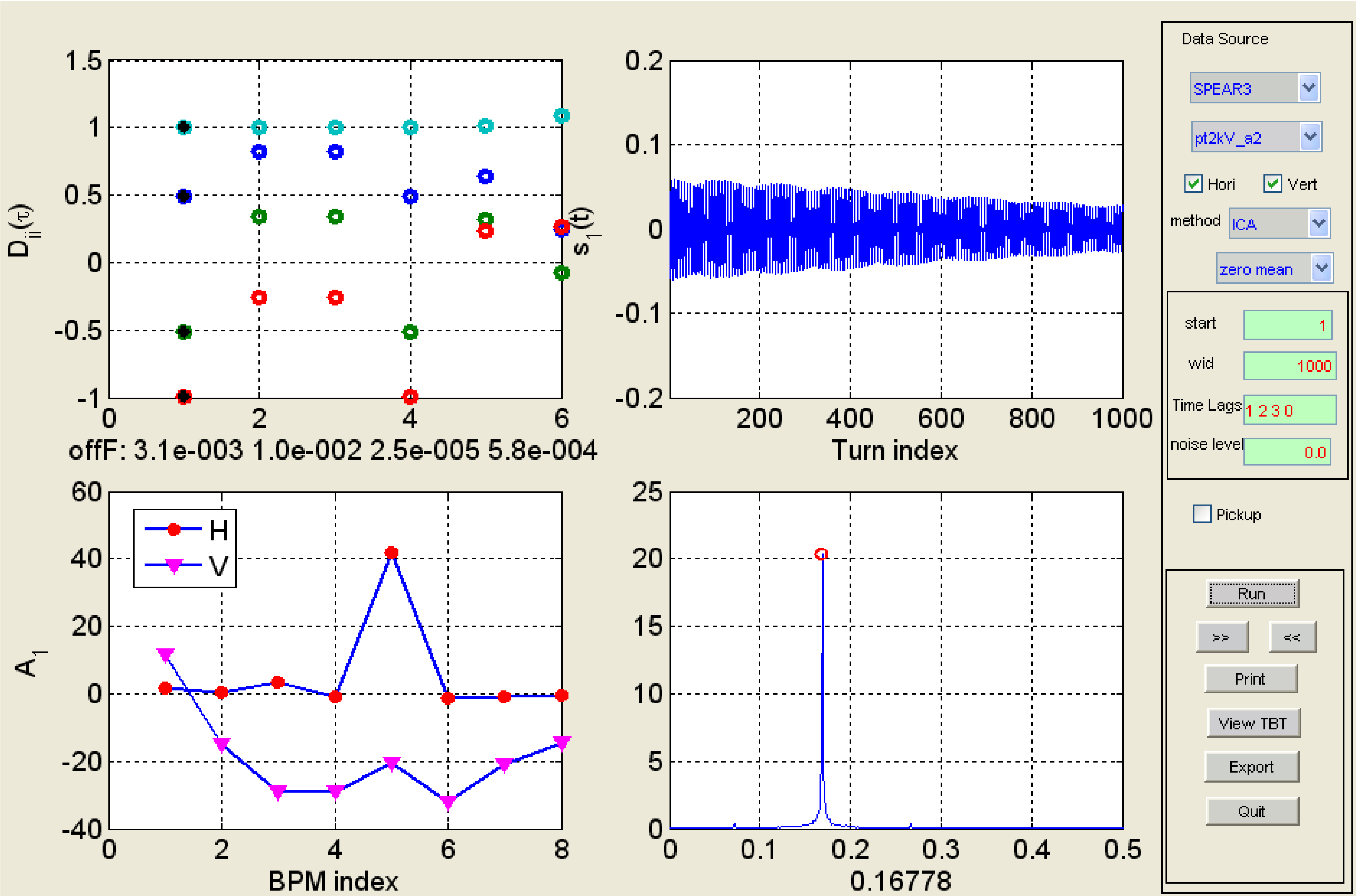
The ICA method can de-couple the normal modes in presence of linear coupling.



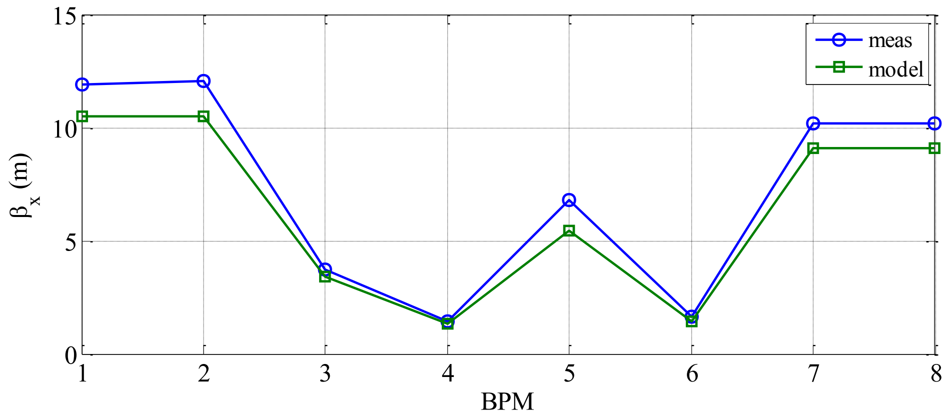


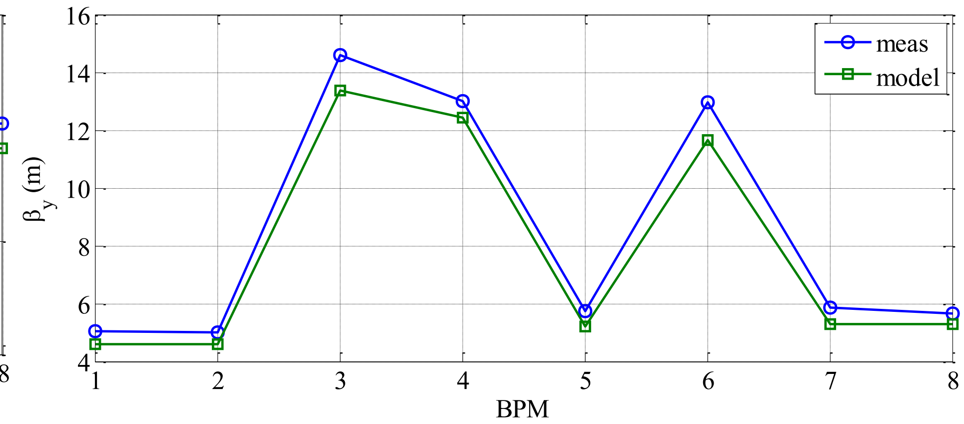
FFT spectra of raw horizontal and vertical BPM signals at section L1. Both BPMs see a mixture of the "plus" mode and "minus" mode.

