USPAS 2015: East Brunswick, Rutgers

Nonlinear Lattice Characterization: Dynamic Aperture, Momentum Aperture, Lifetime, Frequency Map Analysis

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Outline

(*Transverse*) single particle dynamics often determines injection efficiency, lifetime, ...

- Motivation
- Nonlinear Dynamics Dynamic Aperture
 - Tunescans
 - Frequency Maps
 - On energy dynamic aperture: Injection Efficiency
- Lifetime limiting processes
 - Momentum aperture: Touschek Lifetime
- Summary





Motivation

- Particles are lost in accelerators because of finite apertures, potentially limiting
 - Injection efficiency, or
 - Beam lifetime

- Limiting apertures can be *physical* or *dynamic*:
 - Vacuum chamber \rightarrow physical aperture
 - Nonlinear single particle dynamics → dynamic (energy) aperture
- Loss process typically involves two steps:
 - Scattering process (or injection) launching particles to large amplitudes outside core of beam
 - Resonant or diffusive processes (nonlinear dynamics) leading to growth of oscillation amplitudes



ALS What determines the momentum aperture?



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$$U(\omega) = \frac{U(0)}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{\omega}{Q\omega_0}\right)^2}}$$

- Driven harmonic oscillator
 - periodic excitations
 - frequency of excitation determined by external source
- Betatron oscillations

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- Excitation due to field error, fixed in space (and usually not time dependent)
- Excitation frequency is determined by ٠ oscillation frequency of beam particles
- Both result in similar driven resonances









Betatron Resonances







• Resonances can occur when the tunes satisfy: $mV_x + nV_y = q$

where m, n and q are integers

- Generally resonances are weaker the higher their order
- Integer resonances driven by dipole errors, half-integer by quadrupole errors, third-integer by sextupoles, ...

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Resonances in Phasespace

 A quadupole perturbation (i.e. kick linearly dependent on position) quickly increase betatron amplitude near half-integer resonance, sextupole perturbation (i.e. kick depends quadratically on position) drives third-integer resonance.





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Tune shift with amplitude

Particle tune get shifted with amplitude





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Tune shift with energy

 Particle tune get shifted with particle energy/momentum – Chromaticity

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- Sextupoles are used to correct Chromaticity (linear or low order)
 - However, higher order terms often remain (even when using many sextupole families)
- Example: Double Bend Achromat, 2 sextupole families, +/-2% energy







• ALS consists of 12 sectors

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12-fold periodicity Suppression of resonances

$$mv_x + nv_y = 12 \times q$$

where m, n and q are integers

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Benefits of Periodicity



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Resonance Excitation

Resonances can lead to irregular and chaotic behavior for the orbits of particles which eventually will get lost by diffusion in the outer parts of the beam.

Rule of thumb => Avoid low order resonances (<~ 12th for protons and <~ 4th for electrons)

One can study the strength of resonances by using a tracking code or through measurements

=> Tune scans
=> Frequency Map Analysis







Tune scan

When resonances are present they may change the distribution of the beam at large amplitudes.

 In the case of a resonance island →particles may get trapped at large amplitudes

Technique:

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By Introducing a scraper and a loss monitor



Scan the tunes and measure the change in the count rate

Developed by A. Temnykh (Proc. Of the IXth ALL-Union Meeting on Accelerators of Chaged Particles, Dubna, 1984, INP Peport No. INP 84-131



AS Tune scans (with and without large beta beating)

Uncorrected lattice



Corrected lattice

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Three resonances are present:





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AS Profile measurement near 3rd order resonance

Profile measurement

Horizontal phase space







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KAM Theorem

(the basis of frequency map analysis)

According to the KAM (Kolmogorov-Arnold-Moser) theorem, in a phase space that is sufficiently close to an integrable conservative system, many invariant tori will persist. Trajectories starting on one of these tori remain on it thereafter, executing *quasiperiodic motion with a with a fixed frequency vector* depending only on the torus.

⇒ Measuring how quickly frequencies of particle motion change allows quantitative analysis of how irregular a trajectory is







Frequency Map Analysis

Developed by Jacques Laskar

The frequency analysis algorithm (NAFF) is a postprocesser for particle tracking data that numerically computes, over a finite time span, a frequency vector for any initial condition.

Frequency Map: Initial condition ---- Frequency vector

Based on the KAM theorem, frequency map analysis determines whether an orbit is regular or chaotically diffusing.

