Turn-by-turn BPM data analysis

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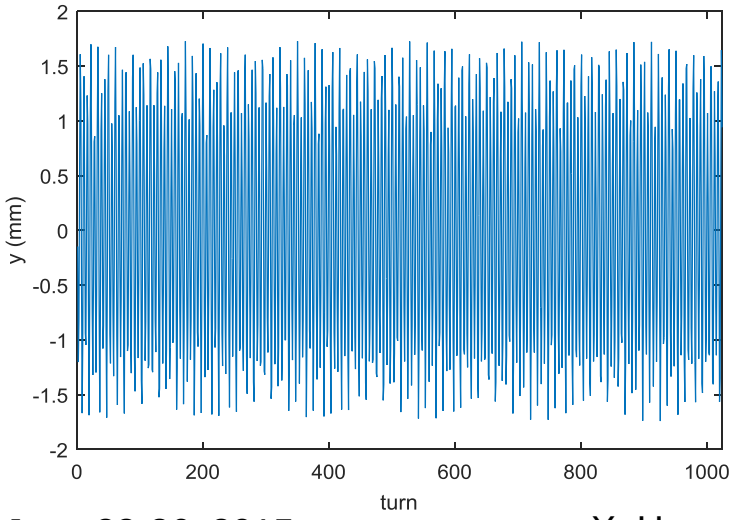
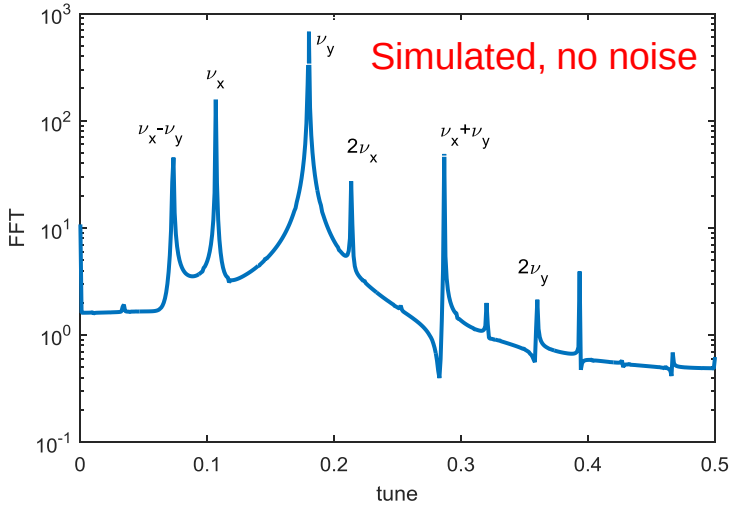
USPAS 2015 Summer – Beam based diagnostics

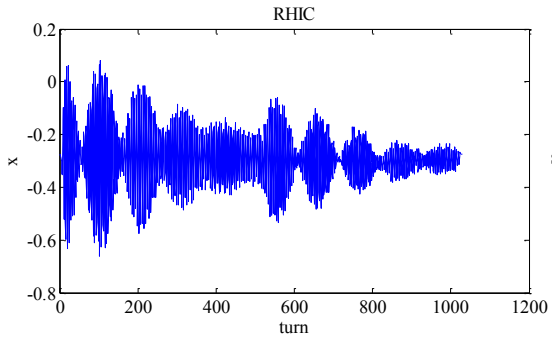
Turn-by-turn BPM data

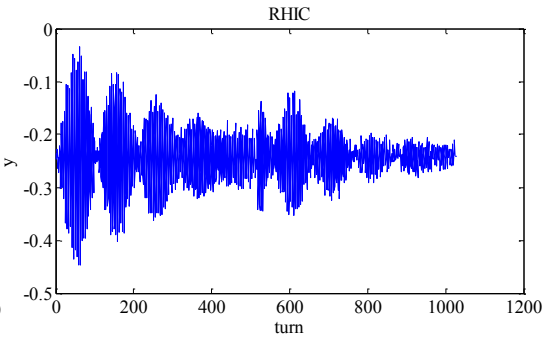
- In addition to the average orbit, TbT BPM data capture the dynamics of beam motion.
- TbT BPM data contain
  - Temporal information:
    - various frequency components from single particle dynamics
    - De-coherence/re-coherence – a collective effect
  - Spatial information: BPMs at different location see different amplitudes and phases of the frequency components.
  - Noise

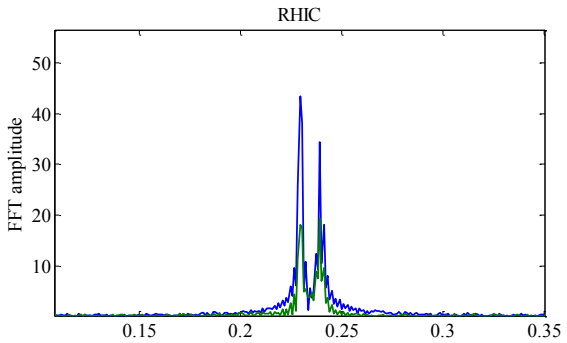


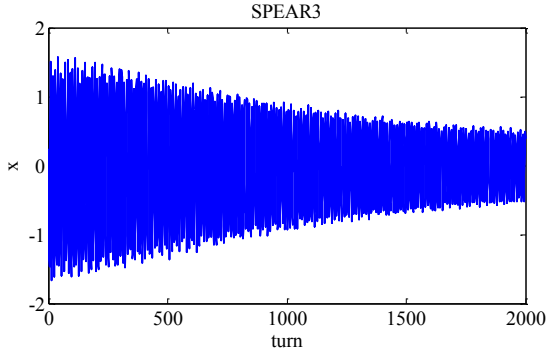


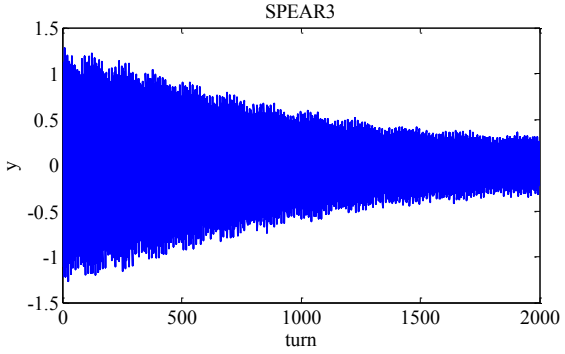
Examples of turn-by-turn BPM data

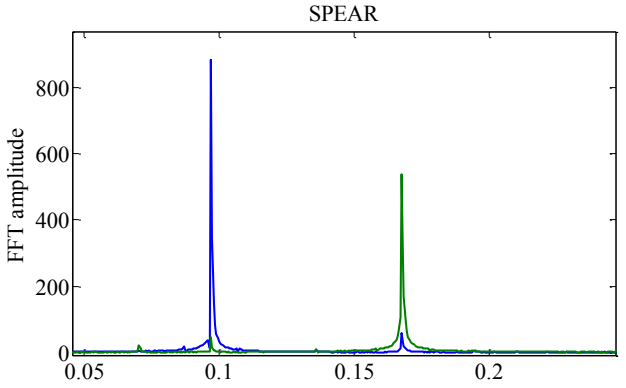


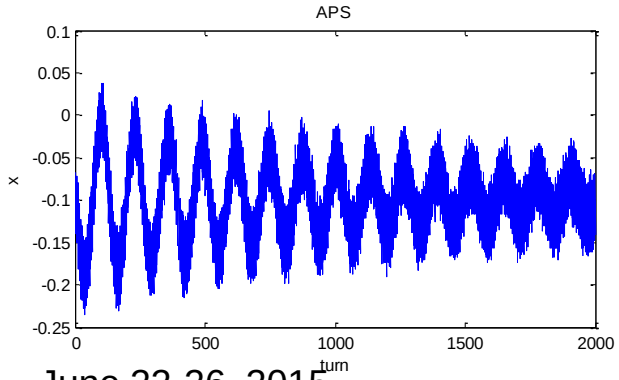


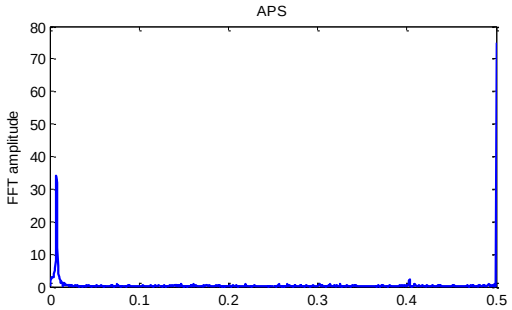






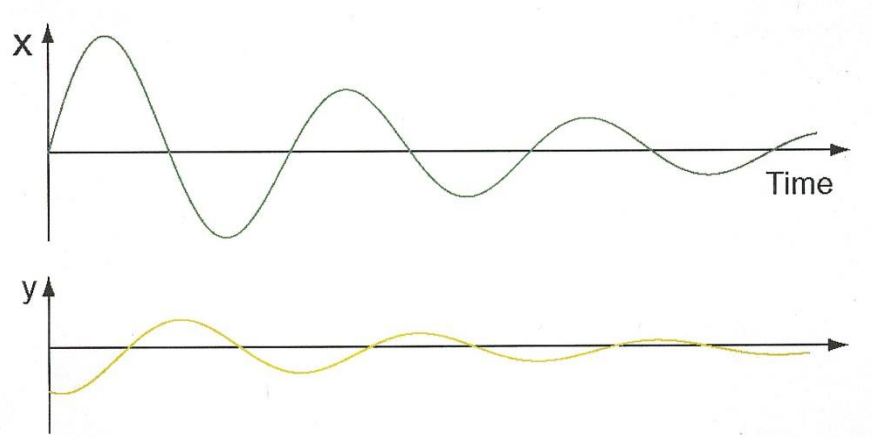






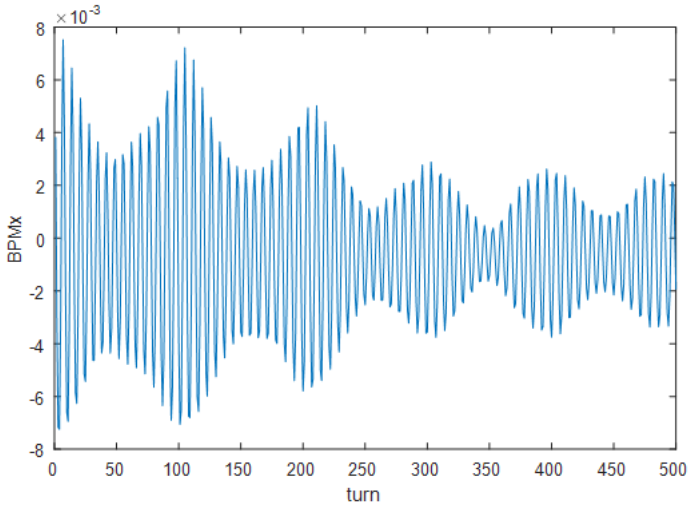
Excitation of beam: impulse excitation

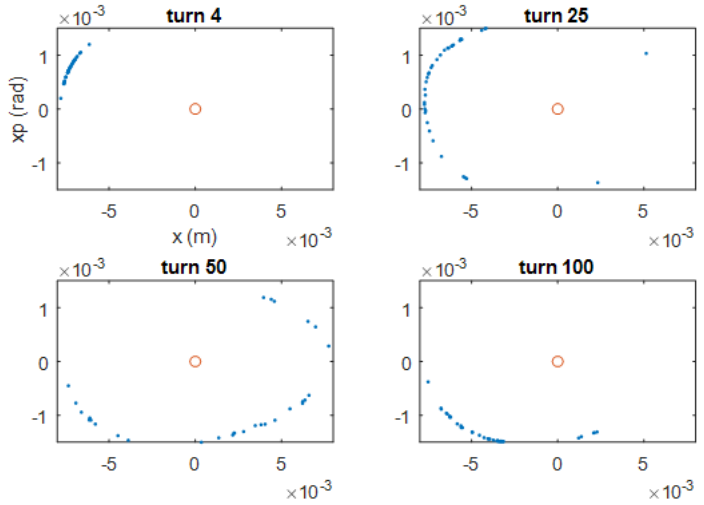
- A fast kick to the beam by a pinger or kicker, or injection steering error.