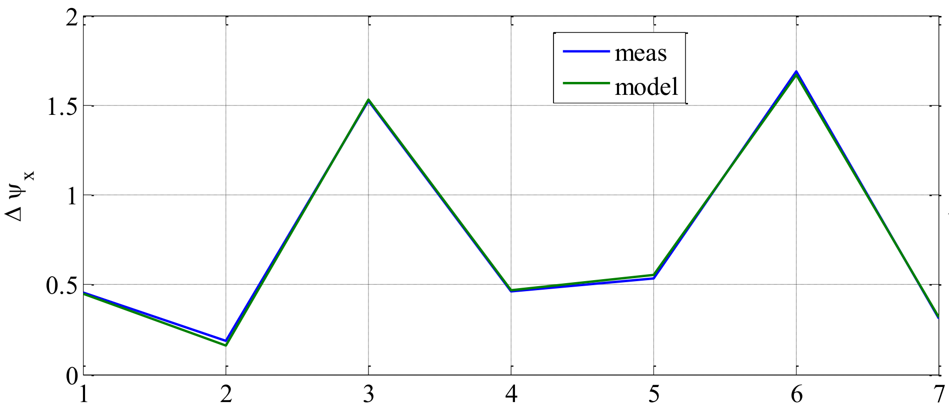
Example: SPEAR3 data

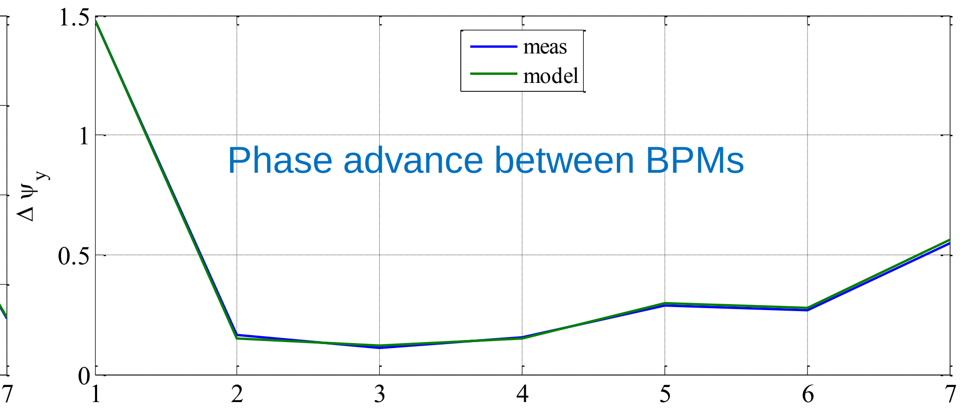


SPEAR3: the measured phase advance



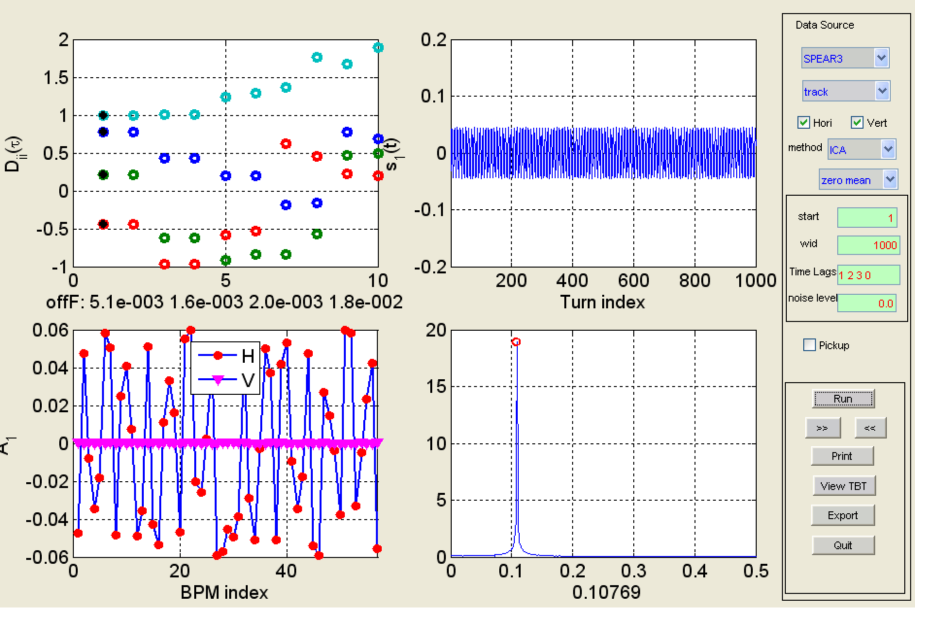


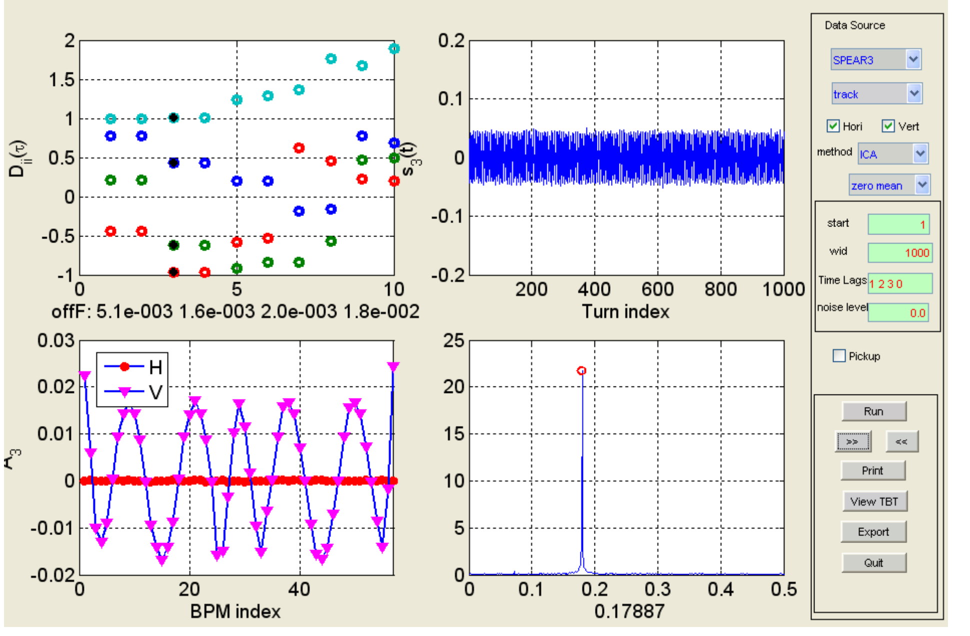




There are BPM gain errors. But the phase advances are in excellent agreement with the model.