Regular orbits → Frequency vector remains fixed in time Nonregular orbits → Frequency vector changes in time



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Tunes and Diffusion Rates

+



FREQUENCY ANALYSIS POSTPROCESSOR



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TRACKING CODE





Frequency Map Analysis





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ALS Vertical orbit diffusion – On-energy example

Particle are lost in the vertical plane

• via nonlinear coupling and diffusion of the trajectory.



Example : Particle launched at 12 mm horizontally and 1 mm vertically and tracked with damping and synchrotron oscillations. (Simulated injection)



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Frequency Map Analysis

Frequency Space

Amplitude Space





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ALS Electromagnetic Beam Position Monitors





Measured Frequency Map



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excellent agreement, using calibrated model (gradient errors), random skew errors, nominal sextupoles

Phys. Rev. Lett. 85, 3, (July 2000), pp.558-561



Fast Decoherence Problem for Experiment



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- Detuning with amplitude causes very fast decoherence for larger amplitudes
- Individual particles are still oscillating with same amplitude (radiation damping time >10k turns)
- Makes frequency analysis difficult
 - Small number of turns
 - Signal not quasiperiodic

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Measured Frequency Map/Beam Loss



- Partial Beam Loss mostly if particles have to pass (radiation damping) through resonance intersection
- Isolated resonances not dangerous.

Side remark: Spectra contain more information than just fundamental frequencies – other resonance lines – resonance strength versus amplitude (see resonance driving term lecture).



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ALS Model independent evaluation of dynamics



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Frequency map analysis allows to model independently evaluate how regular beam motion is



Related Problem: Injection Efficiency





Storage ring physical and dynamic aperture has to be large enough to capture sufficient amount of injected beam – Often limited by dynamic aperture.

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Lifetime: Introduction

- Beam often needs to be stored for as long as possible (stability, flux, luminosity), making lifetime a key performance parameter
- Important scattering processes dominating beam lifetime include:
 - elastic and inelastic gas scattering, intra beam scattering, quantum lifetime (SR), tune resonances, etc.
 - They can increase particle oscillation amplitudes (e.g. scattering, diffusion) ultimately leading to particle loss on physical apertures.
- Damping and excitation play major role in the lepton case.





Definition of Lifetime

- In a loss process, the number of particles lost at the time *t* is proportional to the number of particles present in the beam at the time *t*: $dN = -\alpha N(t)dt \quad with \quad \alpha \equiv constant$
- By defining the lifetime τ as:

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- From the last equation, one can see that the lifetime is defined as the time required for the beam to reduce its number of particles to 1/ e of the initial value.
- Lifetime due to the individual effects (gas, Touschek, ...) can be similarly defined. The total lifetime will be then obtained by summing the individual contributions:

$$\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} + \dots$$

• With this definition, the problem of calculating the lifetime is reduced to the evaluation of the single lifetime components.



ALS Example Lifetimes in Real Accelerators









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Lifetime in different accelerator types

- In proton and heavy ions storage rings no damping is present: Any perturbation can build up and can eventually lead to particle loss.
 - Important electron loss mechanisms are negligible for protons: Touschek + inelastic gas scattering, quantum lifetime.
 - But other effects such as elastic gas scattering, molecule excitation, fluctuations in the magnetic and RF fields, Coulomb scattering (intra-beam scattering), ..., add up to generate a lifetime of the order of typically hundreds of hours.
- In synchrotron light sources Touschek scattering usually dominates and leads to lifetimes of a few hours.
- In colliders, the interaction between the colliding beams, the so-called *beam-beam effect*, often becomes the main mechanism of losses.





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ALS Some types of scattering events influencing beam lifetime

- Electron-Photon Scattering
 - Quantum Lifetime
- Electron-Gas Scattering
 - Gas Lifetime

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- Electron-Electron Scattering
 - Touschek Lifetime



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bending magnet



Quantum Lifetime

- Emission of synchrotron radiation is quantized
 - Transverse distribution of radiation is approximately Gaussian
 - A Gaussian distribution of particles is produced
- Tails of distribution are lost

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 Redistribution on time scale of damping time

$$\tau_{Q_T} \cong \tau_{D_T} \frac{\sigma_T^2}{A_T^2} \exp(A_T^2/2\sigma_T^2) \quad T = x, y$$

Transverse quantum lifetime

where
$$\sigma_T^2 = \beta_T \varepsilon_T + \left(\eta_T \frac{\sigma_E}{E_0}\right)^2$$
 $T = x, y$

$$\tau_{Q_L} \cong \tau_{D_L} \exp(\Delta E_A^2 / 2\sigma_E^2)$$

Longitudinal quantum lifetime





Gas-scattering lifetime

Particles scatter elastically or inelastic with residual gas atoms. This introduces betatron or synchrotron oscillations.