Free oscillation (reveals characteristic tune and linear/nonlinear responses). May have fast decoherence (fewer turns of usable signal)

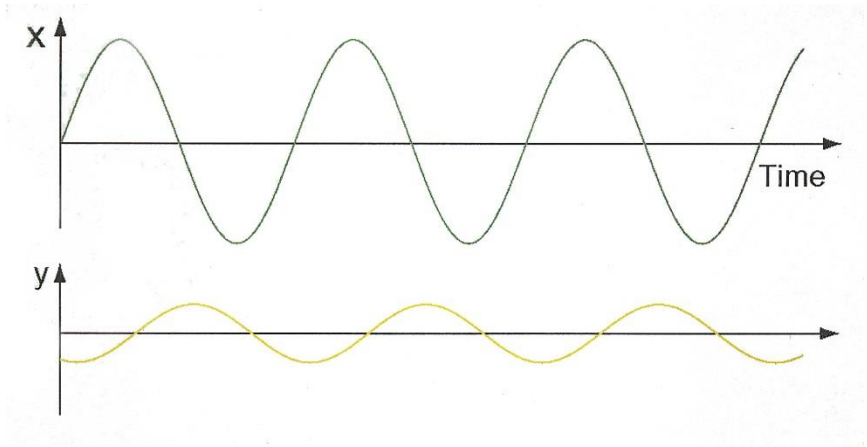
Tune spread due to large chromaticity (and energy spread) or oscillation amplitude (and lattice nonlinearity) causes de-coherence.



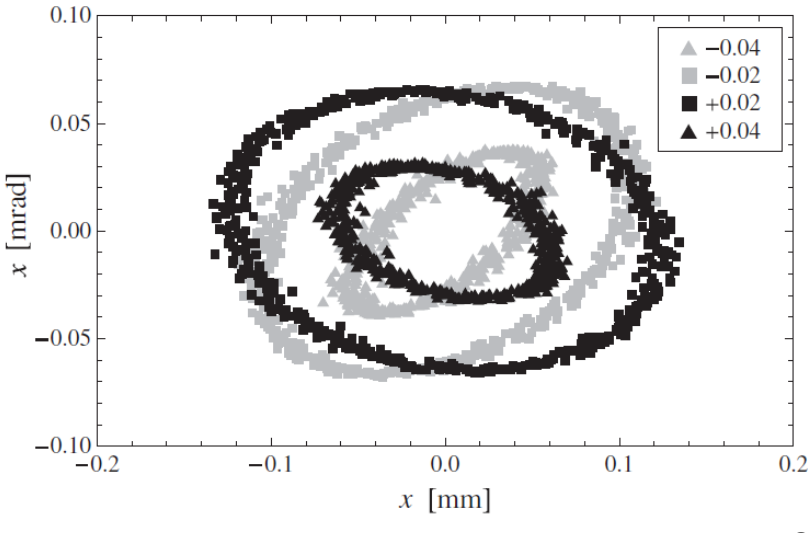


Resonant excitation

- Constantly driving with a weak sinusoidal signal (w/ an AC dipole or stripline kicker). Driving tune is equal or close to betatron tunes.



(1) Driven oscillation (observed tunes beta functions and phase advanced are not altered*)
(2) Driving signal may be phase locked to beam (electron beam).
(3) Steady signal (steady state, but not necessarily single particle motion, may still be affected by de-coherence)



Observed phase space ellipse depends on tune difference $\delta_d = \nu_d - \nu$.

*Miyamoto, et al, PRSTAB, 11 084002 (2008)

Precise tune determination from discrete data

- NAFF (numerical analysis of fundamental frequency)
  - Maximize the spectrum overlap between the sample and that of a pure sinusoidal signal. J. Laskar, et al,1990, Icarus 88, 266-291

$$F(\bar{\nu}) = \left| \sum_{n=1}^{N} s_n e^{-i2\pi\bar{\nu}n} W(n) \right|$$

- Interpolated FFT
  - Interpolation with peak frequency and its highest neighbor.

$$\nu_{\text{int}} = \frac{1}{N}\left[k - 1 + \frac{A(k)}{A(k-1) + A(k)} \right], k-1 \le N\nu \le k$$

Asseo CERN PS Note 87-1

For $N \gg 1$

$$\nu_{Fint} = \frac{k}{N} + \frac{1}{\pi}\text{atan}\left(\frac{|\phi(\nu_{k+1})|\sin(\frac{\pi}{N})}{|\phi(\nu_k)| + |\phi(\nu_{k+1})|\cos(\frac{\pi}{N})} \right)$$

R. Bartolini, et al, EPAC 96

These methods can achieve accuracy $\propto \frac{1}{N^2}$, or $\propto \frac{1}{N^4}$ when with data windowing.

Calculation of amplitude and phase

- With the tune determined, amplitude and phase of the frequency component can be obtained by

$C = \sum_n x(n) \cos 2\pi n\nu, \qquad S = \sum_n x(n) \sin 2\pi n\nu,$

Then amplitude and phase are

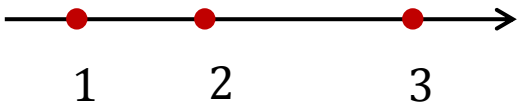
$A = \frac{2\sqrt{C^2+S^2}}{N}$, and $\psi = -\cot^{-1}\frac{S}{C}$,

P. Castro, et al, PAC'93

where N is the number of turns. Error of phase $\sigma_\psi = \frac{1}{A}\sqrt{\frac{2}{N}}\sigma_x$

- Measurement of beta function from phase advances.

$$\beta_1^{meas} = \beta_1^{model} \frac{\cot\psi_{12}^{meas} - \cot\psi_{13}^{meas}}{\cot\psi_{12}^{model} - \cot\psi_{13}^{model}}$$



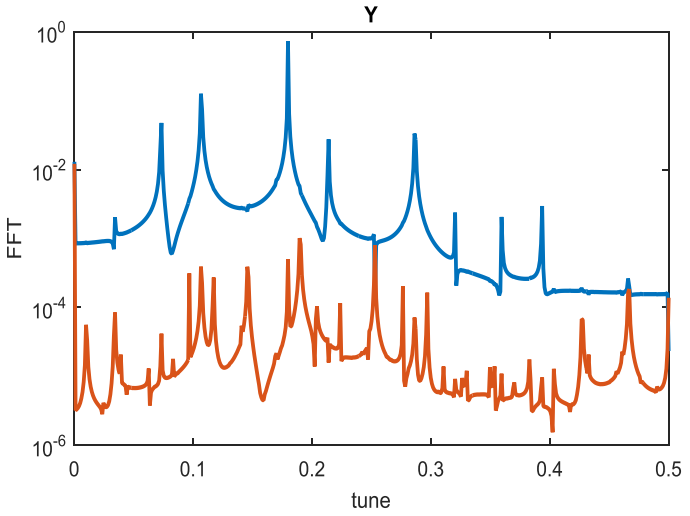
This is because for 2×2 transfer matrix $M_{i\to f}$,

$$\frac{m_{11}}{m_{12}} = \frac{1}{\beta_i}(\cot\psi_{if} + \alpha_i)$$

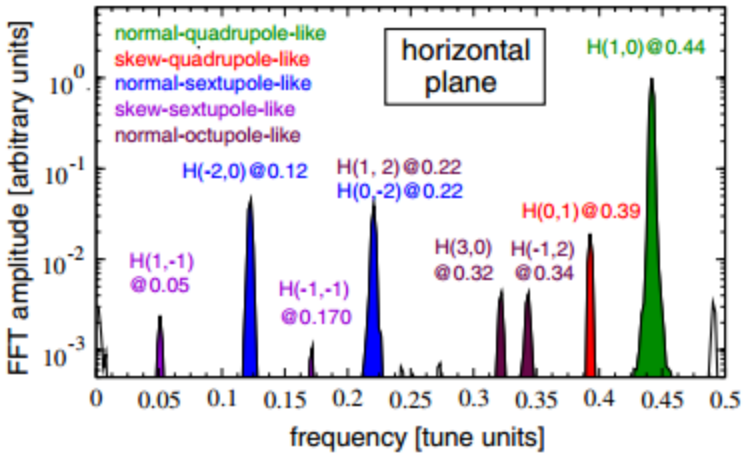
assuming for a short section difference between actual and model matrix is small.

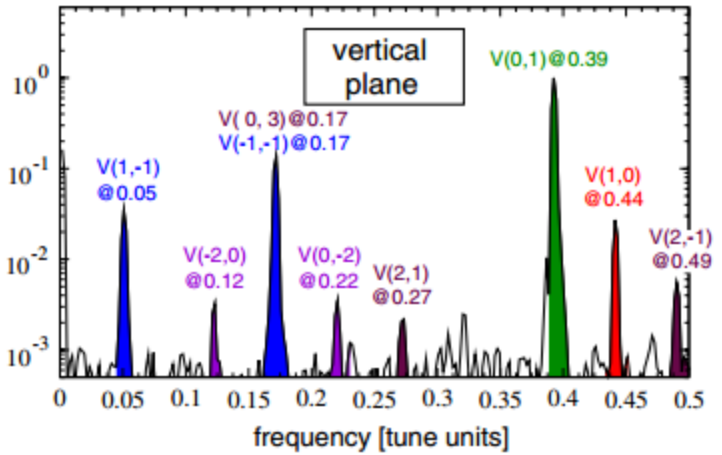
Extracting frequency components

- The frequency components can be iteratively extracted.



The amplitudes and phases of the various frequency components from BPMs at different locations can be used for optics, coupling, and nonlinear dynamics correction.





Experimental data at ESRF

A. Franchi, et al, PRSTAB 17, 074001 (2014)

Coupling correction with resonance driving terms

RDTs of linear coupling, f_{1001} (difference) and f_{1010}(sum)

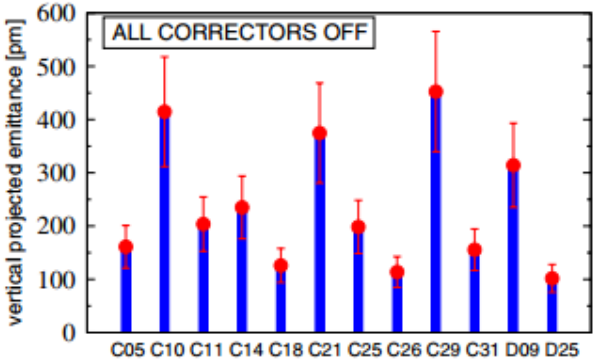
$$\hat{x} - i\hat{p}_x^h = \sqrt{2I_x}e^{i\psi_x} - 2if_{1001}\sqrt{2I_y}e^{i\psi_y} - 2if_{1010}\sqrt{2I_y}e^{-i\psi_y},$$

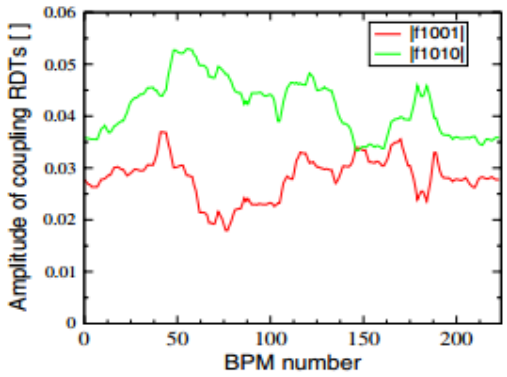
$$\hat{y} - i\hat{p}_y^h = \sqrt{2I_y}e^{i\psi_y} - 2if_{1001}^*\sqrt{2I_x}e^{i\psi_x} - 2if_{1010}\sqrt{2I_x}e^{-i\psi_x},$$

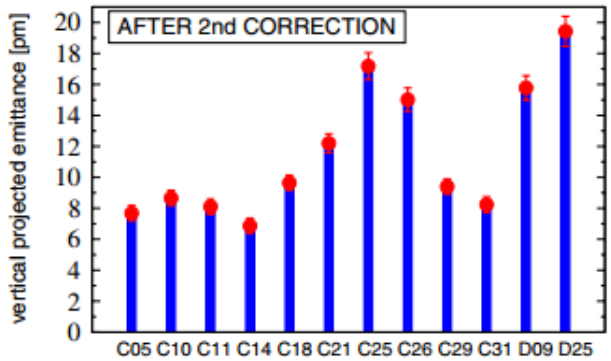
Fitting the model to produce the same RDTs.