Example: Nonlinear modes in tracking data

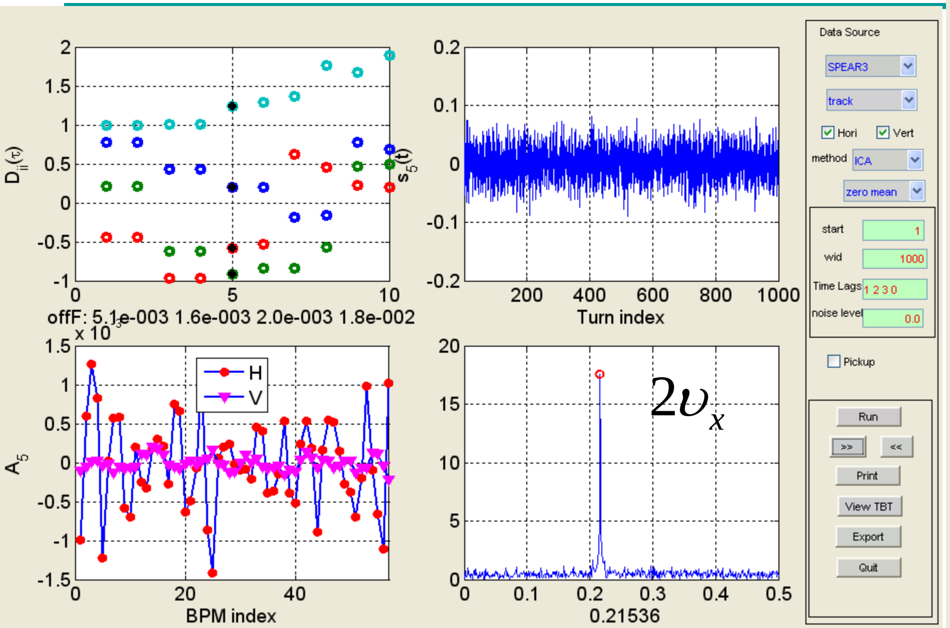


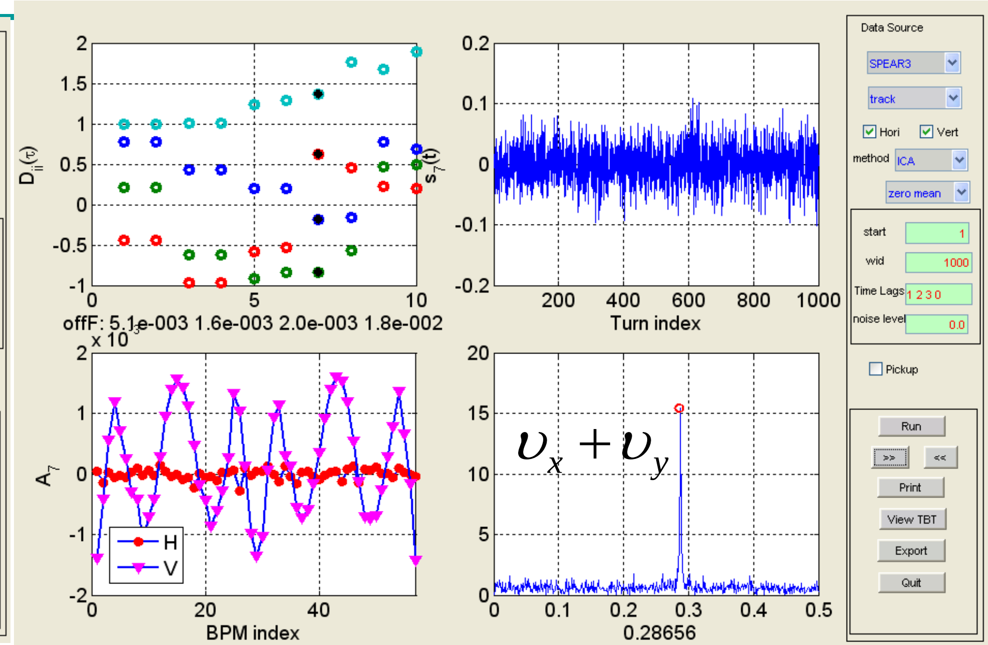


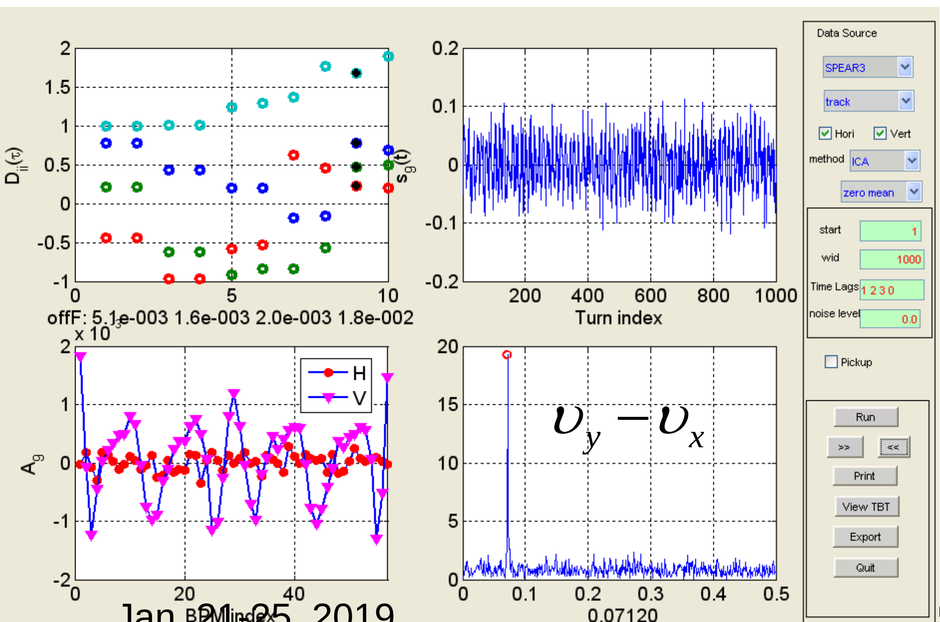
The betatron modes.

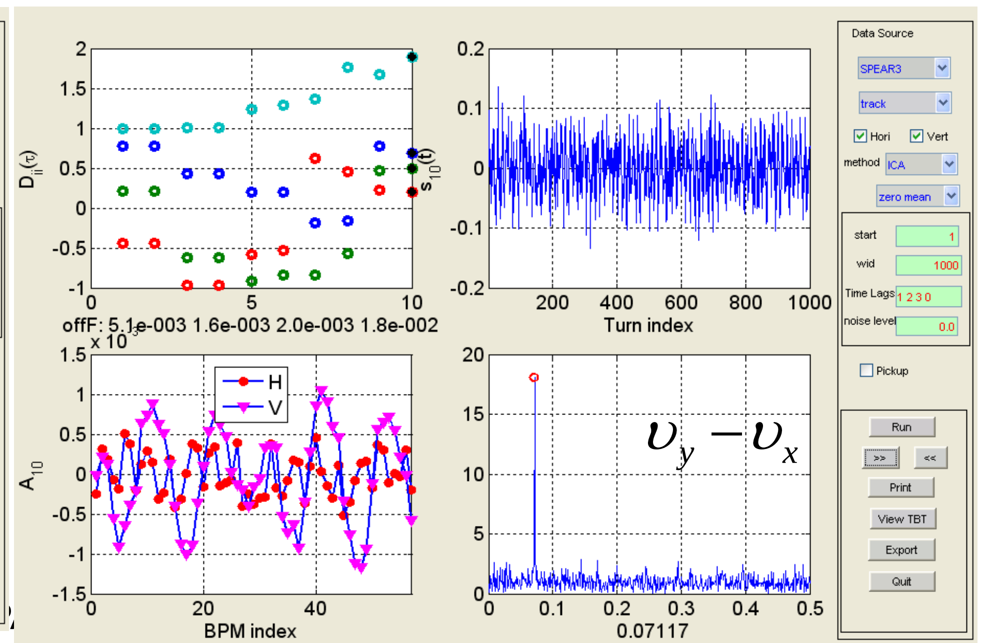
Data from tracking SPEAR3 model. There are 57 BPMs.