The scattering process can be described by the classical Rutherford scattering with differential cross section per atom in cgs units

$$\frac{d\sigma}{d\Omega} = \left(\frac{zZe^2}{2\beta cp}\right) \frac{1}{\sin^4 \frac{\theta}{2}}$$





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Gas-scattering lifetime

If the new amplitudes are outside the aperture the particles are lost.

•The elastic scattering lifetime is proportional to the square of the transverse aperture A:

$$\frac{1}{\tau_{el}} \propto \frac{1}{E^2} \times \left(\frac{\beta_x}{A_x^2} \langle P \beta_x \rangle + \frac{\beta_y}{A_y^2} \langle P \beta_y \rangle \right)$$

 The inelastic scattering lifetime is proportional to the logarithm of the energy/momentum aperture ε :





 τ_{inel}

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Gas Lifetime – Vacuum Requirements

• For electrons one can simplify the formulas for gas Bremsstrahlung lifetime (in the approximation of $\langle Z^2 \rangle \sim 50$):

$$\tau_{Brem[hours]} \cong -\frac{153.14}{\ln(\Delta E_A/E_0)} \frac{1}{P_{[nTorr]}}$$

• In the same approximation, the elastic gas scattering lifetime becomes:

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$$\tau_{Gas[hours]} \cong 10.25 \frac{E_{0[GeV]}^{2}}{P_{[nTorr]}} \frac{\varepsilon_{A[\mu m]}}{\langle \beta_{T} \rangle_{[m]}}$$

• For typical electron ring parameters, one finds that the requirement on vacuum is for dynamic pressures of the order of a few nTorr.



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Touschek Lifetime

Beam direction

Particles inside a bunch perform transverse betatron oscillations around the closed orbit. If two particles scatter they can transform their transverse momenta into longitudinal momenta.

• The first observation was done in the early 60's in Frascati at ADA, the electron-positron accelerator conceived by the Austrian scientist Touschek.

• The Touschek effect is the dominant lifetime contribution in many modern electrons storage rings.







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Touschek Scattering

- Large angle electron-electron scattering
 - \rightarrow single scattering leads to loss
- Calculate scattering cross-section: Møller cross section, which reduces to $d\sigma \prec \frac{1}{\beta^2} \left(\frac{1}{\sin^4 \theta} - \frac{1}{\sin^2 \theta} \right) d\Omega$
- Above formula is correct for non relativistic velocities (in restframe of particle bunch) and if there is no average polarization
 - In reality effect of polarization not negligible (see my talk on energy calibration)
- If the new momenta of the two particles are outside the momentum aperture, ε , the particles are lost. The lifetime is proportional to the square of ε

$$\frac{1}{\tau_{tou}} \propto \frac{1}{E^3} \frac{I_{bunch}}{V_{bunch} \sigma'_{x}} \frac{1}{\varepsilon^2} f(\varepsilon, \sigma'_{x}, E)$$







ALS Transverse Acceptance and Gas Lifetime

• move scraper into beam and record lifetime: acceptance, gas pressure



Assuming different distribution of the gas, i.e. higher pressure in the straight sections: $3*10^{-10}$ mbar Desorption coefficient: $1.75*10^{-12}$ mbar/mA



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- Calculate RF voltage dependent Touschek lifetime based on calibrated machine model (emittance, beamsized, lattice function, sposition dependent dynamic momentum aperture all calculated from calibrated model)
- Compare measurements (green errorbars) with those calibrated calculation
 - Excellent Agreement





ALS Off-energy dynamics: Touschek Lifetime

• Lifetime is crucial performance parameter for light sources \Rightarrow for 3rd generation light sources limit is Touschek lifetime \Rightarrow strong function of momentum aperture ϵ

$$\frac{1}{\tau_{tou}} \propto \frac{1}{E^3} \frac{I_{bunch}}{V_{bunch} \sigma'_{x}} \frac{1}{\varepsilon^2} f(\varepsilon, \sigma'_{x}, E)$$

- Momentum aperture ϵ is often limited by single particle dynamics
- 3rd generation light sources with their strong focusing to achieve small equilibrium emittances (small dispersion) and very strong sextupoles did originally not achieve their design momentum apertures of about 3%.