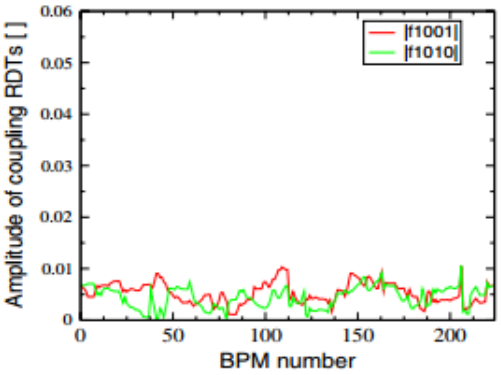
$$\begin{pmatrix} a_1\vec{f}_{1001} \\ a_1\vec{f}_{1010} \\ a_2\vec{D}_y \end{pmatrix}_{\text{meas}} = -\mathbf{M}\vec{J}_c,$$

R. Calaga, et al, PRSTAB 8, 034001 (2005)
A. Franchi, et al, PRSTAB 14, 034002 (2011)









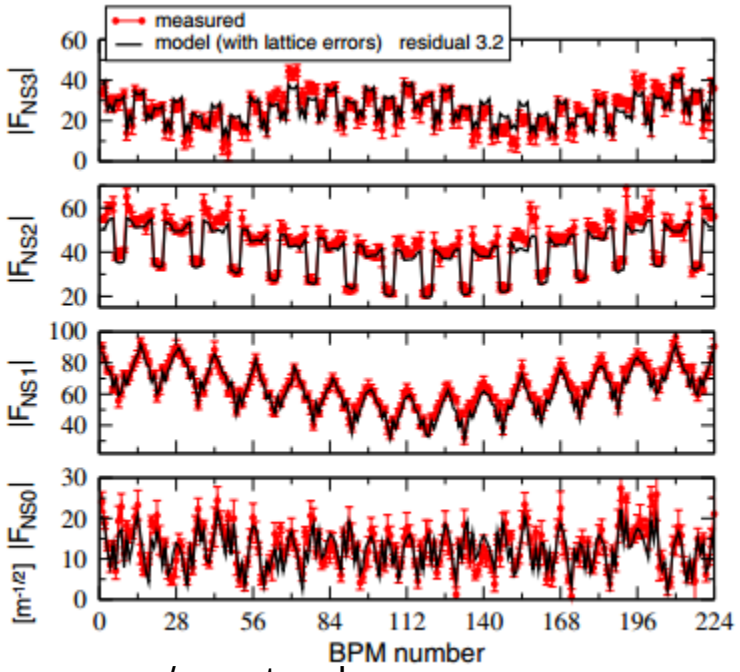
Nonlinear resonance correction

Strengths of spectral lines of TbT data are related to combined RDTs (CRDT).

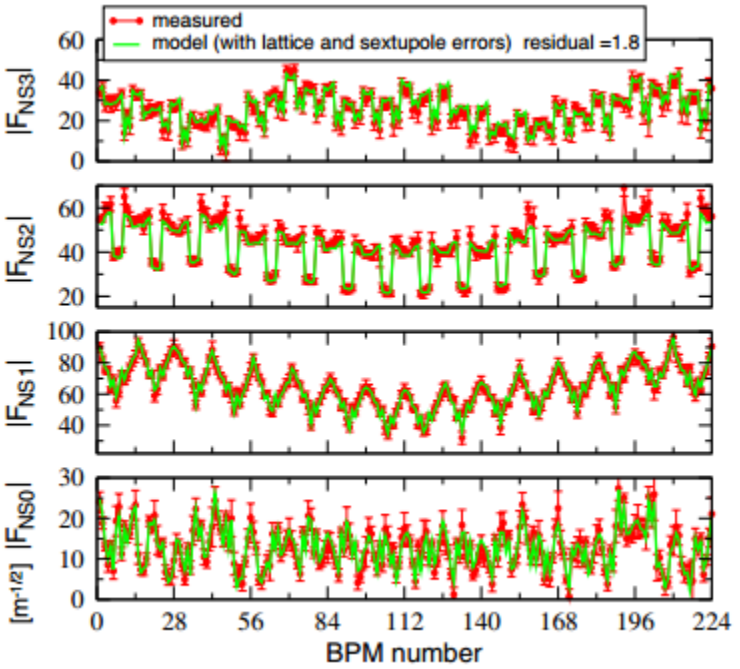
Example

Spectral line	Amplitude	Phase ϕ	Combined RDT	Resonances	Magnetic term
$H(-2,0)$	$(2I_x)\lvert F_{NS3}\rvert$	$q_{F_{NS3}}+\frac{3}{2}\pi-2\psi_{x0}$	$F_{NS3}=3f^{(1)}_{3000}-f^{(1)*}_{1200}$	(1,0),(3,0)	Normal sextupole
$H(0,-2)$	$(2I_y)\lvert F_{NS2}\rvert$	$q_{F_{NS2}}+\frac{3}{2}\pi-2\psi_{y0}$	$F_{NS2}=f^{(1)}_{1020}-f^{(1)}_{0120}$	$(1,-2),(1,2)$	Normal sextupole
$V(-1,-1)$	$(2I_x2I_y)^{1/2}\lvert F_{NS1}\rvert$	$q_{F_{NS1}}+\frac{3}{2}\pi-\psi_{x0}-\psi_{y0}$	$F_{NS1}=2f^{(1)}_{1020}-f^{(1)*}_{0111}$	$(1,2),(1,0)$	Normal sextupole
$V(1,-1)$	$(2I_x2I_y)^{1/2}\lvert F_{NS0}\rvert$	$q_{F_{NS0}}+\frac{3}{2}\pi+\psi_{x0}-\psi_{y0}$	$F_{NS0}=2f^{(1)}_{0120}-f^{(1)}_{0111}$	$(1,-2),(1,0)$	Normal sextupole

Amplitudes and phases of CRDTs can be fitted to the ideal model.



w/o sextupole errors



w/ sextupole errors

Amplitudes of normal sextupole driven lines. A. Franchi, et al, PRSTAB 17, 074001 (2014)

Fitting transfer matrix from TbT BPM data

When angle coordinates can be obtained from two BPMs separated by a short section with known or calibrated elements (such as a simple drift), the transfer matrix between two points can be determined.

$$x'_{1,2} = (x_2 - x_1)/L, \qquad y'_{1,2} = (y_2 - y_1)/L,$$ for a drift.

Least square fitting ($\mathbf{M}_{21} = \mathbf{M}_{1\to 2}$):

$$\mathbf{M}_{21} = \mathbf{X}_2\mathbf{X}_1^T(\mathbf{X}_1\mathbf{X}_1^T)^{-1}$$

$\mathbf{X}_{1,2}$ are $4 \times N$ matrix, with each column being $[x, x', y, y']^T$ for a pass. If $\mathbf{X}_1$ and $\mathbf{X}_2$ are at the same location, but separated by one turn, then we get the one-turn transfer matrix.

The transfer matrix by fitting this way is not strictly symplectic. One may fit 10 parameters that construct a 4×4 symplectic matrix.

X. Huang, et al, PRSTAB 13, 114002 (2010)

Model independent components and independent component analysis for TbT data processing

- In the previous method data processing is performed for each individual plane (x or y) of each BPM
- A better approach is to treat all BPM data collectively and coherently, because
  - All BPMs observe the same signals.
  - Number of BPMs is typically much bigger than the number of signals.
  - Statistical advantages can be achieved when proper methods are used to extract the signals from the large number of samples.
- Methods taking the coherent approach include
  - Model independent analysis (principal component analysis)
  - Independent component analysis.

J. Irwin, et al, PRL 82, 1684 (1999)

X. Huang, PRSTAB, 8, 064001, 2005

A model of BPM turn-by-turn data

- The turn-by-turn beam position signal is a combination of various source signals.

$$x_i(t) = \sum_j a_{ij} s_j(t) + n_j(t)$$ For the i'th BPM

or $$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$$ $\mathbf{A}$ is the mixing matrix

There are only a few meaningful source signals, such as betatron oscillation and synchrotron oscillation.

Form a matrix of the BPM data

$$\mathbf{x} = \begin{pmatrix} x_1(1) & x_1(2) & \cdots & x_1(T) \\ x_2(1) & x_2(2) & \cdots & x_2(T) \\ \vdots & \vdots) & \ddots & \vdots \\ x_m(1) & x_m(2) & \cdots & x_m(T) \end{pmatrix}$$

m BPMs and T turns

X. Huang, PRSTAB, 8, 064001, 2005

Betatron modes via singular value decomposition

It has been proven that when the BPM reading contains only one betatron mode, i.e.

$$x_m(t) = \sqrt{2J(t)\beta_m}\cos(\phi(t)+\psi_m)$$

Note the constant orbit offsets are always removed for each BPM. This is called "centering".

then there are only two non-trivial SVD eigen-modes

$$\mathbf{x} = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathrm{T}} = s_+\mathbf{u}_+\mathbf{v}_+^{\mathrm{T}} + s_-\mathbf{u}_-\mathbf{v}_-^{\mathrm{T}}$$

U,V are orthogonal matrices, S is a block-diagonal matrix.

$$u_{+,m} = \frac{1}{s_+}\sqrt{<J>\beta_m}\cos(\phi_0+\psi_m),$$

$$u_{-,m} = \frac{1}{s_-}\sqrt{<J>\beta_m}\sin(\phi_0+\psi_m)$$

$$v_+(t) = \sqrt{\frac{2J(t)}{T<J>}}\cos(\phi(t)-\phi_0),$$

$$v_-(t) = -\sqrt{\frac{2J(t)}{T<J>}}\sin(\phi(t)-\phi_0)$$

u: spatial vector

v: temporal vector

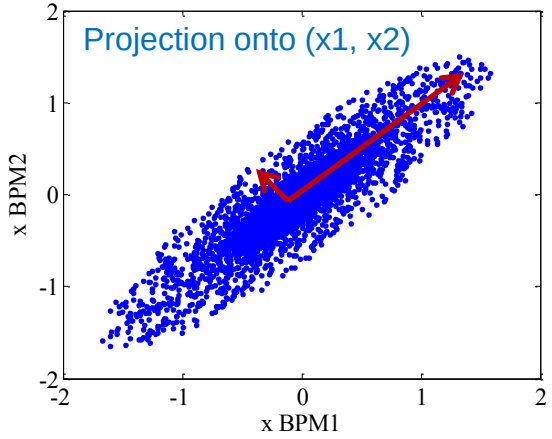
Beta function and betatron phase advance can be calculated from the spatial vector.