Example: coupling and nonlinear modes in tracking data

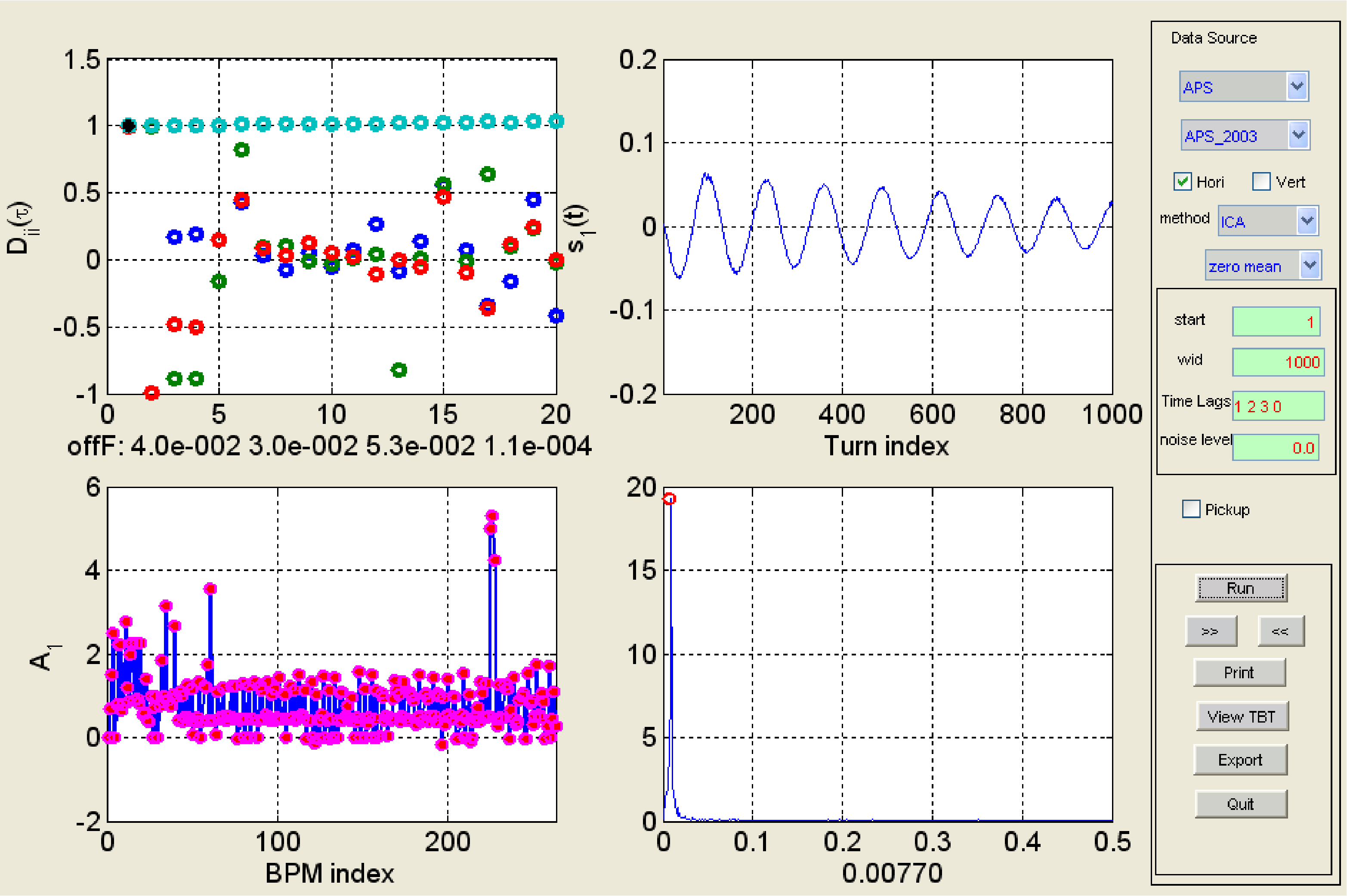


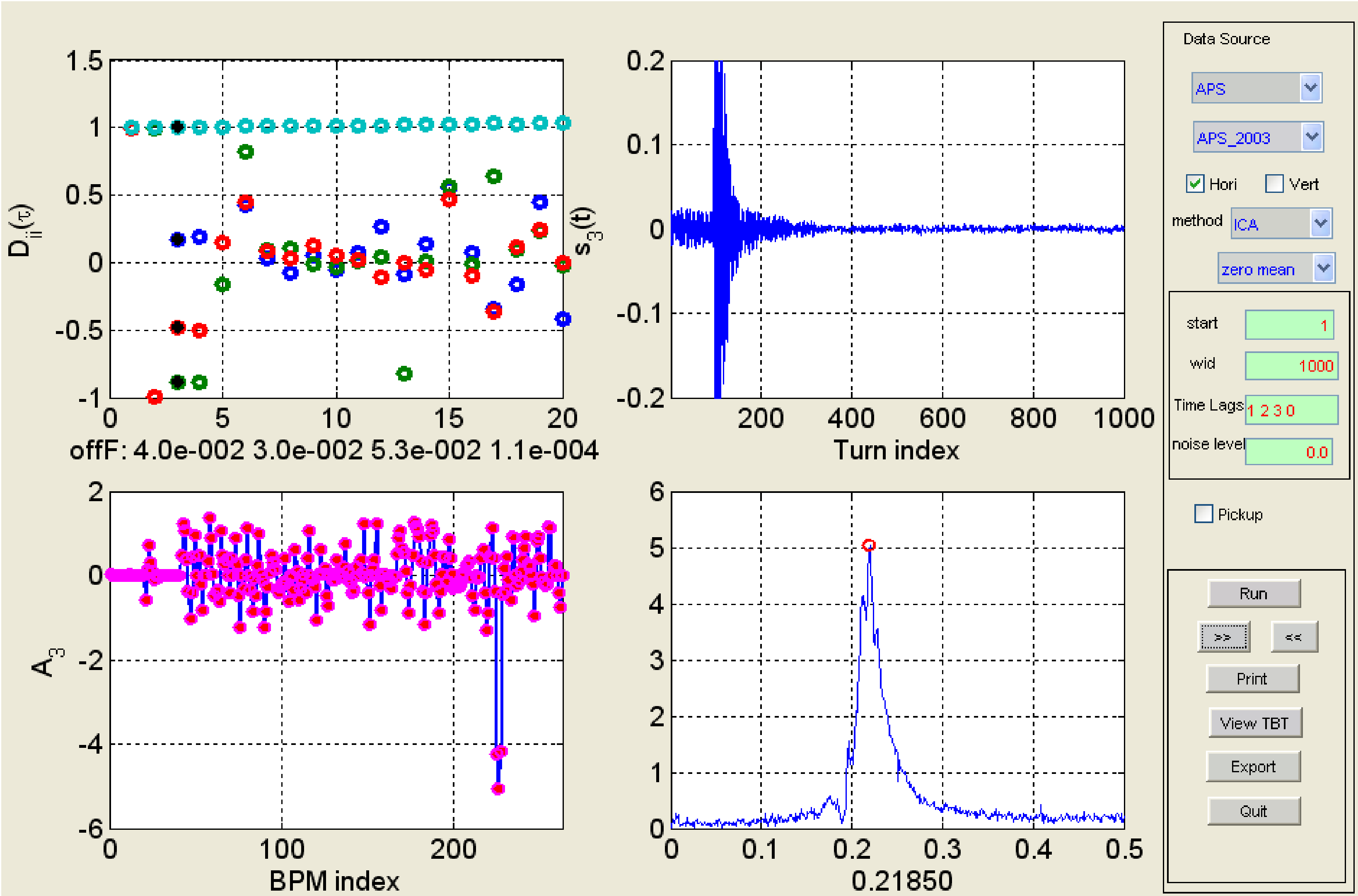






Application: APS data





$D_{ii}(\tau)$
offF: 4.0e-002 3.0e-002 5.3e-002 1.1e-004
$s_3(t)$
Turn index
A_3
BPM index
0.21850
Data Source
APS
APS_2003
Hori
Vert
method
ICA
zero mean
start
1
wid
1000
Time Lags
1 2 3 0
noise level
0.0
Pickup
Run
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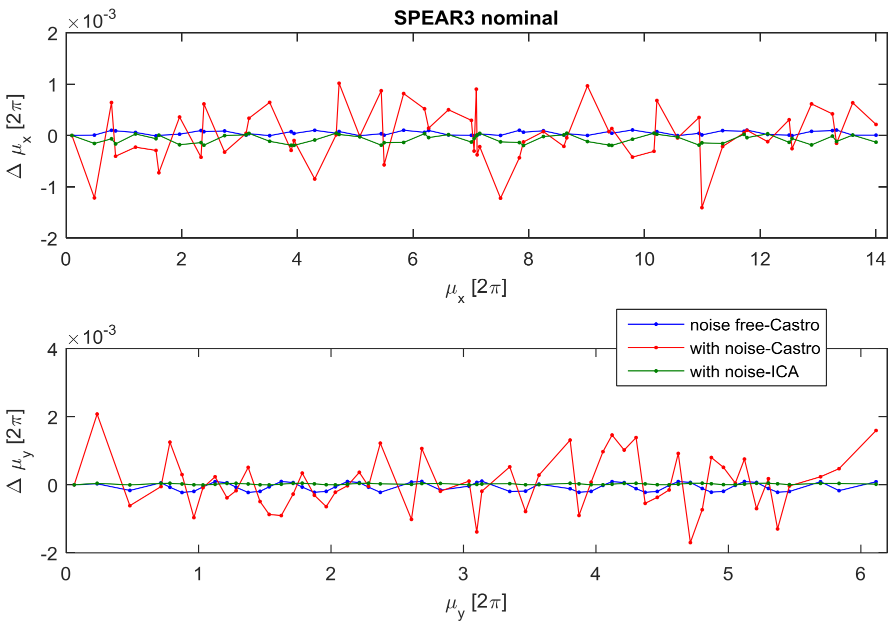
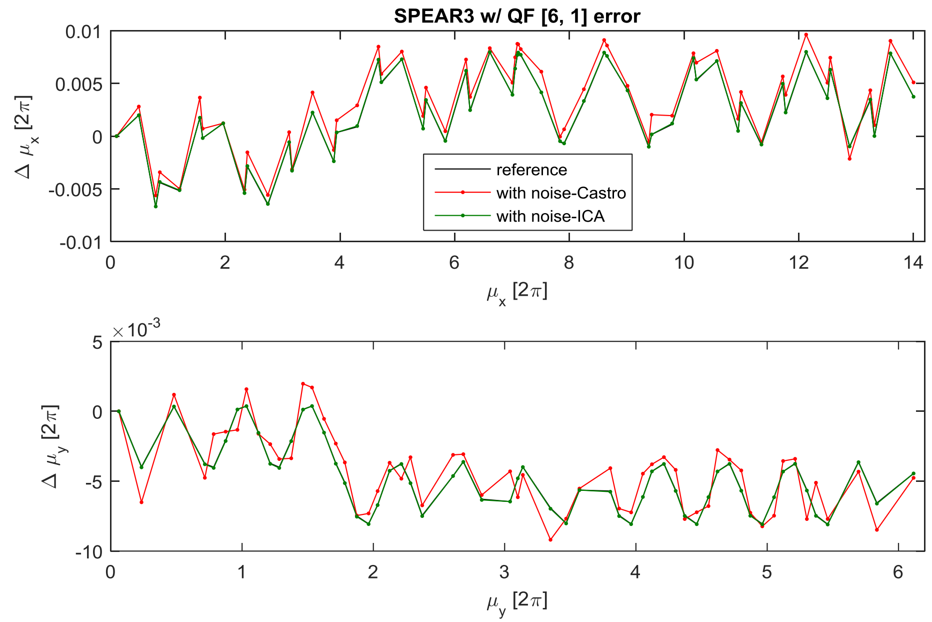
Print
View TBT
Export
Quit

Comparison of precision of phase measurements

Phase measurement with Castro's method and ICA from simulated data (SPEAR3)



Nominal SPEAR3 lattice



With introduced quadrupole error

Phase measurement with ICA has much higher precision than the single-BPM approach.

High precision phase measurement is beneficial to optics, coupling and nonlinear dynamics correction with TbT BPM data.

Optics and coupling correction with ICA results

TbT data with coupling with ICA mode separation

$$x_n = A\cos\Psi_1(n) - B\sin\Psi_1(n) + c\cos\Psi_2(n) - d\sin\Psi_2(n),$$
$$y_n = a\cos\Psi_1(n) - b\sin\Psi_1(n) + C\cos\Psi_2(n) - D\sin\Psi_2(n),$$

TbT BTPM data are related to normal mode coordinates through a transformation matrix P, with $X = P\Theta$

$$X = \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} \quad \Theta = \begin{pmatrix} \sqrt{2J_1}\cos\Phi_1 \\ -\sqrt{2J_1}\sin\Phi_1 \\ \sqrt{2J_2}\cos\Phi_2 \\ -\sqrt{2J_2}\sin\Phi_2 \end{pmatrix}$$

The P-matrix can be derived from the one-turn transfer matrix.