ALS What determines the momentum aperture





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ALS Longitudinal variation of momentum aperture



- Because of variation in H-function, momentum aperture will vary around the ring (depending on scattering location)
- Not necessarily symmetric for positive and negative momentum deviation (asymmetric bucket)



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ALS example: RF amplitude



 Momentum aperture in ALS is clearly impacted by dynamics

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 Sensitivity to chromaticity is at first surprisingly large (sextupole strength only different by a few percent).



AS Touschek Scattering – Tune Shift – Particle Loss



- Particle losing/gaining energy horiz. oscillation (dispersion/H-function) + long. Oscillation
- Particle changes tune

- Synchrotron oscillations (chromaticity)
- Radiation damping (detuning with amplitude and chromaticity)
- During damping process particle can encounter region in tune space where motion gets resonantly excited.



Measurement principle



- Experimentally very difficult to exactly simulate Touschek scattering (simultanous kicks) – also difficult to measure tunes during synchrotron oscillations
 - Some positive results (Y. Papaphilippou et al.)

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 Still possible to locate loss regions when scanning only transverse amplitude while keeping energy offset fixed



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Measurement Detail



- Use single turn kicker to excite beam with increasing amplitude
- Use current monitor to record relative beam loss after kick
- Use turn-by-turn BPMs to record oscillation frequencies

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AS Aperture Scan for 3 Different Chromaticities

Small horiz. ChromaticitySmall horiz.Small vert.Large vert.

Large horiz. Large vert.

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ε > 3 % straight2.65 % arcs

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 ϵ = 2.6 % straight ϵ = 2.6 % straight 1.75 % arcs 1.9 % arcs



AS Aperture Scan for 3 Different Chromaticities

Small horiz. ChromaticitySmall horiz.Small vert.Large vert.

Large horiz. Large vert.



ε > 3 % straight2.65 % arcs

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 $\epsilon = 2.6 \%$ straight $\epsilon = 2.6 \%$ straight 1.75 % arcs 1.9 % arcs



ALS Results agree well with Simulations



 Simulations reproduce shift of beam loss area caused by the coupling resonance to higher momentum deviations

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Lifetime vs. Vertical Physical Aperture

- Found dynamic+momentum 'aperture' and (small) vertical physical aperture very closely linked for ALS momentum aperture collapses around 40-50 $\sigma_{\rm v}$
- Since both are very important performance parameters studied link further:
- Performance (Brightness) of undulators/wigglers (both permanent magnet and SC) depends on magnetic gap
- Strong incentive to push physical aperture as low as possible
- Evolution at the ALS from 15 mm
 via 9 and 8 mm to now 5.5 mm –
 enabled by better understanding and
 optimization





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Summary: Lifetime Limiting Processes

Elastic Scattering

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$$\frac{1}{\tau_{el}} \propto \frac{1}{E^2} \times \left(\frac{\beta_x}{\Delta_x^2} \langle P \beta_x \rangle + \frac{\beta_y}{\Delta_y^2} \langle P \beta_y \rangle \right)$$
(1)

- Touschek Effect
- Quantum Lifetime

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$$\frac{1}{\tau_{tou}} \propto \frac{1}{E^3} \frac{I_{bunch}}{V_{bunch}} \frac{1}{\varepsilon} f(\varepsilon, \sigma'_x, E)$$
(2)

$$\frac{1}{\tau_q} \propto \frac{\Delta^2}{\sigma^2} \times \exp(-\frac{\Delta^2}{2\sigma^2})$$
(3)

• Inelastic Scattering $\frac{1}{2} \propto \langle P \rangle \times \ln(\varepsilon)$

$$\frac{1}{\tau_{inel}} \propto \langle T \rangle \times \mathrm{III}(\varepsilon)$$

$$\frac{1}{\tau} = \frac{1}{\tau_{el}} + \frac{1}{\tau_{tou}} + \frac{1}{\tau_{ql}} + \frac{1}{\tau_{inell}}$$



C. Steier, Beam-base

C. Steier, Beam-based Diagnostics, USPAS 2015, 2015/6/22-25



(4)



Summary (Lifetime)