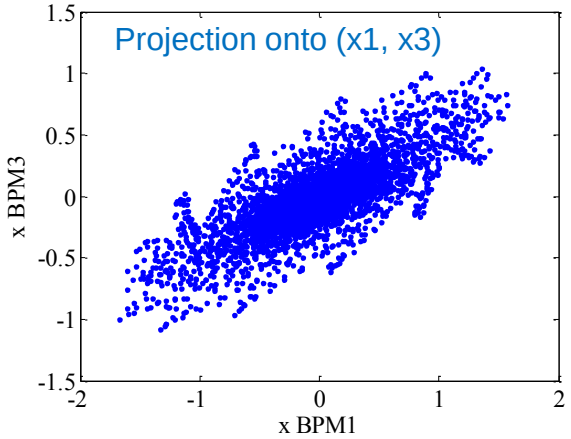
$$\psi_m = \tan^{-1}\left(\frac{s_-u_{-,m}}{s_+u_{+,m}}\right)$$

$$\beta_m = \frac{1}{<J>}[(s_+u_{+,m})^2 + (s_-u_{-,m})^2]$$

Chun-xi Wang, et al. PR-STAB 6, 104001 (2003).

What does SVD do?





The BPM data can be viewed as T points in the m-dimensional space.

$$P(t) = (x_1(t), x_2(t), \cdots, x_m(t))$$

These points form an hyper-ellipsoid. What SVD does is to identify its principal-axes. This is called principal component analysis (PCA).

PCA: with a linear orthogonal transformation to obtain a set of linearly un-correlated components (variables) which holds (successively) the largest variances.

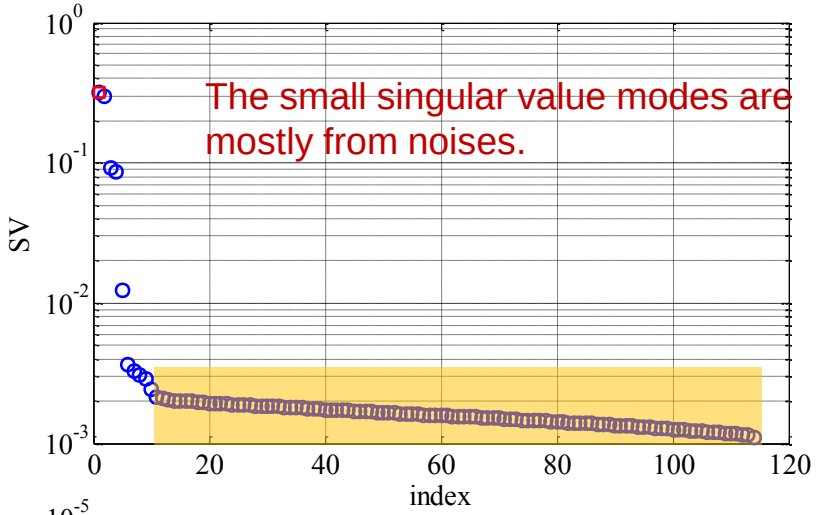
$$\mathbf{x}\mathbf{x}^T = \mathbf{U}\Sigma\mathbf{U}^T, \quad \Sigma = \mathbf{S}\mathbf{S}^T$$

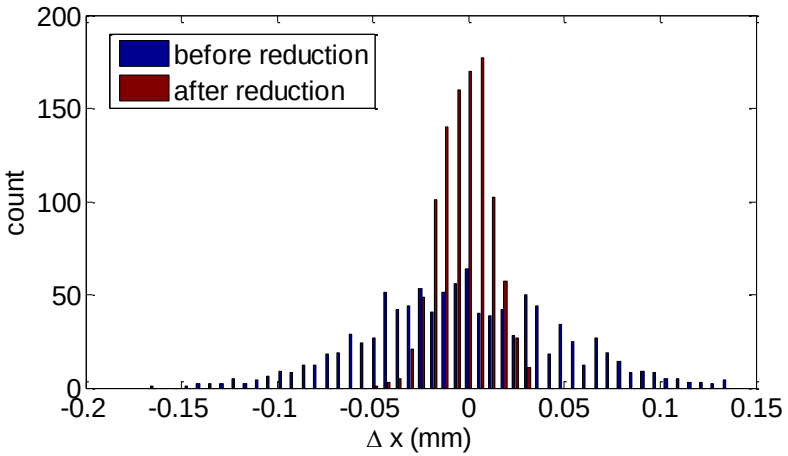
The U matrix diagonalize the covariance matrix.

The results in the previous slide states: with only one betatron mode in the BPM data, the hyper-ellipsoid degenerates to an ellipse (2D).

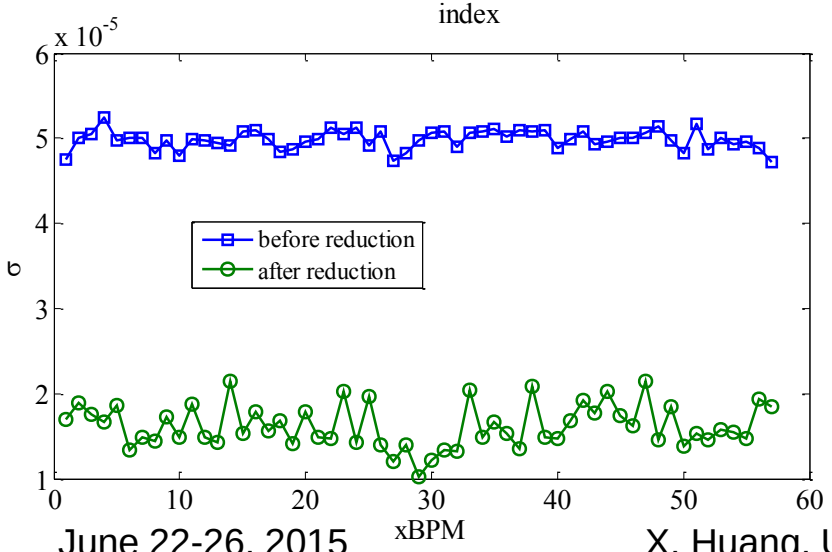
Noise reduction with SVD

As the random noises are distributed in all eigen-modes while the signals are concentrated in the leading eigen-modes, noise can be reduced by re-constructing the data after removing the noise-only (with small singular values) modes.





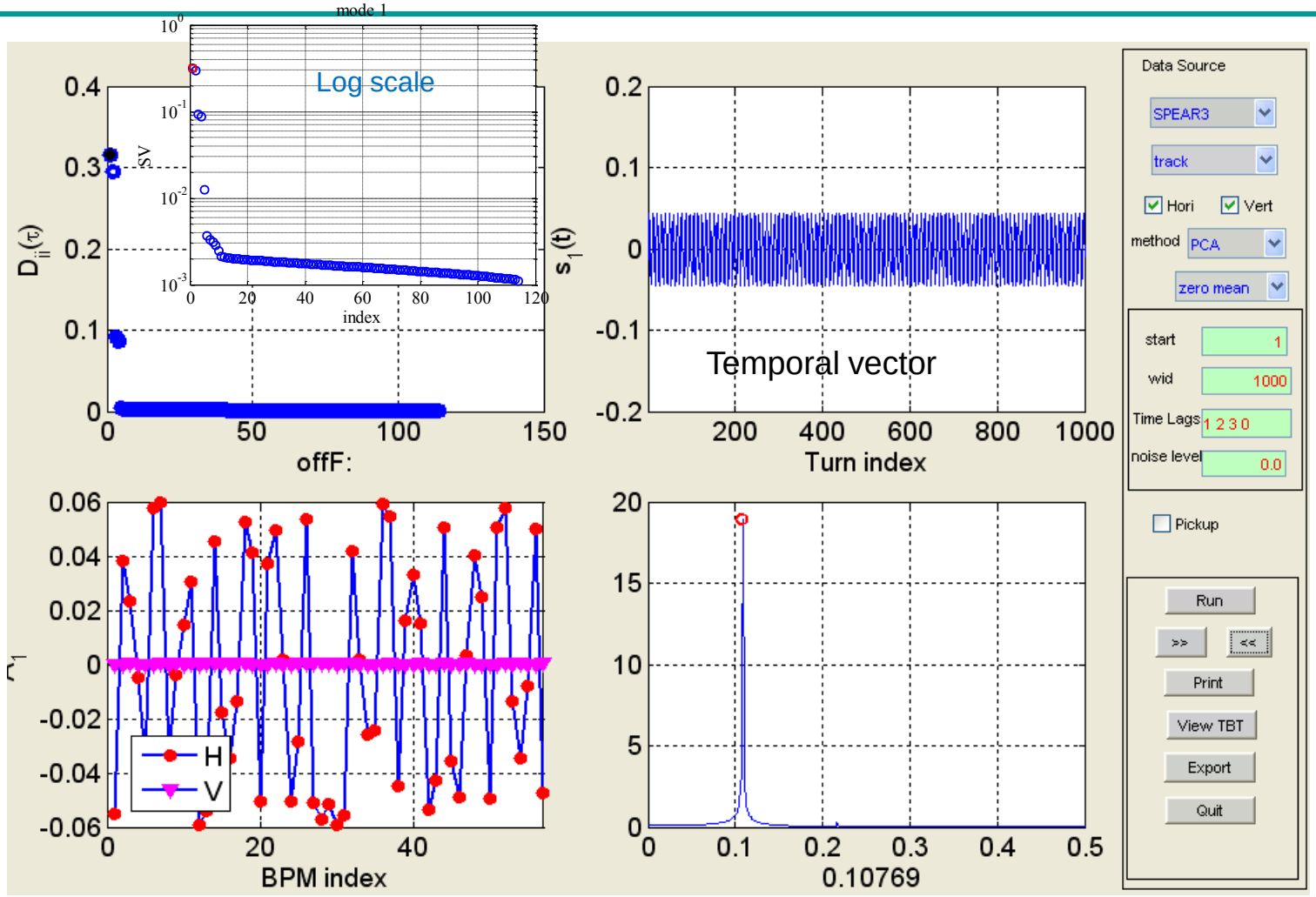
Keep 10 out of 114 modes.



The noise level (sigma) is reduced (keeping p out of 2m modes) to

$$\sigma_n = \sigma\sqrt{\frac{p}{2m}}$$

Example of SVD analysis

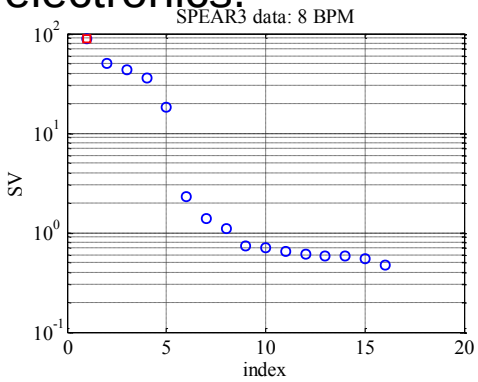


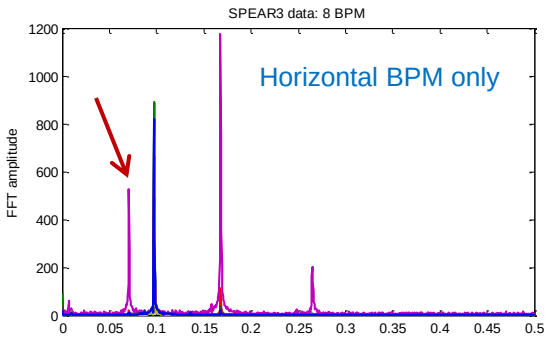
Spatial vector

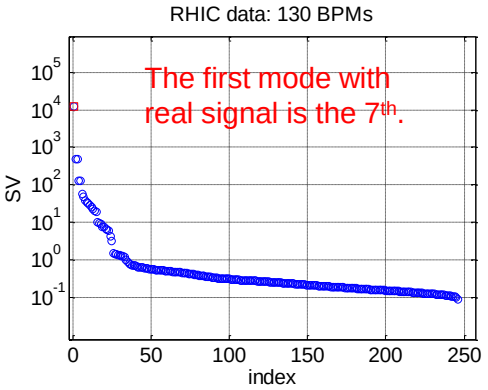
This data set is from tracking the SPEAR3 lattice with added random noise (sigma=0.05 mm). You will play with this program (and the data sets) in the computer-lab class.