Therefore

$$x = p_{11}\sqrt{2J_1}\cos\Phi_1 + \sqrt{2J_2}(p_{13}\cos\Phi_2 - p_{14}\sin\Phi_2),$$
$$y = \sqrt{2J_1}(p_{31}\cos\Phi_1 - p_{32}\sin\Phi_1) + p_{33}\sqrt{2J_2}\cos\Phi_2,$$

where by definition

$$p_{12} = p_{34} = 0$$

By comparison:

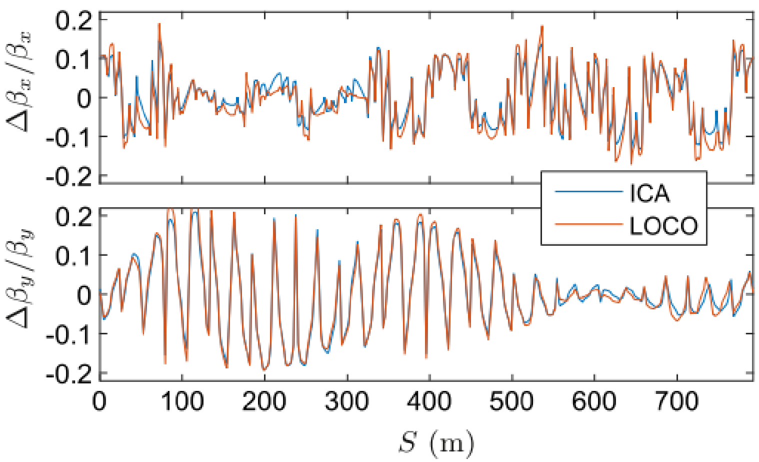
$$\sqrt{A^2+B^2} = \sqrt{2J_1}\,p_{11}, \qquad \sqrt{c^2+d^2} = \sqrt{2J_2}\sqrt{p_{13}^2+p_{14}^2},$$

$$\sqrt{C^2+D^2} = \sqrt{2J_2}\,p_{33}, \qquad \sqrt{a^2+b^2} = \sqrt{2J_1}\sqrt{p_{31}^2+p_{32}^2}.$$

$$\tan^{-1}\frac{B}{A} = \mathrm{Mod}_{2\pi}(\Phi_1),$$
$$\tan^{-1}\frac{d}{c} = \mathrm{Mod}_{2\pi}(\Phi_2 + \tan^{-1}\frac{p_{14}}{p_{13}}),$$
$$\tan^{-1}\frac{b}{a} = \mathrm{Mod}_{2\pi}(\Phi_1 + \tan^{-1}\frac{p_{32}}{p_{31}}),$$
$$\tan^{-1}\frac{D}{C} = \mathrm{Mod}_{2\pi}(\Phi_2).$$

Fitting the model with the ICA results

- The lattice model can be fitted to minimizing the differences between the amplitude and phase of the coupled modes.
  - Dispersion measurements are also included for fitting.
  - BPM gains and rolls can be fitted.



Comparison of beta beating from fitting results of LOCO and TbT data taken at the same time for NSLS-II ring (before correction).

Fitted lattice parameters by ICA and LOCO before and after corrections.

Parameters	Before		After	
	ICA	LOCO	ICA	LOCO
rms $\Delta\beta_x/\beta_x$	0.0678	0.0780	0.0051	0.0194
rms $\Delta\beta_y/\beta_y$	0.0937	0.0991	0.0038	0.0110
rms ΔD_x	0.0167	0.0227	0.0056	0.0045
rms ΔD_y	0.0080	0.0085	0.0030	0.0046
Mean ϵ_y/ϵ_x	0.0147	0.0129	0.0027	0.0031

Optics correction with fitted ICA results has been demonstrated on NSLS-II. Results were similar to LOCO.

X. Huang, X. Yang, IPAC 2015
X. Yang, X. Huang, NIMA 828 (2016) 97-104

Optics correction comparison study

- A comparison of performance of several optics correction method was done at NSLS-II.

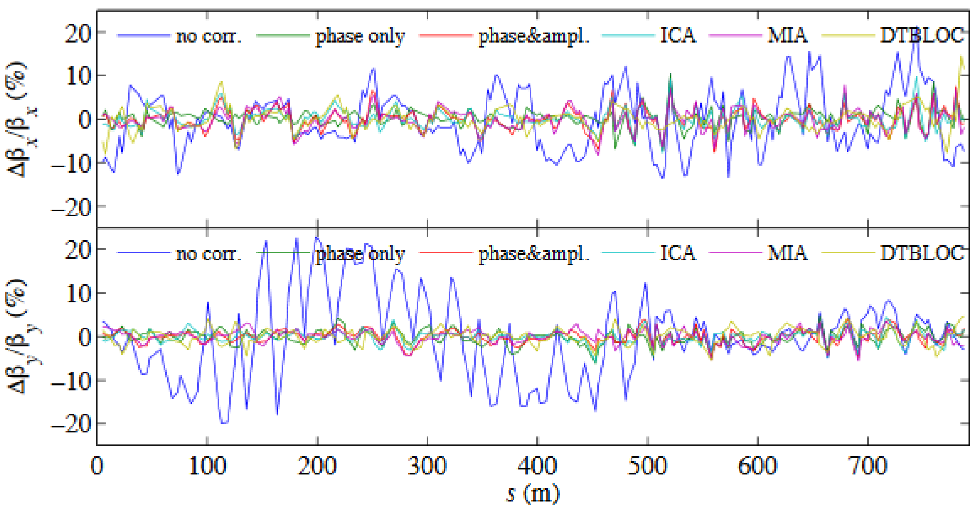


Table 1: Residual Errors

Algorithm	$\Delta\beta_x/\beta_x$ %	$\Delta\beta_y/\beta_y$ %	$\Delta\psi_x$ °	$\Delta\psi_y$ °	$\Delta\eta_x$ mm	η_y mm
no corr.	8	10	4.5	3.5	18	8
LOCO	2.1	1.4	0.5	0.2	2.6	4.4
phase only[1]	2.3	1.8	0.6	0.5	39	9.9
phase&amp.[1]	2.8	1.7	0.7	0.9	11	7.8
ICA	2.6	1.6	0.5	0.4	5.0	2.3
MIA	2.8	1.7	0.7	1.0	5.4	6.8
DTBLOC	3.0	1.9	0.4	0.8	2.3	4.5

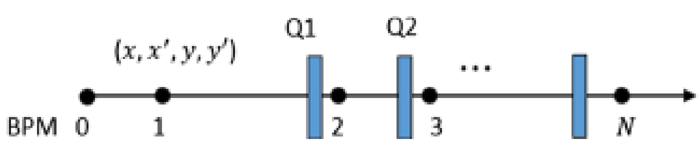
[1] no dispersion corrected

V. Smaluk, et al, IPAC 2016

While the optics correction results are similar, only LOCO and the ICA method can correct optics and coupling simultaneously.