- Lifetime limiting processes are one main performance limitation for accelerators
- Particle scattering (gas, intra beam, inter beam, ...) usually launches particles to large amplitudes leading to particles loss
- Understanding the Nonlinear Dynamics that affects the motion of this large amplitude particles is essential to quantify these processes
 - And be in a position to improve things, i.e. make beam lifetimes longer



ALS Further Reading (Nonlinear Dynamics)

- A. Wrulich, *Single Beam Lifetime*, CAS 5th General accelerator physics course, CERN 94-01
- M. Sands, *The Physics of Electron Storage Rings. An Introduction*, SLAC Report 121 UC-28 (ACC) (1970)
- W. Decking, and D. Robin, in *Proceedings of the AIP Conference 468*, Arcidosso, Italy, 1998 (Woodbury, New York, 1999), 119–128.
- W. Decking, and D. Robin, in *Proceedings of the 18th Particle Accelerator Conference*, New York, 1999 (IEEE, Piscataway, NJ, 1999), 1580–1583.
- J. Safranek, Nucl. Instr & Methods, A388, 27 (1997)
- D. Robin, J. Safranek, and W. Decking, Phys. Rev. ST Accel. Beams 2, 044001 (1999).
- D. Robin, C. Steier, J. Laskar, and L. Nadolski, Phys. Rev. Lett., 85, 3, 558 (2000).
- J. Laskar, *Icarus*, 88, 266-291 (1990).
- H.S. Dumas, and J. Laskar, Phys. Rev. Lett., 70, 2975–2979 (1993).
- J. Laskar, in *Proceedings of 3DHAM95 NATO Advanced Study Institutes*, S'Agaro, 1995 (Kluwer Academic Publishers, Dordrecht, The Netherlands, 1999), 134–150.
- C. Steier, D. Robin, J. Laskar, and L. Nadolski, in *Proceedings of the 7th European Particle Accelerator Conference*, Vienna, 2000 (Austrian Academy of Sciences Press, Vienna, 2000), 1077–1079.
- J. Laskar, Physica D, 67, 257-281 (1993)
- C. Steier, et al. Phys. Rev. E 65, 056506 (2002).



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Backup Slides



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Tune scan summary

Advantages

Quickly and sensitively see excited resonances in the tails and core of the beam as a function of different tunes

Disadvantages

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Probing different machines and not looking at the effect of resonances on one working point and at different amplitudes. This is what one really would like to see.







S Full Frequency Maps for ALS 5cm period EPU



- Frequency maps agree well with simulated ones
 - Higher order detuning with amplitude
 - Additional resonances excited
 - Reduced dynamic aperture

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Frequency map measurements at BESSY-II

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- Beam dynamics highly dependent on EPU row phase.
- Dynamic aperture reduction induced injection losses



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Constant Lifetime?

- The previous model assuming constant lifetime is often too simple for describing real accelerators. In most of the electron storage rings the lifetime actually depends significantly on current:
 - The Touschek effect (discussed later), whose contribution dominates the losses in many of the present electron accelerators, depends on current. When the stored current decreases with time, the losses due to Touschek decrease as well and the lifetime increases.
 - Synchrotron radiation intensity and therefore the release of molecules trapped in the vacuum chamber wall depends on current (gas desorption).
 - For higher currents, the pressure in the vacuum chamber increases (dynamic pressure) resulting in more scattering of the beam with the residual gas and a reduced lifetime.
- For reasonably small variations of the current, the constant lifetime assumption is locally valid and it is widely used.



AS Dependency of Lifetime on Transverse Aperture

Theoretical Results







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AS Dependency of Lifetime on Momentum Acceptance

Theoretical results including bunch length change





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ALS Lifetime versus RF-Bucket Height





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ALS Motivation for off-energy dynamics studies

- Design momentum aperture for newer light sources (like Soleil) 5-6% to achieve reasonable lifetimes
- Even using top-up (quasi continous) injection, lifetime is still an issue:
 - Radiation damage/safety
 - Injection transients are not fully transparent





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S Coupling – Sensitivity to Physical Aperture



- Sensitivity of Touschek lifetime on vertical aperture depends on coupling
- High order coupling resonances scale similar to global/local coupling
- For given emittance ratio one can optimize coupling vs. vertical dispersion





ASSimulation Results (Momentum Aperture – Gap)

Emittance increased using vertical dispersion wave ...



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using excitation of coupling resonance



 Tracking results are in good agreement with measured effects, i.e. case with dispersion wave has less yellow and orange areas than the one with excited coupling resonance, indicating less sensitivity to reduced vertical aperture