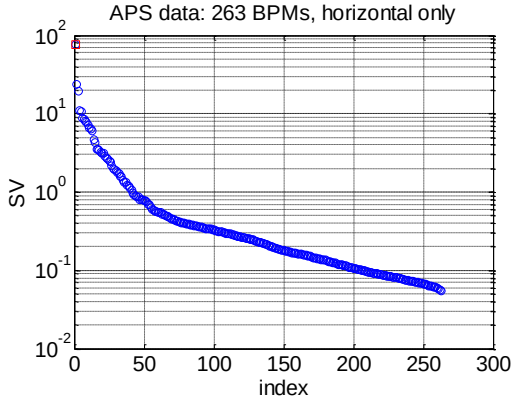
Limitation of the PCA method

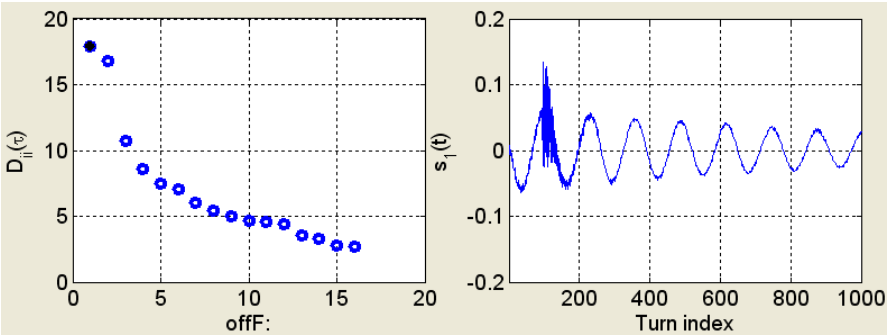
The eigen-modes are determined by the orthogonality and variances (strengths) of the components. If two signals have nearly the same strengths, they will be mixed in the eigen-modes (degeneracy in eigen-analysis). In reality this is common:
(1) Horizontal and vertical betatron modes can be mixed.
(2) Betatron modes can be mixed with the synchrotron mode.
(3) Actual BPM data are often plagued by signal contamination or failing electronics.











The independent component analysis (ICA)

- The source signals are assumed statistically independent.

$$p(x_1, x_2) = p(x_1)\,p(x_2)$$

This is a strong condition that the PCA analysis does not make full use of.

$$E\{h_1(x_1)h_2(x_2)\} = E\{h_1(x_1)\}E\{h_2(x_2)\}$$ For any function h_1,h_2.

PCA only requires the components to be linearly uncorrelated, i.e., the covariance between two variables is zero.

$$E\{x_1x_2\} - E\{x_1\}E\{x_2\} = 0$$

For two Gaussian variables, uncorrelatedness is equivalent to independence. Many ICA algorithms exploit the non-gaussianity of the signals, such as fastICA.

It is possible to use non-gaussianity based methods for BPM data analysis. But we will focus on an algorithm that relies on the time-spectrum of the source signals.

The Principle

- The source signals are assumed to be narrow-band with non-overlapping spectra, so their un-equal time covariance matrices are diagonal.

$$\langle \mathbf{s}(t)\mathbf{s}(t+\tau)^T \rangle = \mathrm{diag}[\rho_1(\tau), \rho_2(\tau), \ldots, \rho_n(\tau)]$$

Since $\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$

$$\mathbf{C}_x(0) \equiv \langle \mathbf{x}(t)\mathbf{x}(t)^T \rangle = \mathbf{A}\mathbf{C}_s(0)\mathbf{A}^T + \sigma^2\mathbf{I}$$

$$\mathbf{C}_x(\tau) \equiv \langle \mathbf{x}(t)\mathbf{x}(t+\tau)^T \rangle = \mathbf{A}\mathbf{C}_s(\tau)\mathbf{A}^T, \tau \neq 0$$

The mixing matrix A diagonalizes the un-equal time sample covariance matrices simultaneously.

The Algorithm* - 1

- Diagonalize the equal-time covariance matrix (data whitening)

D_1,D_2 are diagonal

Set to remove noise

$$\mathbf{C}_x(0) = [\mathbf{U}_1, \mathbf{U}_2] \begin{bmatrix} \mathbf{D}_1 & \\ & \mathbf{D}_2 \end{bmatrix} [\mathbf{U}_1, \mathbf{U}_2]^T \quad \text{with} \quad 0 \le \max(\mathbf{D}_2) < \lambda_c \le \min(\mathbf{D}_1)$$

Construct an intermediate "whitened" data matrix

$$\mathbf{z} = \mathbf{D}_1^{-\frac{1}{2}} \mathbf{U}_1^T \mathbf{x} = \mathbf{V}\mathbf{x} \qquad \text{which satisfies} \qquad < \mathbf{z}\mathbf{z}^T >= \mathbf{I}$$

This pre-processing step is just PCA. Matrix z contains the temporal vectors.

* The second order blind identification (SOBI) algorithm of A. Belouchrani, et al. in IEEE Trans. Signal Processing, 48, 900, (2003).

The Algorithm - 2

- Jointly diagonalize* the un-equal time covariance matrices of matrix z of selected time-lag constants.

$$\mathbf{C}_z(\tau) = \mathbf{W}\mathbf{C}_s(\tau)\mathbf{W}^T \quad \text{for} \quad \tau = \{\tau_i \mid i = 1,2,\cdots,k\}$$

Then

$$\mathbf{s} = \mathbf{W}^T\mathbf{V}\mathbf{x} \qquad \text{and} \qquad \mathbf{A} = (\mathbf{U}_1\mathbf{D}_1^{\frac{1}{2}})\mathbf{W}$$

The columns of A (spatial vectors) and corresponding rows (temporal vectors) of s are the resulting modes.

*Algorithm for joint diagonalization can be found in J.F. Cardoso and A. Souloumiac, SIAM J. Matrix Anal. Appl. 17, 161 (1996)

Linear Lattice Functions Measurements

- There are two betatron modes because each BPM sees different phase.

The betatron component $x = A_{b1}s_1 + A_{b2}s_2$

Beta function and phase advance

$$\beta = a(A_{b1}^2 + A_{b2}^2) \qquad \psi = \tan^{-1}\left(\frac{A_{b1}}{A_{b2}}\right)$$

- There is one synchrotron mode.

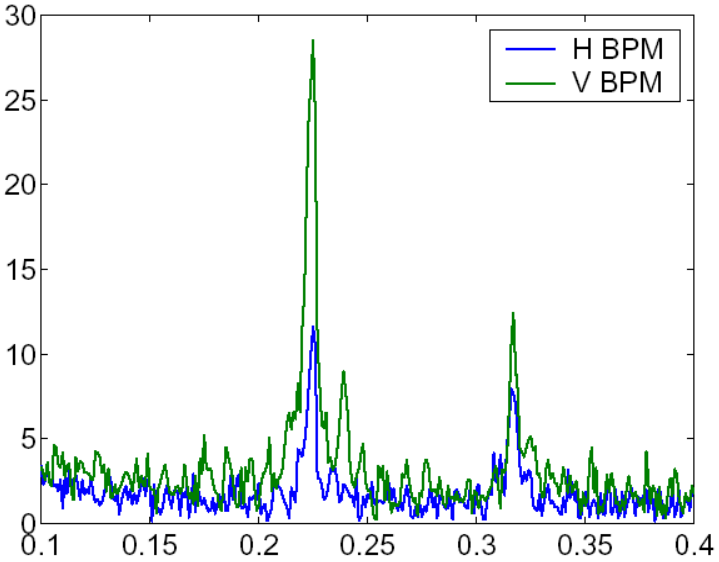
The synchrotron component $x = A_l s_l$

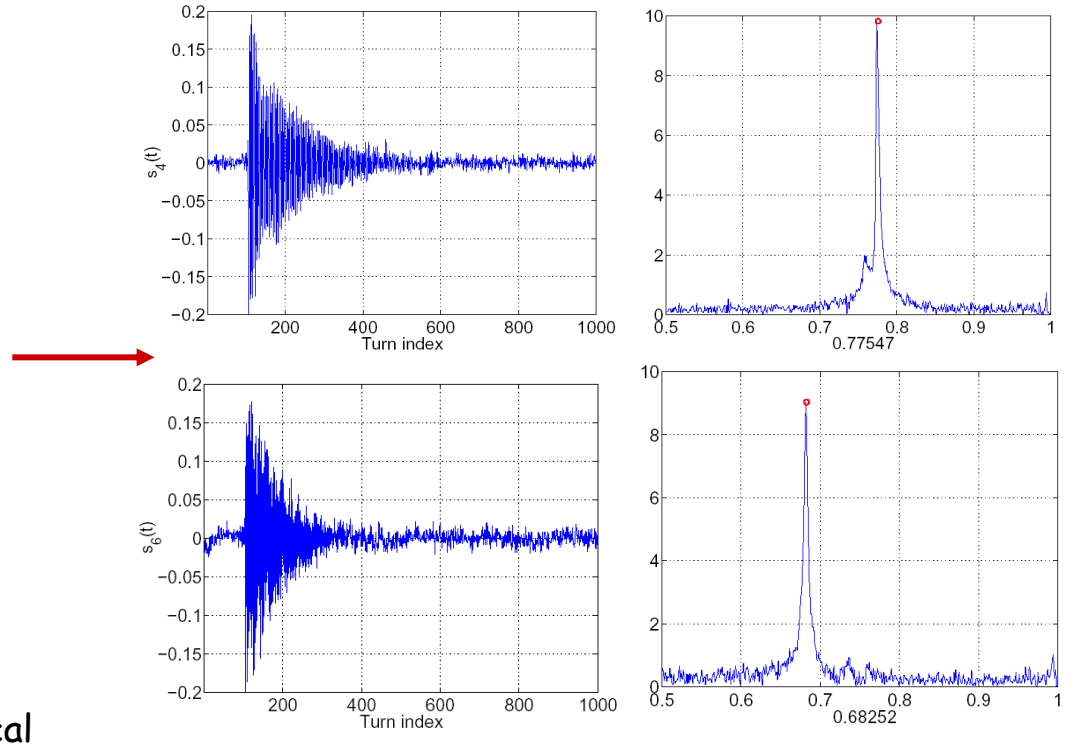
Dispersion function and momentum deviation