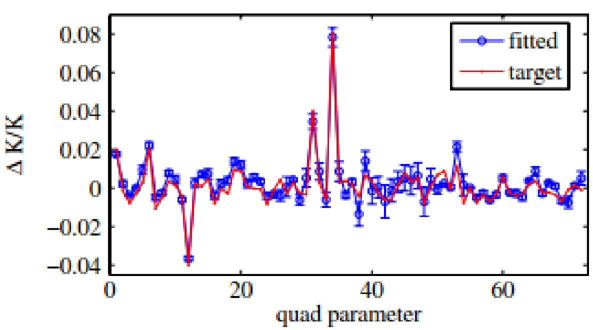
Fitting TbT data directly for optics and coupling correction

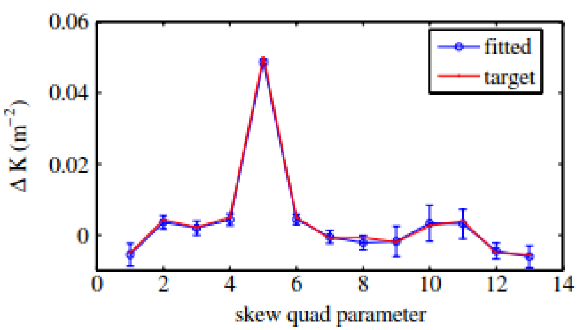
- Turn-by-turn beam motion can be predicted with lattice model if the initial phase space coordinates (x, x’, y, y’) are known.
  - x’, y’ can be determined with two BPMs separated with a drift.
- Tracked TbT data can be fitted to measurements to determine the optics and coupling errors.
  - BPM gains and rolls and be included in fitting

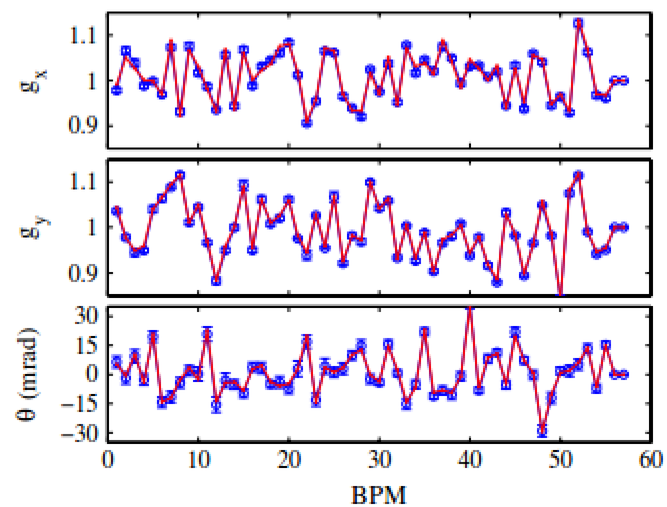


$$\chi^2 = \sum_{n=1}^{N}\sum_{i=2}^{M+1}\left[\left(\frac{x_i(n) - \tilde{x}_i[\mathbf{p};\mathbf{X_1(n)}]}{\sigma_{xi}}\right)^2 + \left(\frac{y_i(n) - \tilde{y}_i[\mathbf{p};\mathbf{X_1(n)}]}{\sigma_{yi}}\right)^2\right],$$

- Simulation results



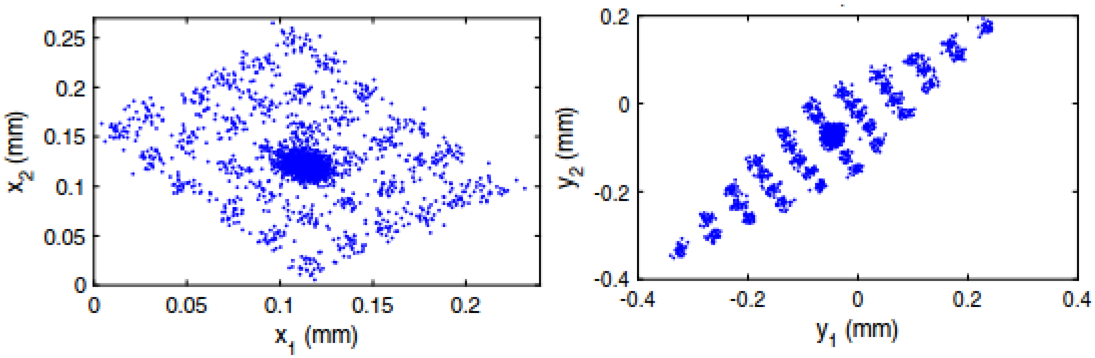




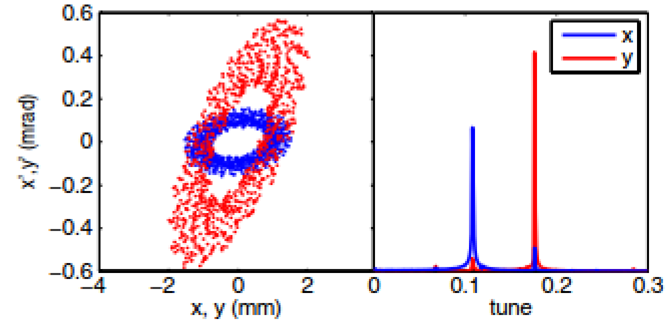
X. Huang, J. Sebek, D. Martin, PRSTAB 13, 114002 (2010)

Fitting linac trajectory data for optics correction

- Steering the beam trajectory in a linac samples the machine optics, similar to TbT data in a ring.



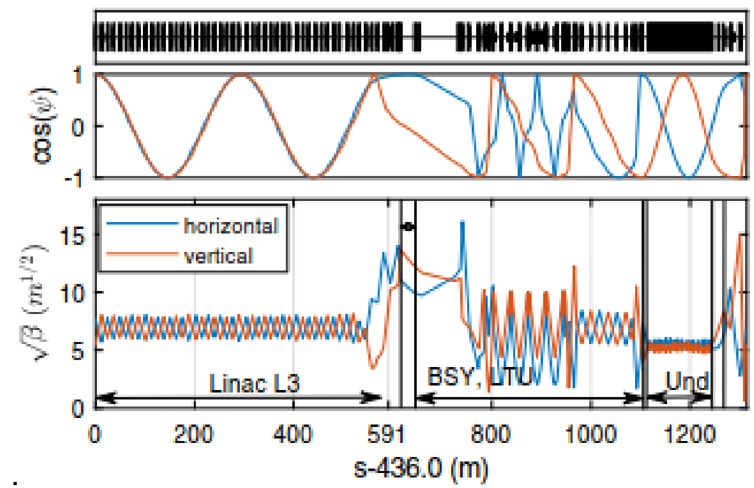
LCLS trajectory scan.



TbT BPM data in SPEAR3

- Trajectory scan data in a linac (or transport line) can be fitted in the same fashion as fitting TbT data to derive quadrupole errors.
- This has been applied to LCLS data.

LCLS optics downstream of BC2



T. Zhang, X. Huang, T. Maxwell, PRAB 21, 092801 (2018)

Local analysis of phase space coordinate data

- The (x, x’, y, y’) data between two location can be used to fit the transfer matrix.
  - In a storage ring, TbT data of (x, x’, y, y’) can be fitted for the one-turn transfer matrix and in turn the Courant-Snyder parameters (i.e., α, β).