$$D_x = bA_l \qquad \delta = \frac{s_l}{b}$$

Example: de-coupling

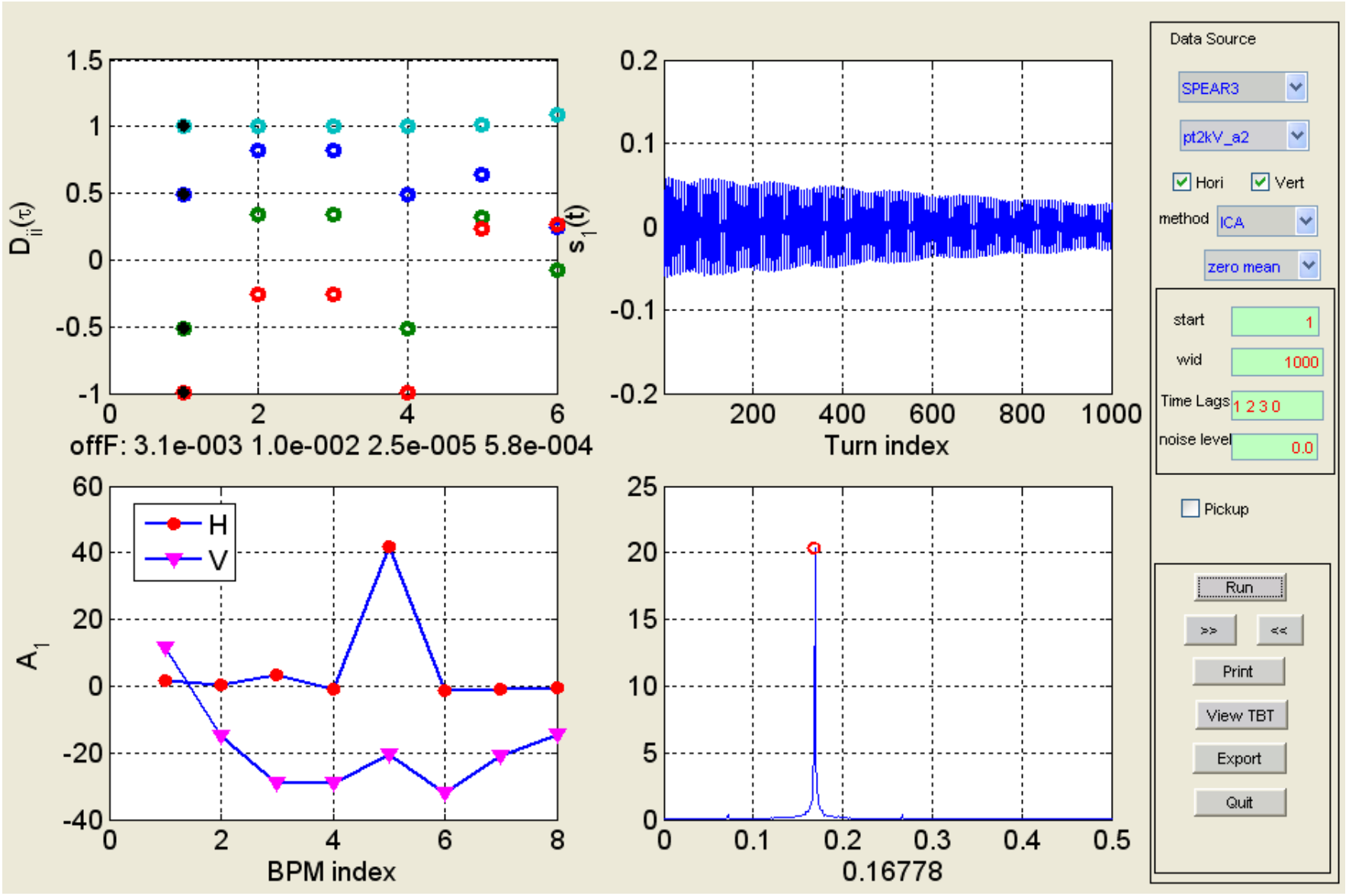
The ICA method can de-couple the normal modes in presence of linear coupling.



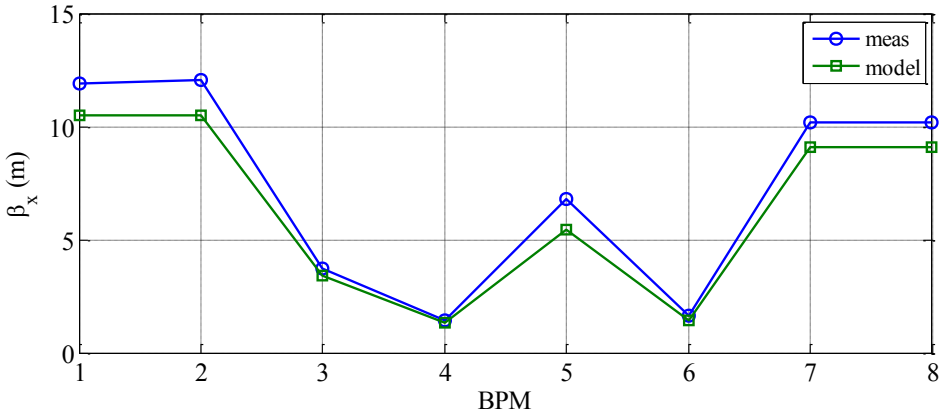


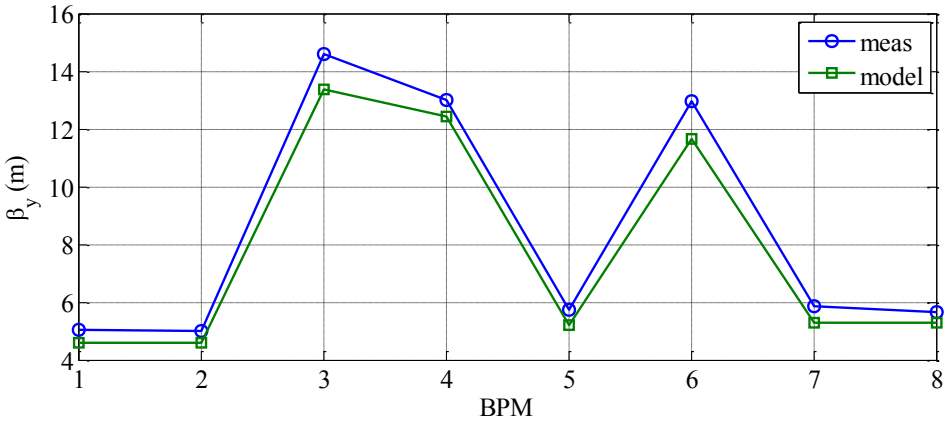
FFT spectra of raw horizontal and vertical BPM signals at section L1. Both BPMs see a mixture of the "plus" mode and "minus" mode.

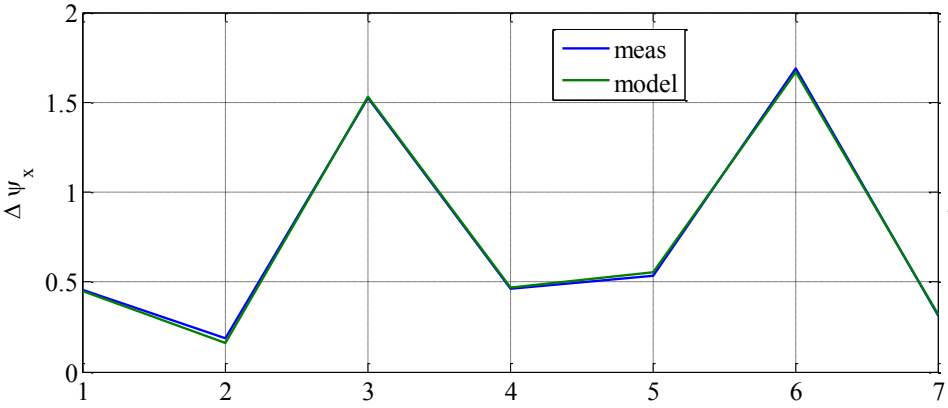
Example: SPEAR3 data

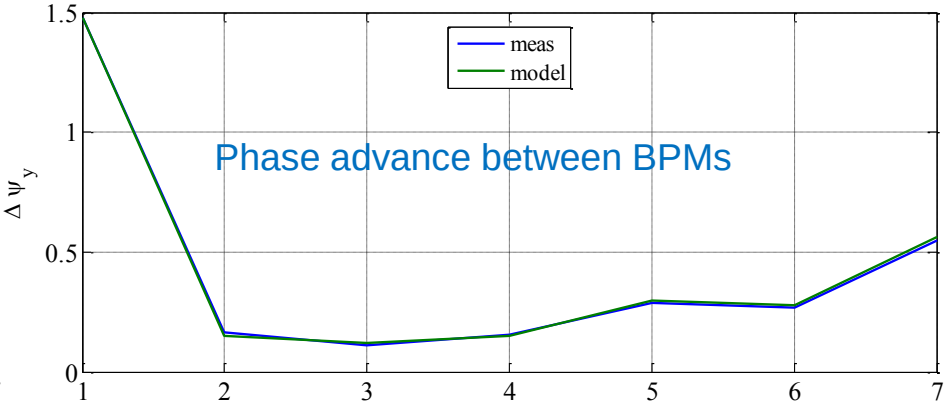


SPEAR3: the measured phase advance



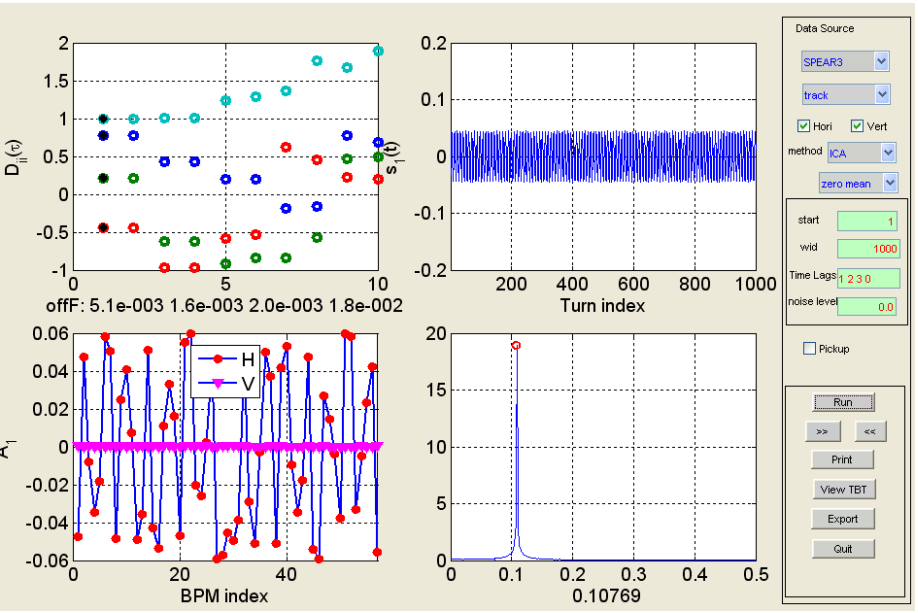


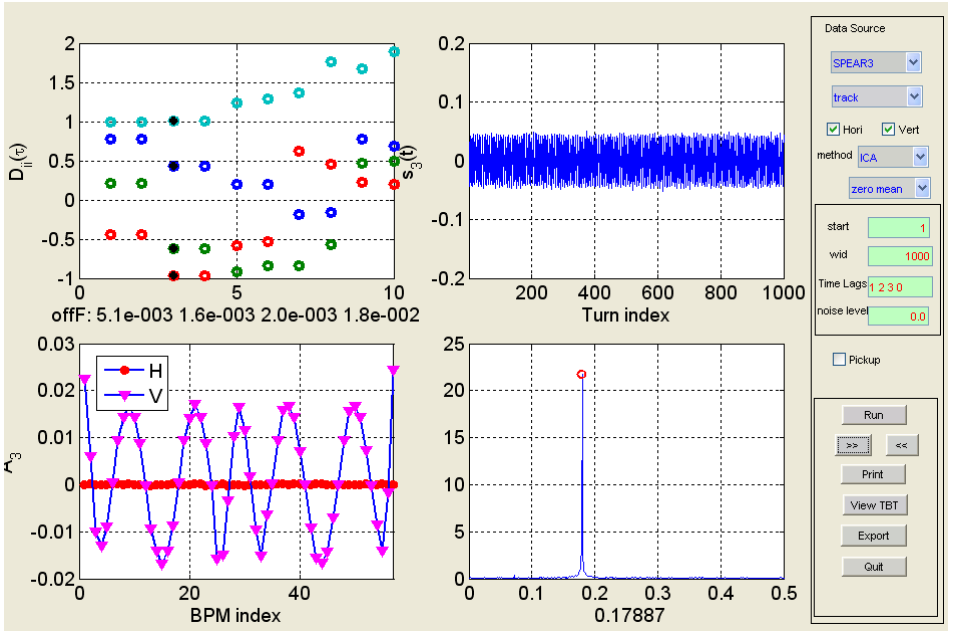




There are BPM gain errors. But the phase advances are in excellent agreement with the model.

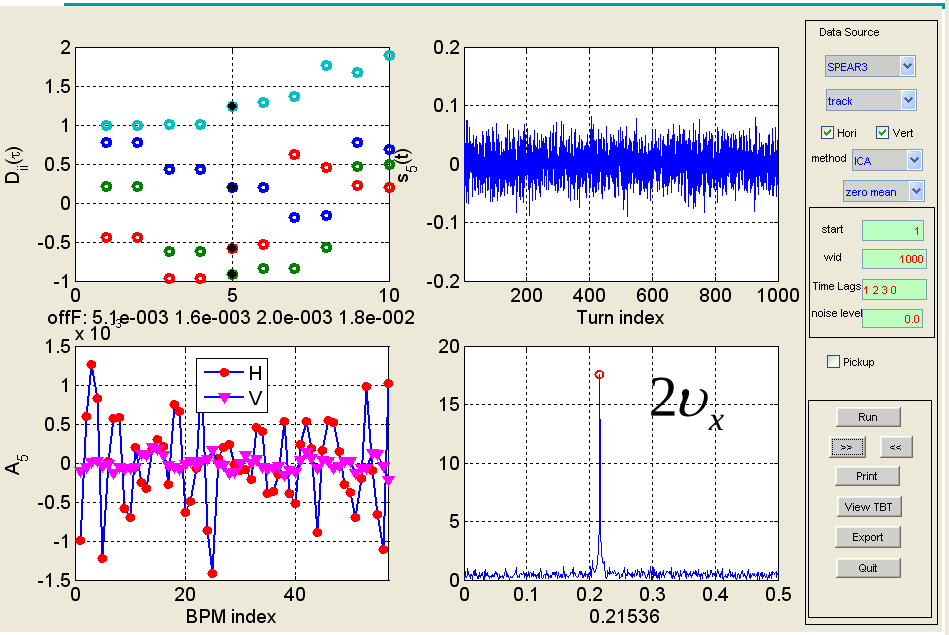
Example: Nonlinear modes in tracking data

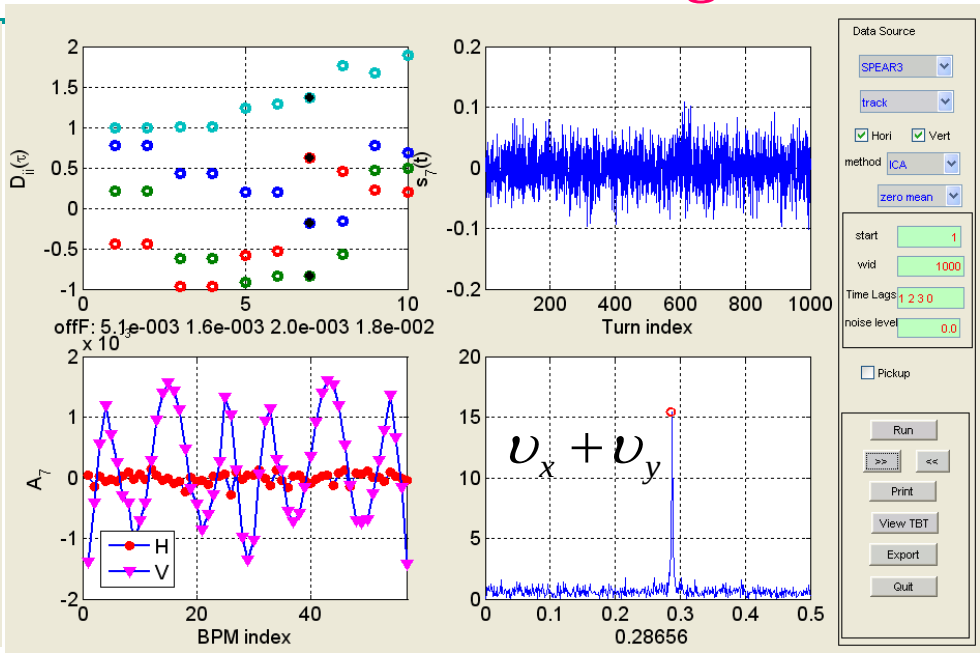


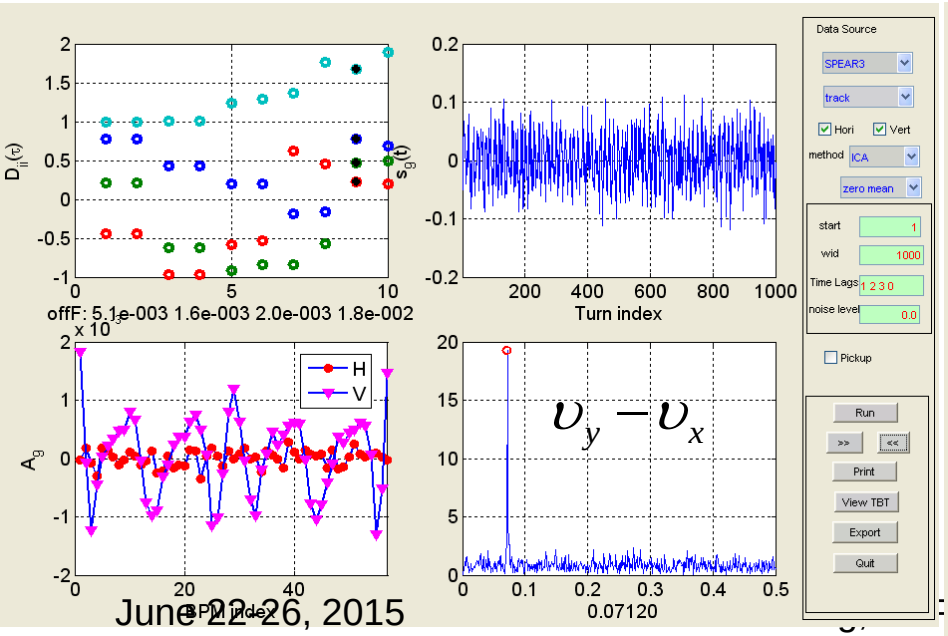


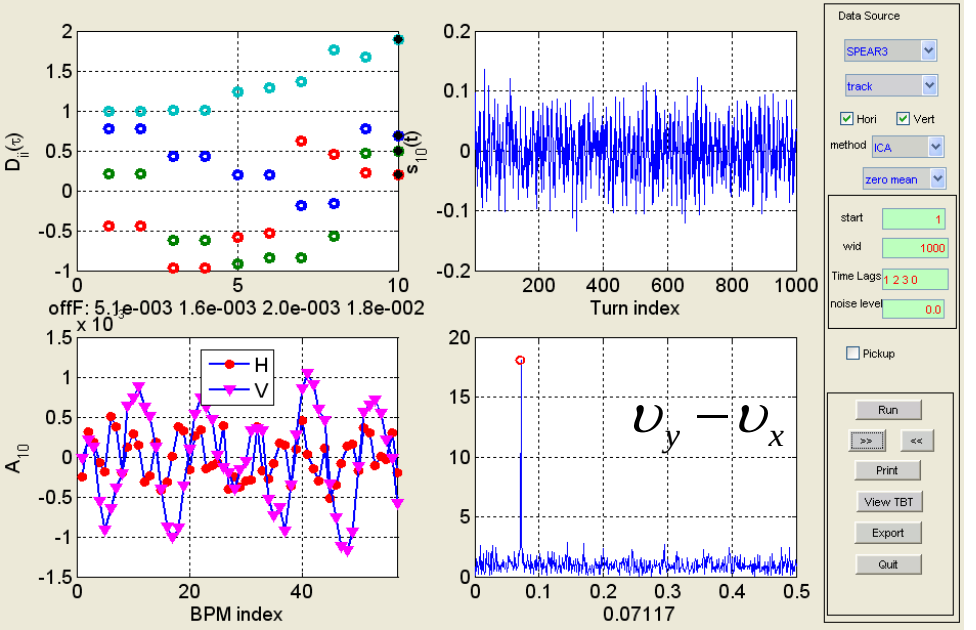
The betatron modes.
Data from tracking SPEAR3 model. There are 57 BPMs.