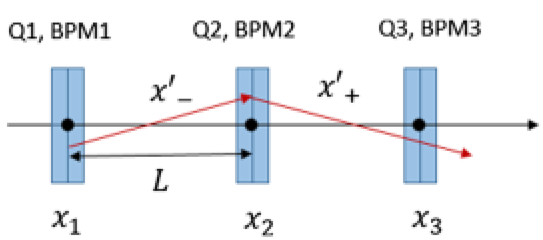
Fitting matrix elements directly
$$\mathbf{M}_{21} = \mathbf{X}_2\mathbf{X}_1^T(\mathbf{X}_1\mathbf{X}_1^T)^{-1}.$$

Fitting parameters to build symplectic matrix
$$\mathbf{M}_{21} = \begin{pmatrix} \gamma\mathbf{I} & \mathbf{C} \\ -\mathbf{C}^+ & \gamma\mathbf{I} \end{pmatrix}\begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{pmatrix}\begin{pmatrix} \gamma\mathbf{I} & -\mathbf{C} \\ \mathbf{C}^+ & \gamma\mathbf{I} \end{pmatrix},$$

$$\mathbf{A} = \begin{pmatrix} p_1 & p_2 \\ \frac{p_1p_3-1}{p_2} & p_3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} p_4 & p_5 \\ \frac{p_4p_6-1}{p_5} & p_6 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} p_7 & p_8 \\ p_9 & p_{10} \end{pmatrix}, \quad \mathbf{C}^+ = \begin{pmatrix} p_{10} & -p_8 \\ -p_9 & p_7 \end{pmatrix}, \quad \gamma = \sqrt{1-\|\mathbf{C}\|},$$

X. Huang, J. Sebek, D. Martin, PRSTAB 13, 114002 (2010)

- In special occasions, the x’, y’ data may be used to directly determine quadrupole gradients.



Quadrupole and BPM layout in the SLAC Linac

$$x'_- = \frac{(x_2-x_1)}{L}\left(\frac{3}{2}-\frac{E_2}{2E_1}\right),$$

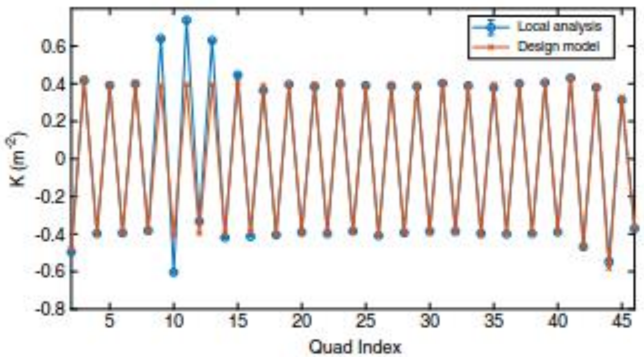
$$x'_+ = \frac{(x_3-x_2)}{L}\left(\frac{1}{2}+\frac{E_3}{2E_2}\right),$$

$$\Delta x'_2 \equiv x'_+ - x'_- = [KL_q]_2 x_2$$

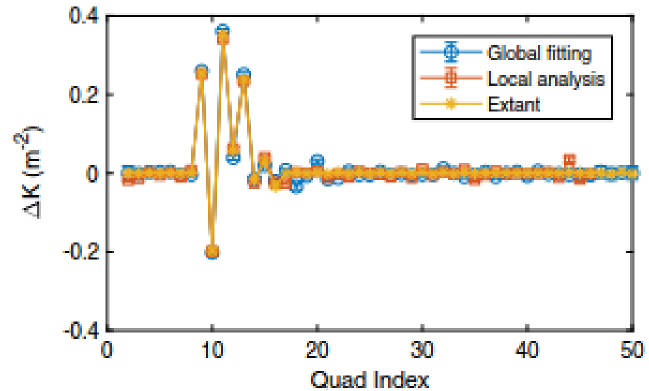
Fitting $\Delta x'_2$ vs. x_2, the quadrupole strength $K\Delta l$ can be determined.

T. Zhang, X. Huang, T. Maxwell, PRAB 21, 092801 (2018)

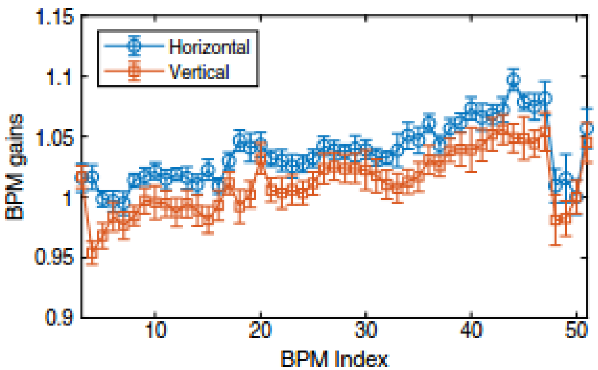
Application to LCLS



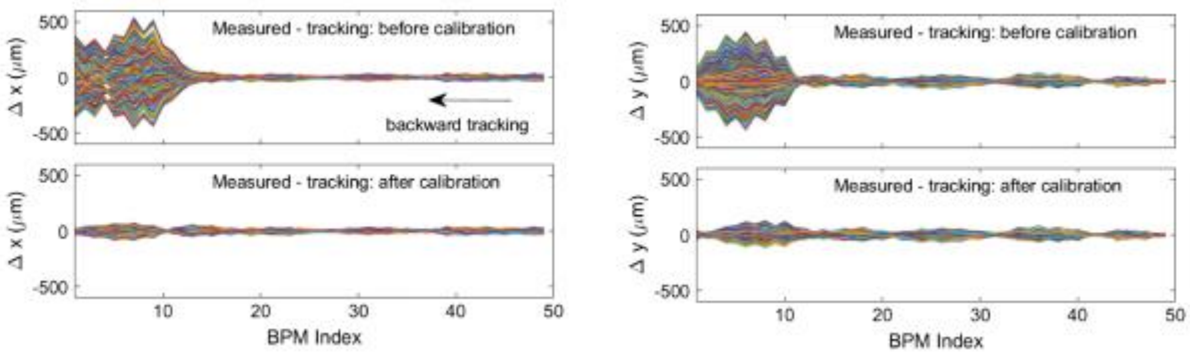
Fitted quadrupole strength (local analysis) vs. design model. Quad 9-13 were tuned away from the model for matching.



Fitted model vs. Extant model (using setpoints and magnet calibration data)



Fitted BPM gains. The linear trend may indicates beam energy error.



Difference between measured and tracked trajectories before and after fitting.

Summary

- Betatron tunes and phase advances can be precisely determined with turn-by-turn BPM data.
- Model independent analysis and independent component analysis improve the accuracy of phase advance data analysis by utilizing data from all BPMs.
  – ICA is more suited to separate coupled motion or data with bad BPMs.
- Beta function can be derived from turn-by-turn BPM data.
  – Three-BPM or N-BPM method
  – From betatron mode oscillation amplitude (MIA/ICA)
- Fitting phase advance and beta function to model can determine the optics errors for correction.
- Directly fitting TbT or trajectory scan data can also determine the optics errors for rings or linacs.