Example: coupling and nonlinear modes in tracking data

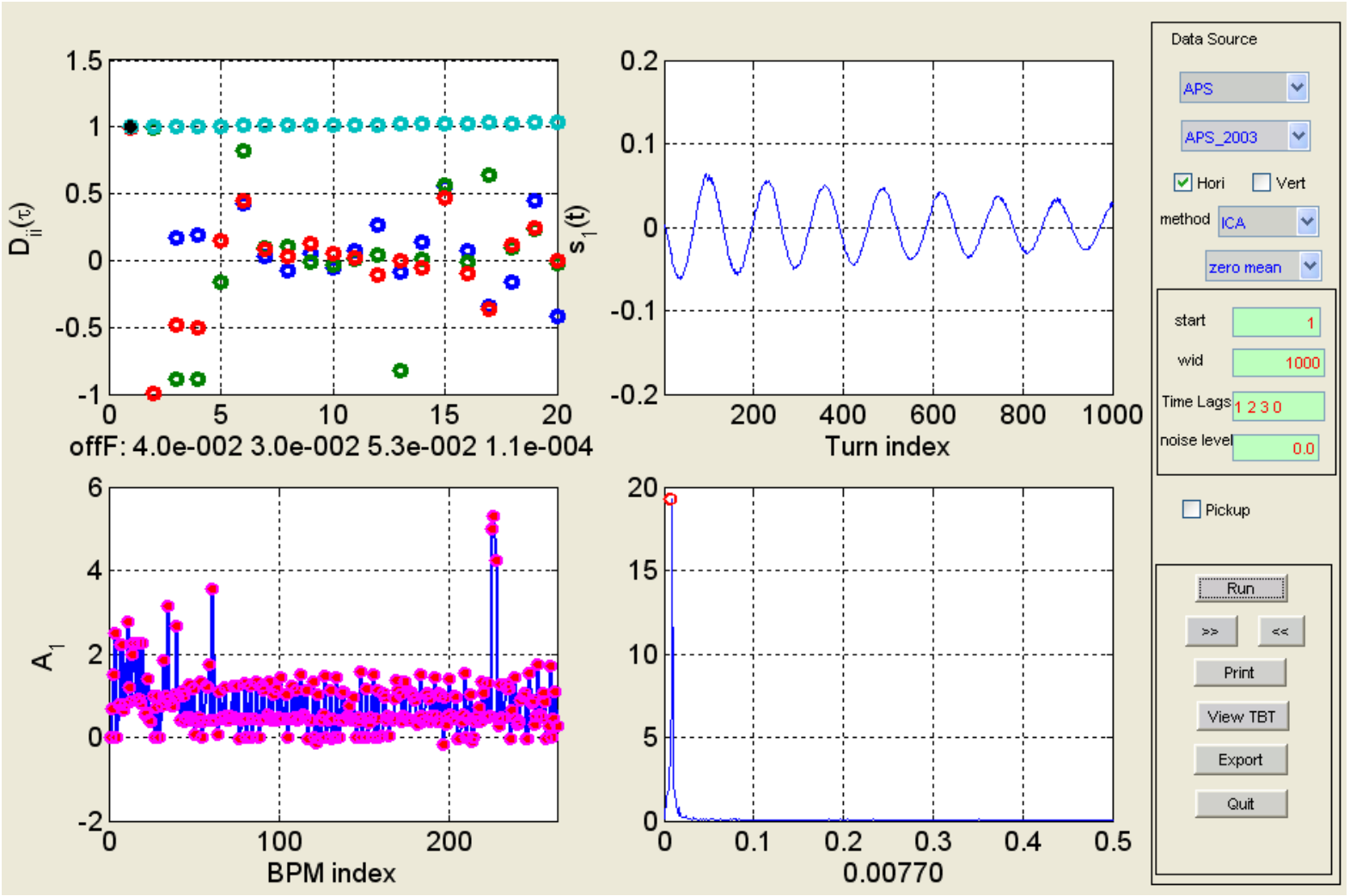


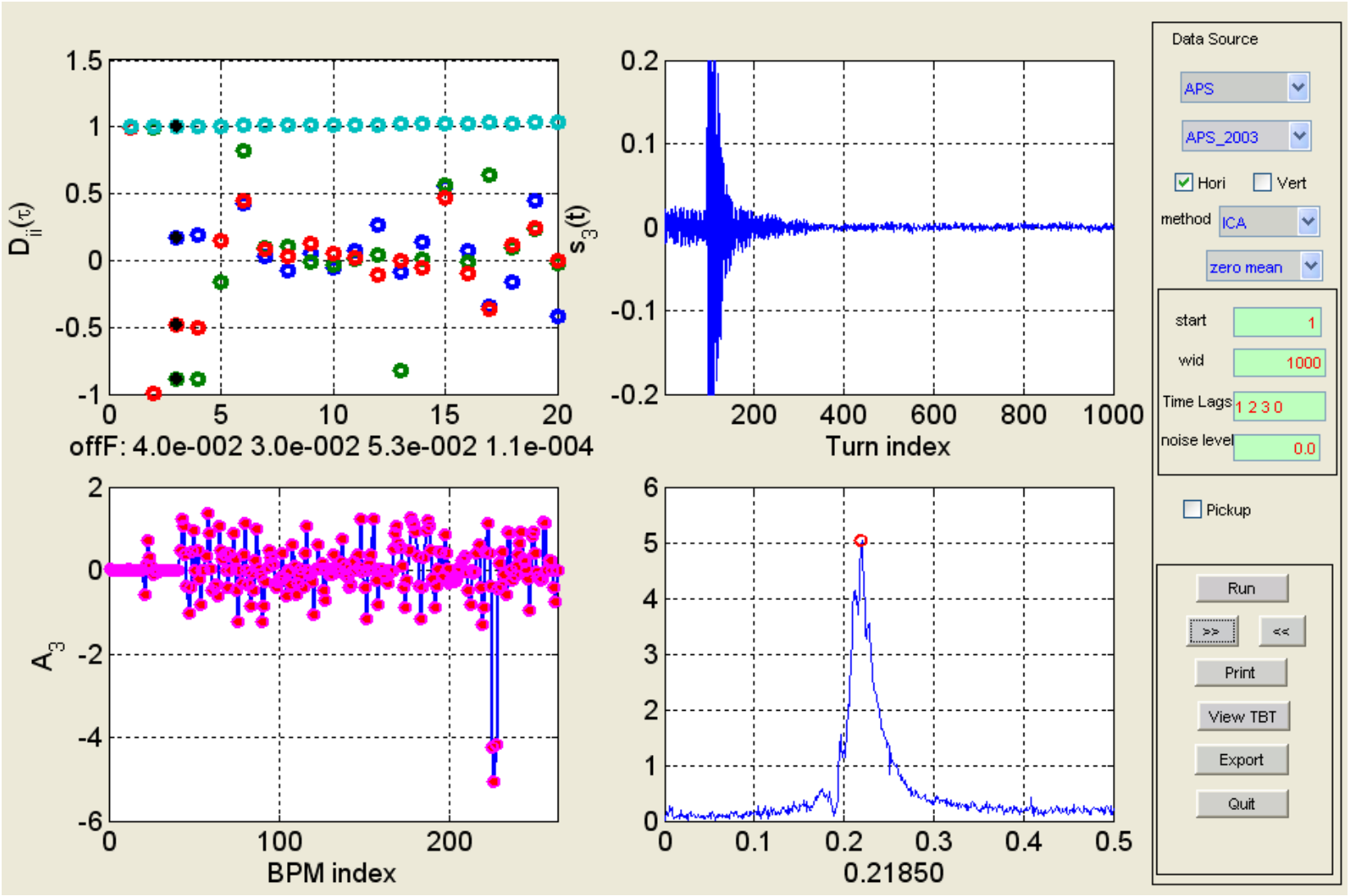






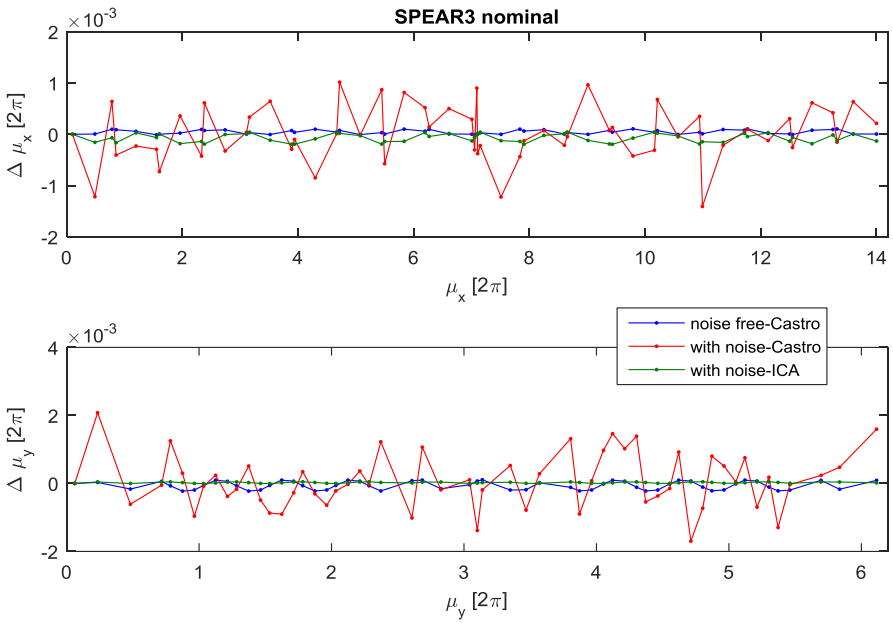
Application: APS data



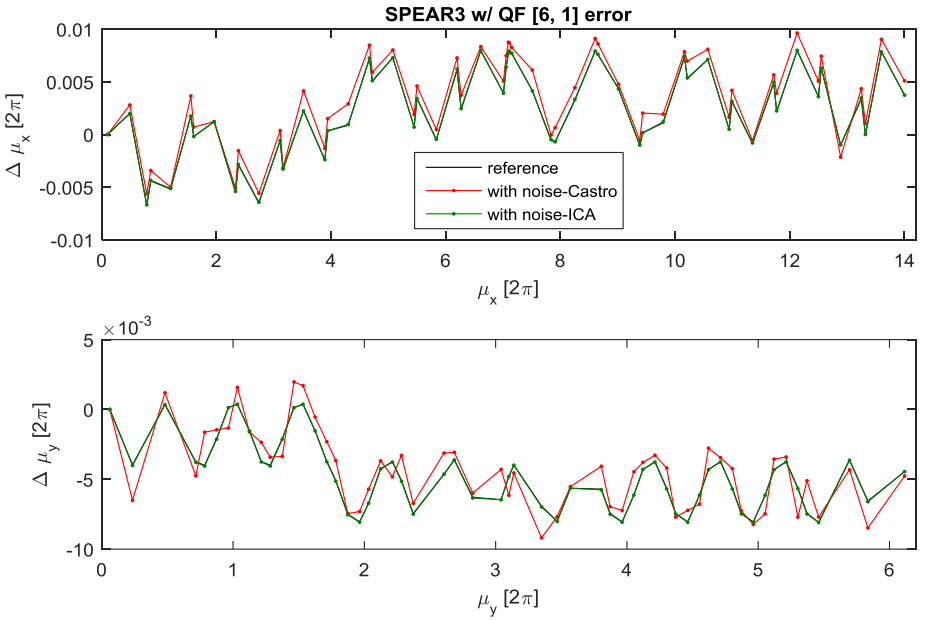

Data Source
APS
APS_2003
Hori
Vert
method
ICA
zero mean
start
1
wid
1000
Time Lags
1 2 3 0
noise level
0.0
Pickup
Run
>>
<<
Print
View TBT
Export
Quit
offF: 4.0e-002 3.0e-002 5.3e-002 1.1e-004
Turn index
BPM index
0.21850

Comparison of precision of phase measurements

Phase measurement with Castro's method and ICA from simulated data (SPEAR3)



Nominal SPEAR3 lattice



With introduced quadrupole error

Phase measurement with ICA has much higher precision than the single-BPM approach.

High precision phase measurement is beneficial to optics, coupling and nonlinear dynamics correction with TbT BPM data.

Optics and coupling correction with ICA results

TbT data with coupling with ICA mode separation

$$x_n = A\cos\Psi_1(n) - B\sin\Psi_1(n) + c\cos\Psi_2(n) - d\sin\Psi_2(n),$$
$$y_n = a\cos\Psi_1(n) - b\sin\Psi_1(n) + C\cos\Psi_2(n) - D\sin\Psi_2(n),$$

TbT BTPM data are related to normal mode coordinates through a transformation matrix P, with $X = P\Theta$

The P-matrix can be derived from the one-turn transfer matrix.

$$X = \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} \quad \Theta = \begin{pmatrix} \sqrt{2J_1}\cos\Phi_1 \\ -\sqrt{2J_1}\sin\Phi_1 \\ \sqrt{2J_2}\cos\Phi_2 \\ -\sqrt{2J_2}\sin\Phi_2 \end{pmatrix}$$

Therefore

$$x = p_{11}\sqrt{2J_1}\cos\Phi_1 + \sqrt{2J_2}(p_{13}\cos\Phi_2 - p_{14}\sin\Phi_2),$$
$$y = \sqrt{2J_1}(p_{31}\cos\Phi_1 - p_{32}\sin\Phi_1) + p_{33}\sqrt{2J_2}\cos\Phi_2,$$

where by definition

$$p_{12} = p_{34} = 0$$

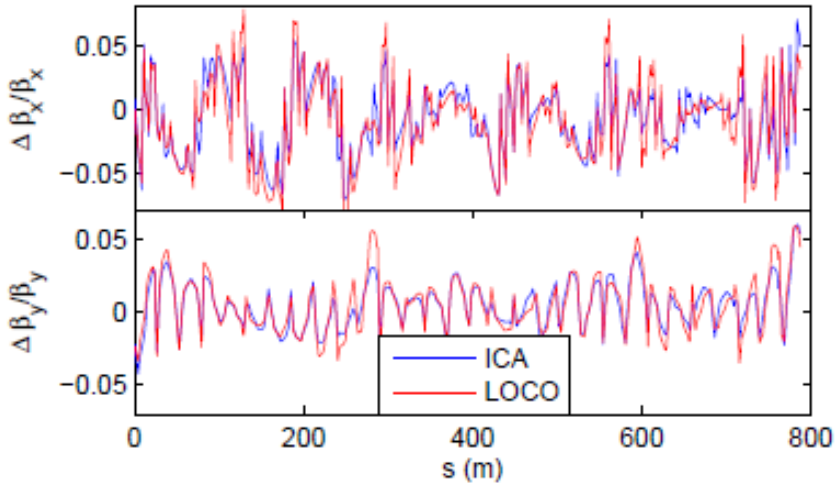
By comparison:

$$\sqrt{A^2+B^2} = 2J_1p_{11}, \quad \sqrt{c^2+d^2} = 2J_2\sqrt{p_{13}^2+p_{14}^2},$$
$$\sqrt{C^2+D^2} = 2J_2p_{33}, \quad \sqrt{a^2+b^2} = 2J_1\sqrt{p_{31}^2+p_{32}^2}.$$

$$\tan^{-1}\frac{B}{A} = \mathrm{Mod}_{2\pi}(\Phi_1),$$
$$\tan^{-1}\frac{d}{c} = \mathrm{Mod}_{2\pi}(\Phi_2 + \tan^{-1}\frac{p_{14}}{p_{13}}),$$
$$\tan^{-1}\frac{b}{a} = \mathrm{Mod}_{2\pi}(\Phi_1 + \tan^{-1}\frac{p_{32}}{p_{31}}),$$
$$\tan^{-1}\frac{D}{C} = \mathrm{Mod}_{2\pi}(\Phi_2).$$

Fitting the model with the ICA results

- The lattice model can be fitted to minimizing the differences between the amplitude and phase of the coupled modes.
  - Dispersion measurements are also included for fitting.
  - BPM gains and rolls can be fitted.



X. Huang, X. Yang, IPAC 2015

Comparison of beta beating from fitting results of LOCO and TbT data taken at the same time for NSLS-II ring.